Robust registration with IRLS and RANSAC

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General frameworks

- Register two sets of points
  - where correspondence is known exactly
    - eg barcode, etc. reference points
  - where correspondence is estimated, but quite well
    - eg two images, interest points
  - where correspondence might be hard to estimate
    - but registration is possible
    - eg two lidar images of about the same stuff

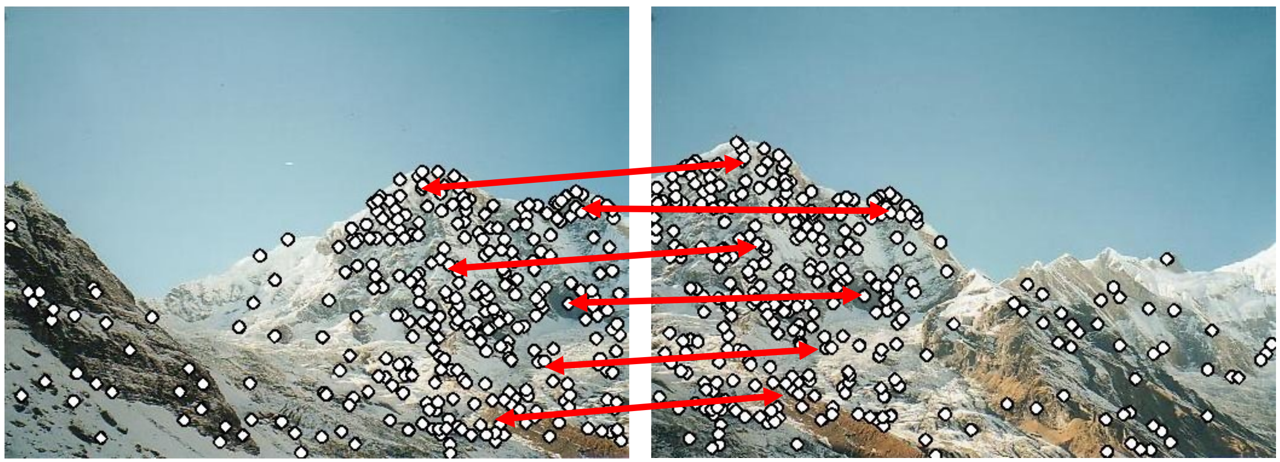
Application: image mosaics

- Find interest points in image A and image B

- Build correspondences:
  - For each a in A find best matching b in B using descriptor
  - For each b in B find best matching a in A using descriptor
  - For consistent pairs, if descriptors are sufficiently similar
    - declare correspondence

