- Some constructions for point sets
- assume we have N points in R^d; we wish to build rep'ns that capture geometry, and are perm invariant.

- Construct a family of graphs

$G^0 = \{\text{point set}, \emptyset\}$

$G^1 = \{\text{"}, \text{1 edge between closest}\}$

$\vdots$

$G^{N^2} = \{\text{"}, \text{all edges.}\}$

These are undirected

notice $E^0 \subset E^1 \subset E^2 \subset \cdots E^{N^2}$

and $E^1 = E^0 \cup \{1 \text{ edge}\}$ etc.

<u>Describing these graphs</u>

two natural topological descriptors

1: $|C| = \#$ of connected components $= \cancel{\#}\, B_0(G)$

↑ 0th Betti number of G.

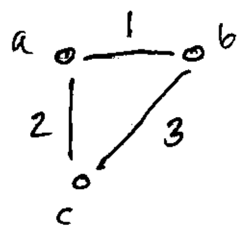
2: $|E| - |V| + |C| = \#$ of linearly indep cycles

$= B_1(G)$

↑ 1st Betti number of G.

→ linearly indep. cycles.

→ count mod 2, and add/subtract edges, vertices



$1+2+3$ is a cycle

<u>because</u>

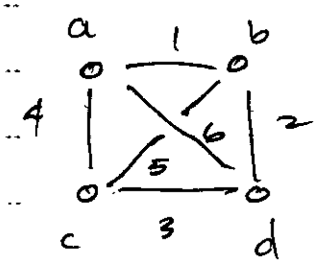
$$\text{boundary}(1+2+3) = 0$$

$$= a + b + b + c + c + a$$

$$= 2a + 2b + 2c$$

$0 \pmod 2$ etc

eg:



<u>cycles</u>: 126 ; 145 ; 436 ; 235

but notice

$$\underbrace{126 + 145 + 436}_{\downarrow} = 235$$

$1+1 \to \phi$, $6+6 \to 0$, $4+4 \to 0$

there are 3 linearly indep cycles.

a) The number of linearly indep cycles in a connected graph (counting mod 2) is $|E| - |V| + 1$

[$\longrightarrow$ Subtract a spanning tree
count remaining edges]

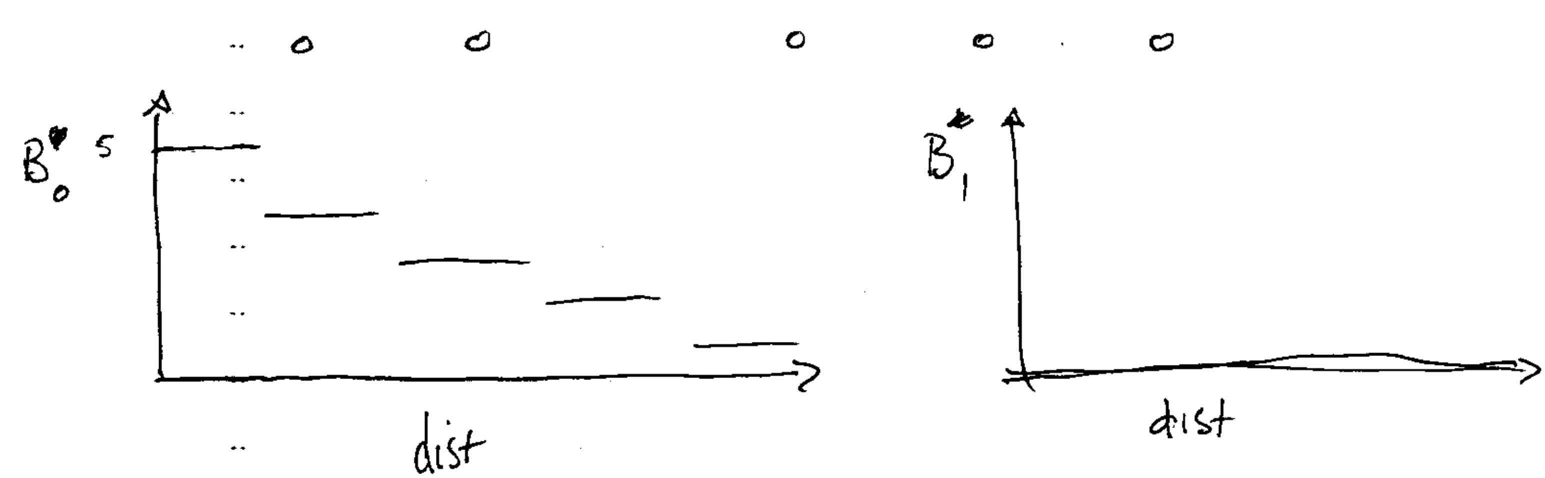
6) " graph w/ $|C|$ connected components is

