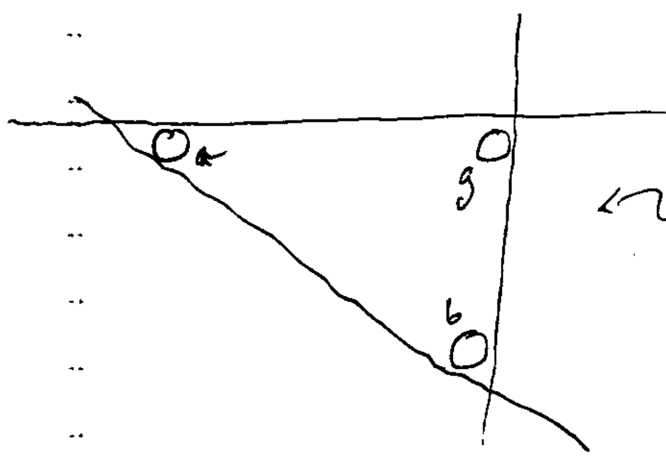
Taking cvx's into account:

- we have K cvx's, each has r faces.
- each face is a "point"
- particularly interesting cases

  I - faces that are tangent (ish) to many convexes

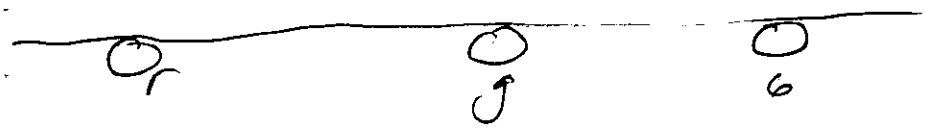
  II - faces that are "far" from other faces

examples of I, 2D



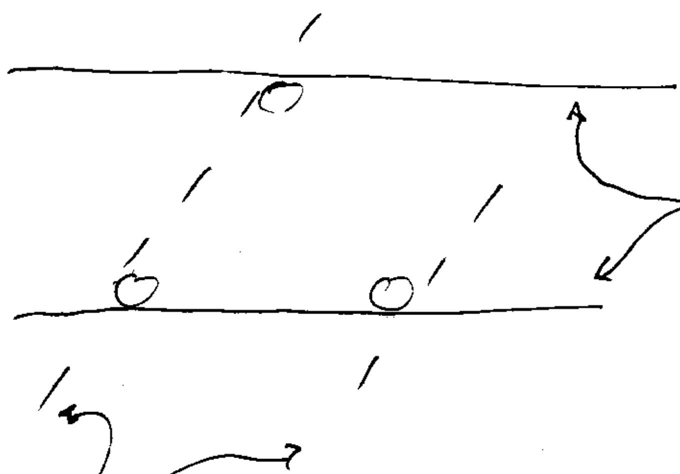
o - are the convexes

/ - are bitangent planes



notice that for the first geometry, no face tangent to 3 ; second, there is one. This difference is worth knowing about.

<u>examples of II</u>

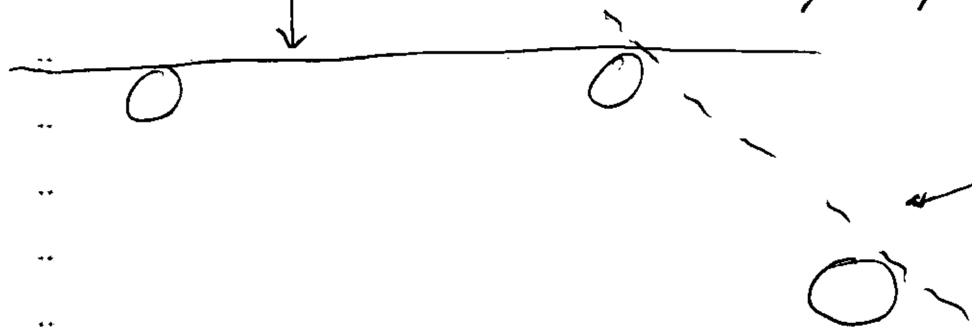


This pair of planes is interesting

as is this

we need to extend the graph construction

there are multiple pairs like this



<u>there is one example construction</u>

- Think of each convex as a "bag" of points (= planes)
- each convex has a different color
- choose a distance d.