- Notice: you should get many correspondences BUT some are wrong

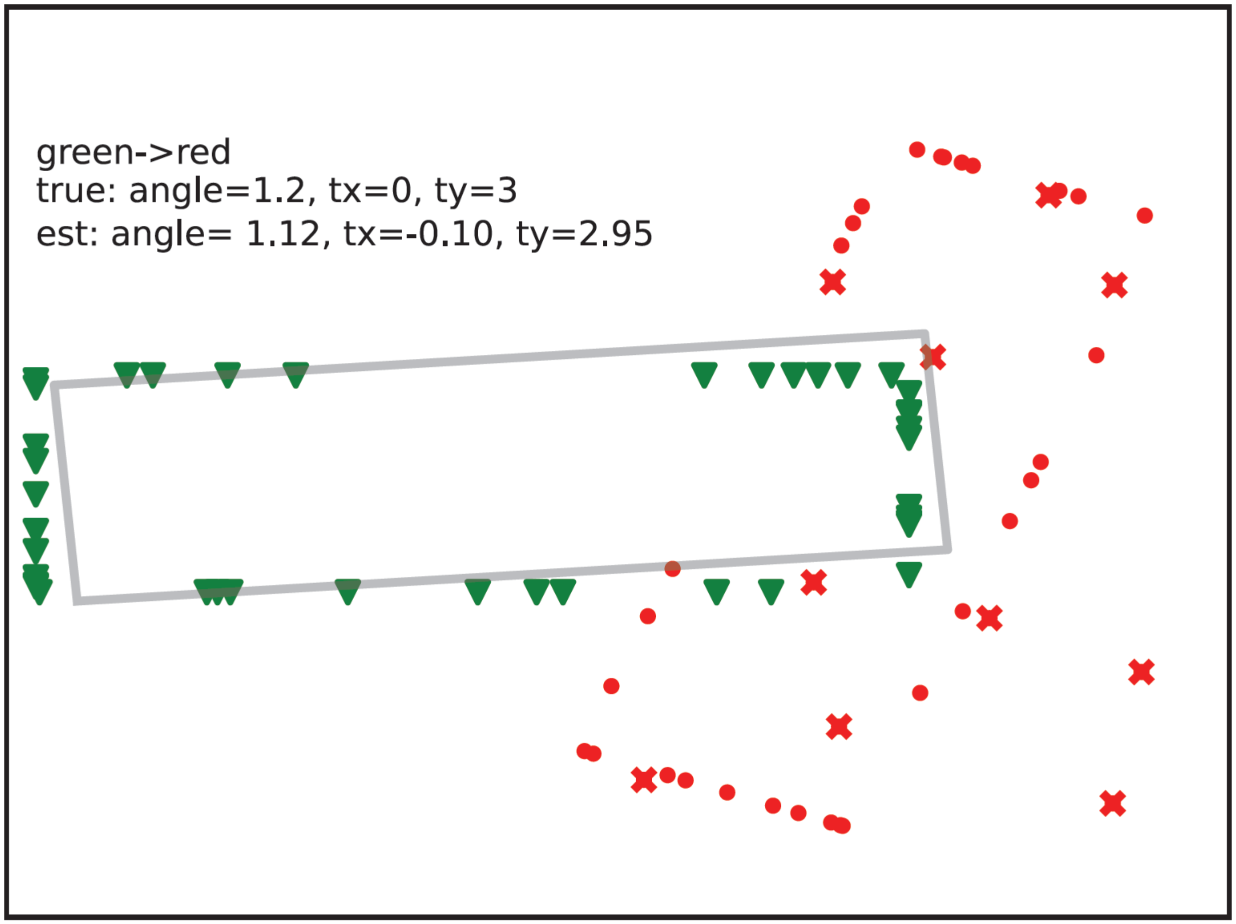
Recall...



General remarks

- Very like line fitting and line fitting recipes apply
- The objects we are working with are now corresponding pairs
  - (point in A, point in B)
- Outliers are usually correspondences that are wrong
  - there could be lots

Outliers affect least squares



- Green triangles – target pts lying on gray rectangle
- Red dots – source
- (target points transformed, then noise added)
- Red x – outliers on source
- Gray rectangle – transformation applied to true rectangle underlying red points
- Notice:
- transformation is disrupted by outliers

IRLS

- Start with initial transformation
  - get weights, scale from transformation

- Iterate:
  - estimate transformation using weights, scale
  - estimate scale using transformation
  - estimate weights using scale, transformation

- We *know* that one stationary point is the true minimum

- No other guarantees I’m aware of, but quite well behaved

IRLS applies

The IRLS recipe can be applied with very little modification to registration. Choose a robust cost function from Section 13.2.1 or elsewhere. Recall this cost applies to the residual. Write θ for the parameters of the transformation $\mathcal{T}_\theta$, and the residual is now

$$r(\mathbf{x}_i, \mathbf{y}_i, \theta) = \sqrt{(\mathbf{x}_i - \mathcal{T}_\theta(\mathbf{y}_i))^T(\mathbf{x}_i - \mathcal{T}_\theta(\mathbf{y}_i))}.$$

The square root ensures that minimizing the least squares criterion is equivalent to