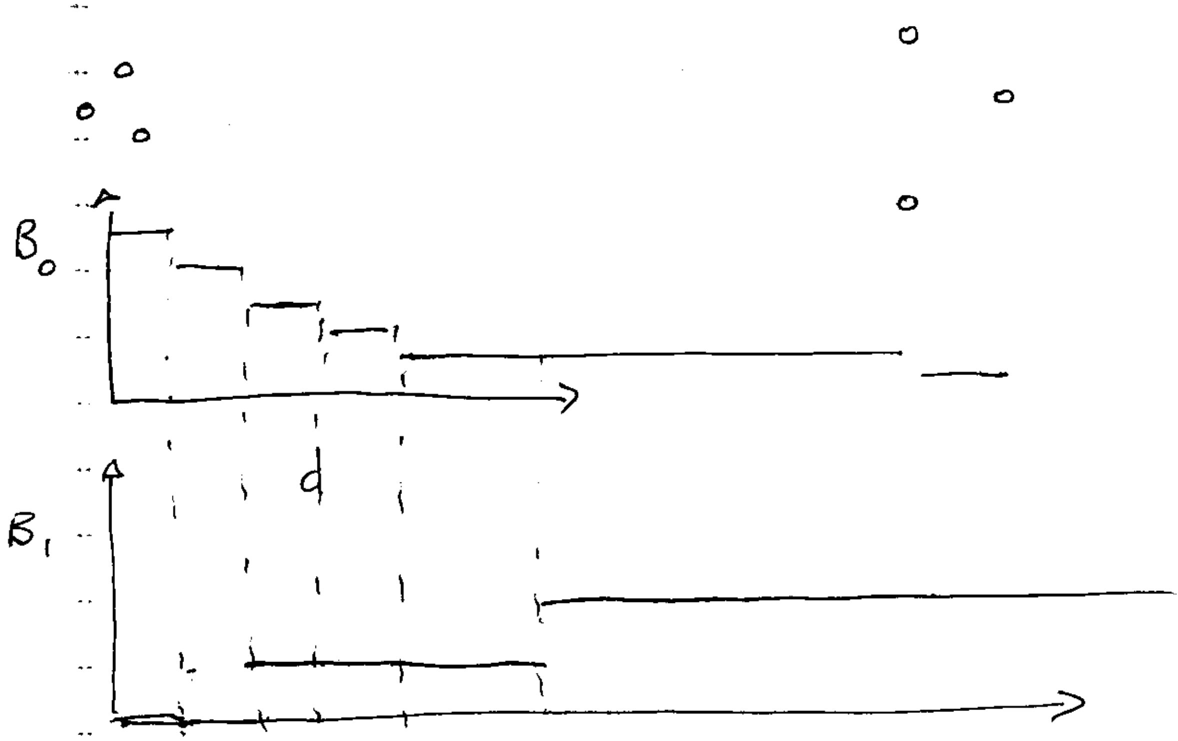
$$|E| - |V| + |C|$$

– as above, check. !

- These two numbers have interesting behavior which captures some geometry

- They are perm invariant





- Some rules.

  – add 1 edge.

  <u>EITHER</u>

  - $|C|$ goes down by 1
  - $|E|$ goes up AND $|C|$ does not go down

<u>So</u>.

EITHER

B_0 goes down by 1

OR

B_1 goes up by 1.

(This is worth remembering - helps build repn.)

Q: - can we get a good summary repn out of this

eg:

$$\left.\begin{array}{l} \displaystyle\int_{d_0}^{d_1} B_0(s)\, ds \\ \\ \displaystyle\int_{d_0}^{d_1} B_1(s)\, ds \end{array}\right\} ?$$

Barcodes (later)

Extend this construction in a variety of ways.

- to primitives

a) each prim is a param object
⇒ each prim is a point in appropriate param space
⇒ need a distance

Box:

axis aligned

$(x_{bl}, y_{bl}, z_{bl}, x_{tr}, y_{tr}, z_{tr}) = p$

$dist(p_1, p_2) = \|p_1 - p_2\|$ ← this is a metric, we're fine

non-a-a

(p, R)

p ↑ for a-a

R ← Rotation

$$dist[box1, box2] = \|p_1 - p_2\| + \|I - R_1 R_2^T\|_F$$

Frobenius norm $\|M\|_F = \sqrt{\sum_{i,j} m_{ij}^2}$

<u>Consider CVXnet repn's</u>

<u>rep</u>: is union of k convexes.

each convex is r planes

$$\underbrace{M}_{r \times 3} x \geq \underbrace{c}_{r \times 1}$$

<u>notice</u> $\lambda M x \geq \lambda c$ also works for $\lambda > 0$ so

<u>Choose</u> M to have rows that are unit vectors.

~~each~~ $M = \begin{bmatrix} m_1^T \\ m_2^T \\ m_3^T \\ \vdots \end{bmatrix} \qquad c = \begin{matrix} c_1 \\ c_2 \\ \vdots \end{matrix}$

~~Con~~ We could think of each plane as a point. $p = (m, c)$

$$d(P_1, P_2) = \|c_1 - c_2\| + \|m_1 - m_2\|$$

OK, as long as they're normalized

<u>OR</u>

$$d(P_1, P_2) = \|c_1 - c_2\| + \cos^{-1}[m_1^T m_2]$$

<u>But</u> we can distinguish between the planes for different convexs

- we have K different point clouds.
- there are 2 types of perm to worry about

- perm "points" inside cvx
- perm convexes.

~~Can compose co~~

- Compose pointnet construction.
  - for each cvx, compute.

$$V_{ci} = \begin{bmatrix} \max\left[\phi_1(p_{i1}) \cdots \phi_1(p_{ir})\right] \\ \max\left[\phi_2(p_{i1}) \cdots \phi_2(p_{ir})\right] \\ \\ \text{etc} \\ \\ \end{bmatrix}$$

then

$$f_c = \begin{bmatrix} \max\left[\psi_1(V_1), \psi_1(V_2) \cdots \psi_1(V_K)\right] \\ \vdots \\ \max\left[\psi_s(V_1) \cdots \psi_s(V_K)\right] \end{bmatrix}$$

<u>alternative</u>:

- regard the K cvxs as
  $K \times r$ points, w metric
  as above, construct B_0, B_1
- <u>Notice</u>: - this ignores important information
  (which plane belongs to which cvx)