- represent the cvx's with a collection of sets.

$$K_\alpha = \{ \{c_{a_1}, \ldots c_{a_r}\}, \{c_{b_1} \ldots c_{b_s}\}, \ldots \}$$

where $S = \{c_1, c_2 \ldots c_r\}$ is in K_α

iff $\min_{p_a \in c_i, p_b \in c_j} d(p_a, p_b) \leq \alpha.$ for all $i, j \in S$.

(equivalently, $c_1 \ldots c_r$ are in S if they "almost" share a plane.

Q: what is the resulting object?

Q: how to represent it?

Abstract simplicial complexes

A collection $\mathcal{A}$ of subsets of a set A is an abstract simplicial complex if every $\sigma \in \mathcal{A}$ has all subsets $\sigma' \subseteq \sigma$ $\in \mathcal{A}$. The elements of A are the vertices of $\mathcal{A}$.

Each set in $\mathcal{A}$ is a simplex whose dimension is its cardinality.

eg: $A = \{ r, g, b, c, m, y \}$

$$\mathcal{A} = \{ \{r\}, \{r, g\}, \{g, b\}, \{r, g, b\}, \{g\}, \{b\} \}$$

"vertex" edge face vertex

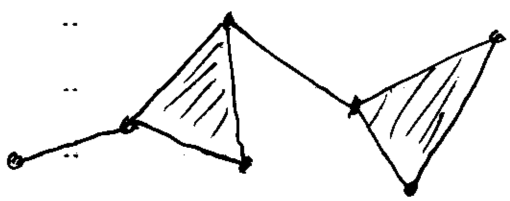
$\{ \{r\}, \{g\}, \{r,g,b\} \}$ $\longrightarrow$ <u>NO</u>

Analogy with <u>simplicial complex</u>

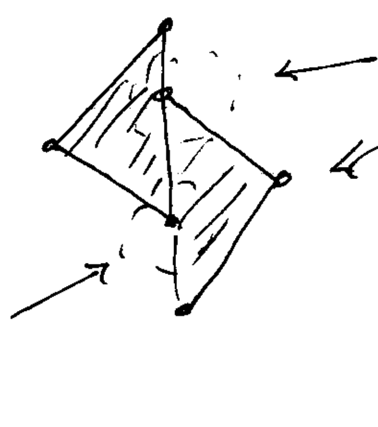
- k simplex is hull ($k+1$ affine indep points).
- 0 simplex $\equiv$ vertex
- 1 simplex $\equiv$ edge
- 2 simplex $\equiv$ triangle.

- a <u>face</u> of a simplex is conv hull of a non-empty subset of pts
- a <u>simplicial complex</u> K is a finite set of simplices such that

- K contains every face of every simplex of K
- for $\sigma, \tau \in K$, either $\sigma \cap \tau = \emptyset$ or $\sigma \cap \tau$ is a face of σ and of τ



← simplicial complex



← NO

the graph construction (above) was
a family of simplicial complexes
→ assume vertices go in only
when connected to an edge.

<u>then</u>:

$$\phi = K^0 \subset K^1 \quad \cdots \quad K^\alpha$$

no edges

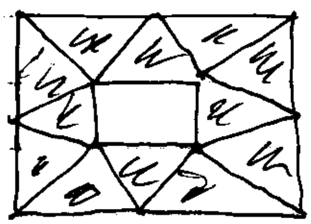
increase dist a bit, insert one edge

etc.

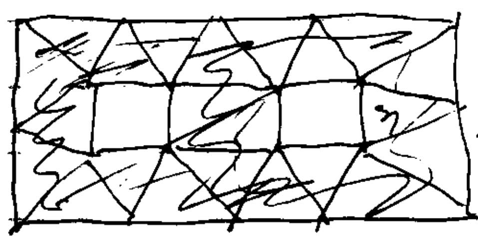
Q: how can we describe such a complex?