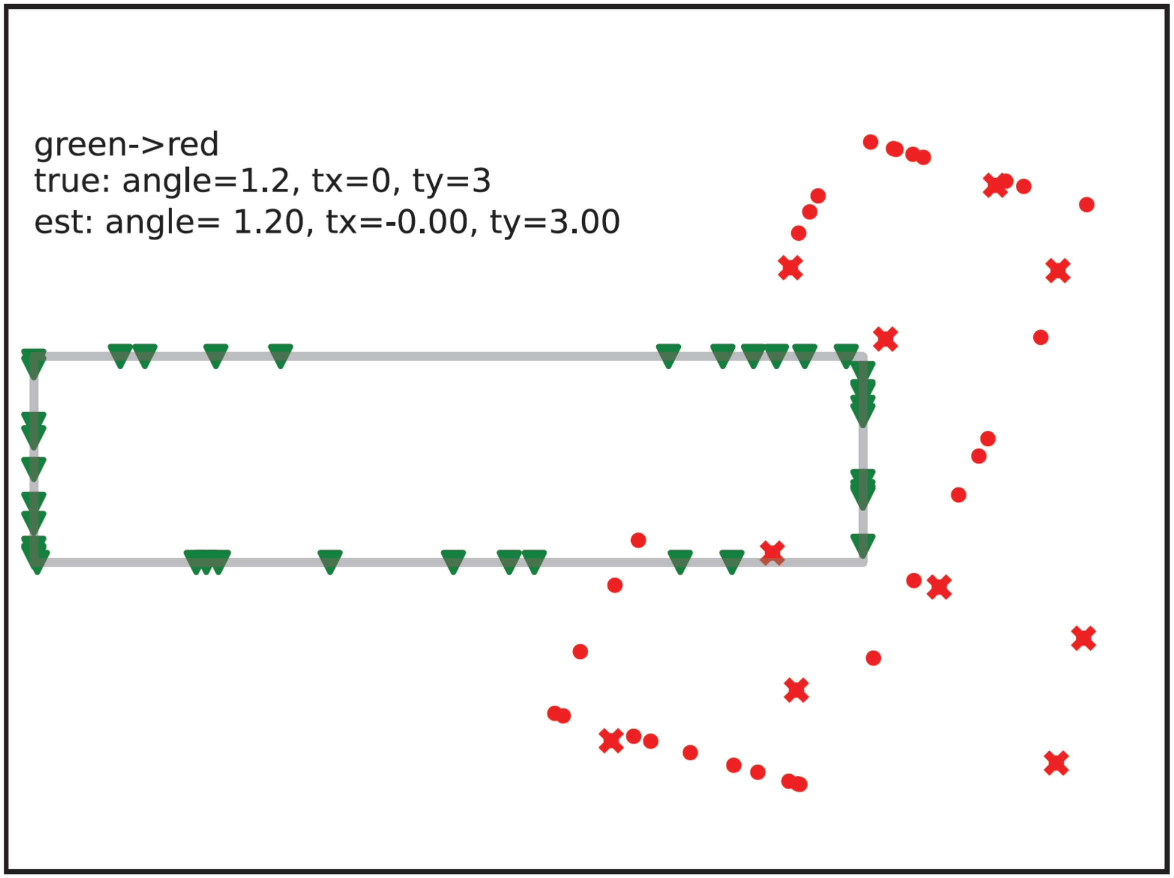
$$(1/2)\sum_i (r(\mathbf{x}_i, \mathbf{y}_i, \theta))^2.$$

For any given θ, the weights are now

$$w_i = \left(\frac{\frac{\partial \rho}{\partial u}}{r(\mathbf{x}_i, \mathbf{y}_i, \theta)} \right).$$

