Back to abstract complexes.

K is a ~~set~~ collection of sets

- a complex if, for $\sigma \in K$,

<u>all</u> $\sigma' \subset \sigma$ have $\sigma' \in K$.

Example constructions:

Delaunay complex.

- point set $P, \in \mathbb{R}^d$
- A k-simplex σ is Delaunay if.
  - its verts are in P
  - there is an open d-ball whose boundary contains verts, but ball contains no other pts of P.
- a Delaunay complex of P is a simplicial complex each of whose

simplices are Delaunay.

- Every non degen (no $d+2$ pts co-spherical) admits a unique Delaunay complex (dual to Voronoi diagram, and MUCH too hard to make).

- <u>Rips complex</u>:
  - P a point set
  - σ a k simplex is

    $$\{p_1, \dots p_{k+1}\}$$

    and is in R^r if

    $$d(p_i, p_j) \leq r \quad \text{for every } p_i \,\&\, p_j \in \sigma$$

~~Witness complex~~:

Rips complex can be big:

$\Omega(n^d)$ ← dimension ← 4? for cvxnet, 7 boxes

↳ # of points

↳ 100's – 1000s for our cases.

Witness complex:

we have L a point set and W ~~a subset of P~~ dense sample of points

eg: dense point cloud.

W = point cloud.

L = small random subset

- Landmarks.

σ an abstract simplex, with vertices in L.

then $w \in W$ is a <u>witness</u> for σ

if $\|w-p\| \leq \|w-q\|$

$\forall p \in \sigma, \quad \forall q \in L-\sigma$

The <u>witness complex</u> is the complex consisting of all simplexes σ st. for any $\tau \subseteq \sigma$, τ has a witness in W

<u>Features</u>

- relatively small and easy to make.

Graph induced complex

- P a point set, G a graph with verts in P.

- $Q \subseteq P$
- $\nu : P \to Q$ $\quad\quad$ $\nu(p) = \operatorname{argmin} d(p, Q)$

  (ie find closest Q to p)

- Graph induced complex contains σ

  $\sigma = \{q_1 \cdot\cdot\, q_{k+1}\}$ iff there is

  a $(k+1)$ clique $p_1 \cdot\cdot\, p_{k+1} \subseteq P$ st

  $\nu(p_i) = q_i$ for each i

G/C can also be constructed in a relatively straightforward way. (Dey notes)

~~Now, how is all this good for us?~~

CVx Net color complex.

have K convexes, r planes in each.

- total of K bags.

$\sigma = \{b_1 \ldots b_{K+1}\}$ is in if there exists some x st

$\min_{w \in b_i} d(w, x) \leq \varepsilon$ for all i

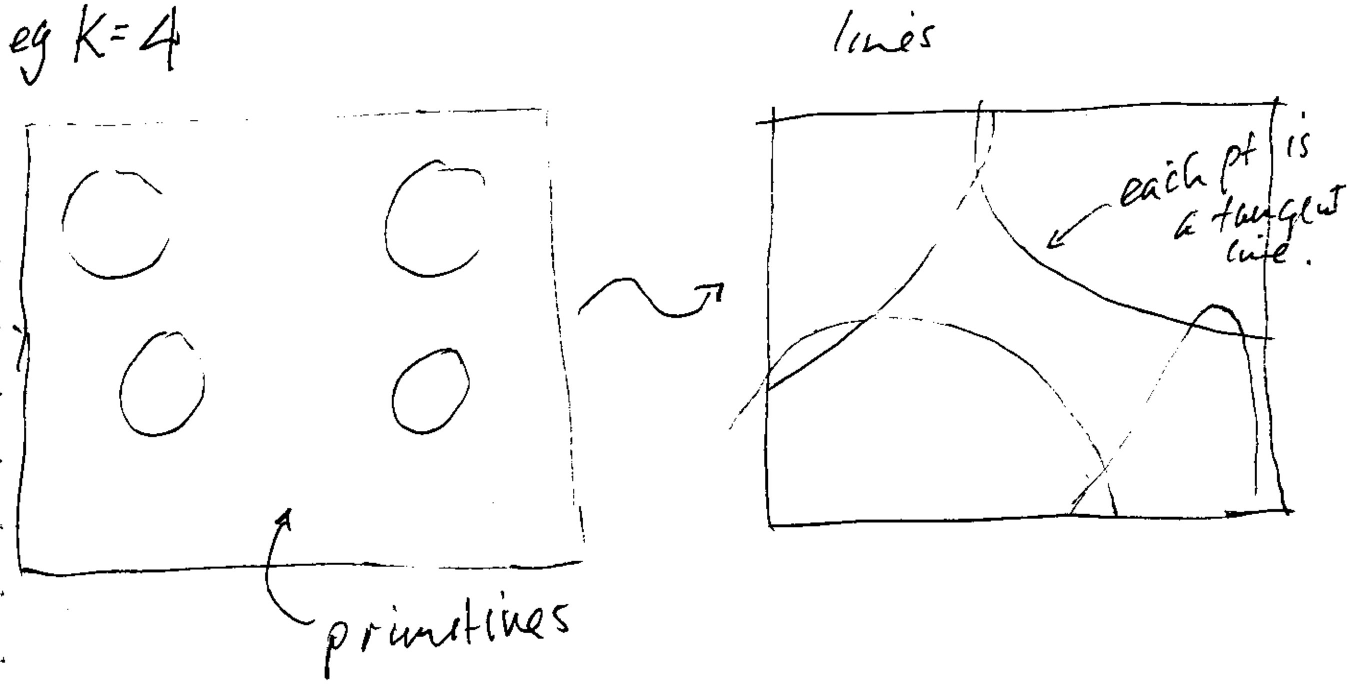
is this a complex?

Yes: if σ is in, so are $\tau \subseteq \sigma$.

What is it like?:

- Consider convex primitives, points in R^2 (easier to draw).

- eg $K=4$

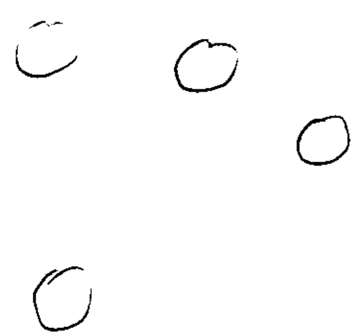


K° =  (Why? - every pair of convexes has bitangent lines)

now increase distance

 — this is filled in if these 3 are "roughly straight"

eg



Notice B_1 has changed — why?

What do we do with this?

- We have permutation invariant constructions for point sets.
- We <u>could</u> infer "topology"
  - → this is persistent homology, barcodes, etc.
- Instead, we could use to construct <u>features</u>
  - the pointnet construction is purely discriminative
  - <u>alternative</u> - compute B_i for a range of complexes
    - <u>What features predict?</u>