~~Simple~~ + Topological rep'ns.

cf. graphs.



This complex has a cycle that is not a boundary



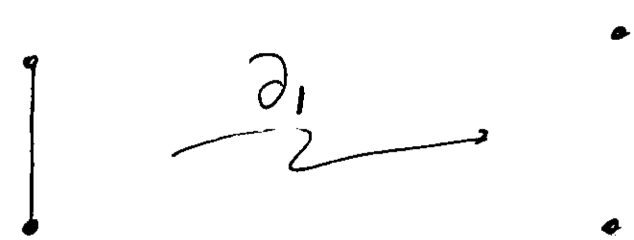
this has at least two

3D pix, too

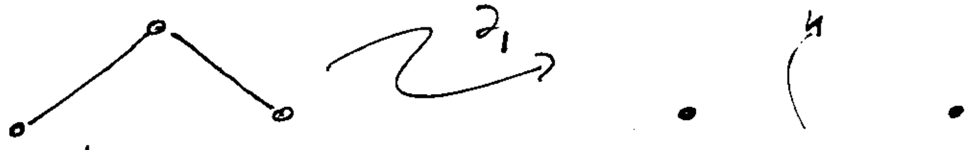
- an n-chain is a weighted sum of n-simplexes. We will use mod 2 weights (but others are possible).

- The boundary of a K-simplex is the sum of all $K-1$ faces.
  [need to adjust this for weights not mod 2]

Examples:

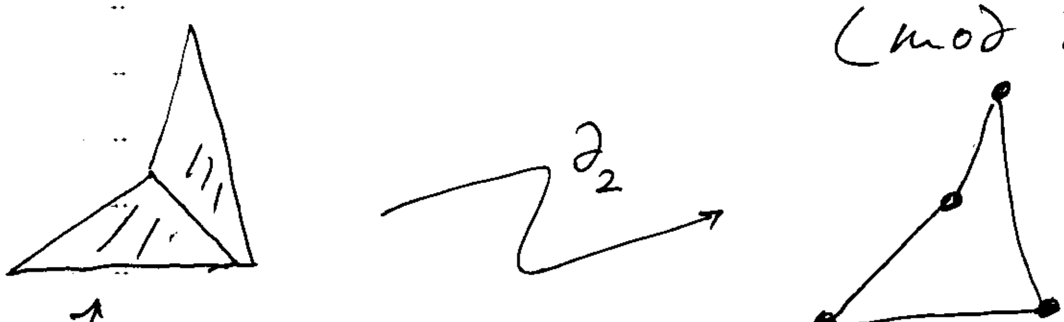


The boundary operator is <u>linear</u>
(mod 2 helps here!)



1-chain

this one disappears,
(mod 2!)



2-chain

<u>Notice that</u>

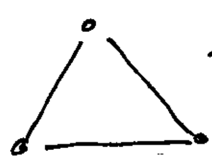
∂_K takes K-chains to $(K-1)$ chains

<u>AND</u> $\partial_{K-1} \circ \partial_K = 0$

(The boundary of a boundary is empty)

<u>CHECK THIS:</u>

Now consider:



This has $\partial_1 = 0$

BUT it isn't itself a boundary.

We <u>Define</u>

$$H_K = \left(\text{space of } K\text{-chains } \sigma \text{ s.t. } \partial_K(\sigma) = 0\right) \Big/ \text{Image}\left(\partial_{K+1}\right)$$

This is Kth simplicial Homology group (over $\mathbb{Z}/2\mathbb{Z}$ - mod 2).

$$B_K = \dim(H_K) = \# \text{ of linearly indep generators}$$

↑ K'th Betti number

We have already seen B_0, B_1 in action.

<u>Examples in Figures</u>

~~There a~~

<u>Repn of cvx net objects</u>

- compute Abstract Simplicial complex as above
- compute $B_0 \cdots B_K$ for various dists
- construct barcode.