IRLS handles few outliers

IRLS, 5 outliers



- Green triangles – target pts lying on gray rectangle
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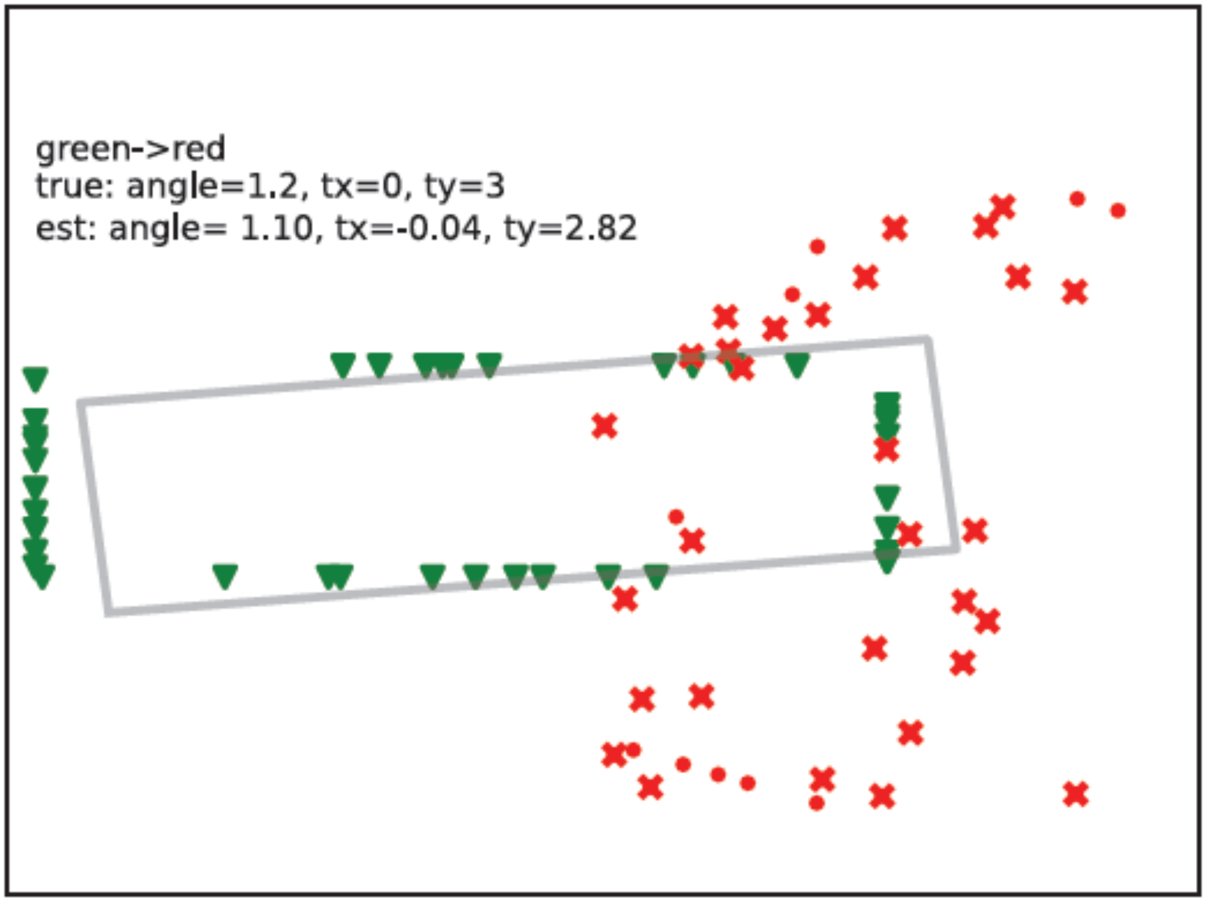
- Notice:
- transformation is NOT disrupted by outliers

IRLS can't do many outliers

IRLS, 30 outliers



- Green triangles – target pts lying on gray rectangle
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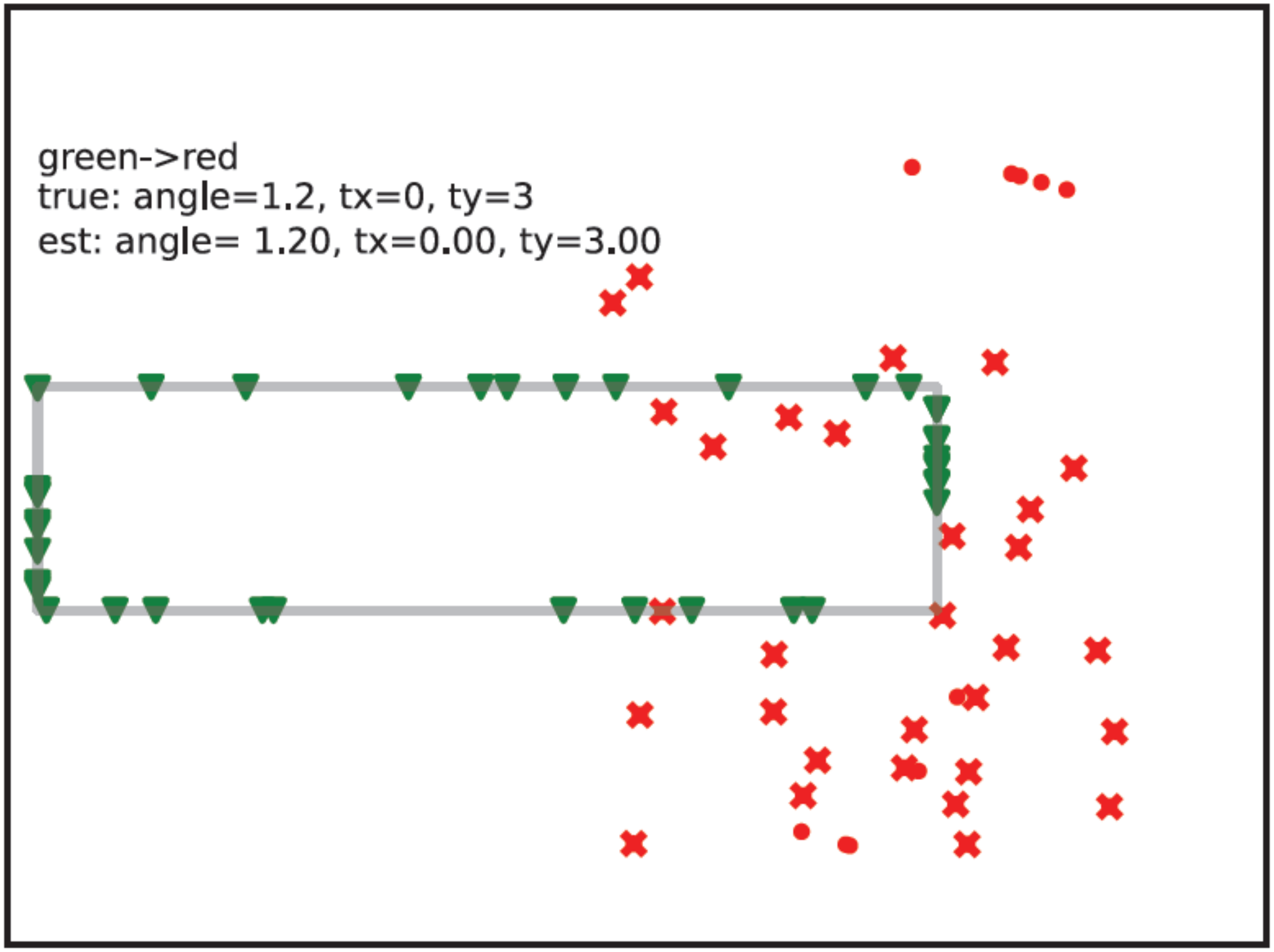
- Notice:
- transformation IS
- disrupted by outliers

RANSAC to the rescue



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- disrupted by outliers

RANSAC

- Affine transformation:
    - d+1 correspondences in d dim
- Projective transformation:
    - d+2 correspondences in d dim
- Euclidean:
    - use 2 for plane (2D)
    - use 3 for 3D

- BUT some such are obvious outliers

- Key Issue: there can be a lot of outliers

Think about this...

16.1. In the lead, I say: " Correspondences that are wrong tend to be badly wrong". Why is this the case?

16.2. Check that I have correctly mapped IRLS (Section 14.2.2) onto registration in Section 16.1.

16.3. Check that I have correctly mapped RANSAC (Section 14.3) onto registration in Section 16.1.2.

16.4. Show how an affine transformation in d dimensions is exactly specified by $d+1$ correspondences (start with $d = 1$).

16.5. Produce a set of two correspondences that can't be exactly registered with a Euclidean transformation in 2D.

16.6. Produce a set of three correspondences that can be exactly registered with a Euclidean transformation in 2D.

16.7. Show that $d + 2$ correspondences are enough to exactly specify a projective transformation in d dimensions.

16.8. Section 16.2 has: " This means that *in the best case* you will need to look at of the order of

$$\frac{1}{[\max(M, N)]^3}$$

samples to see one set of three good samples." Explain.