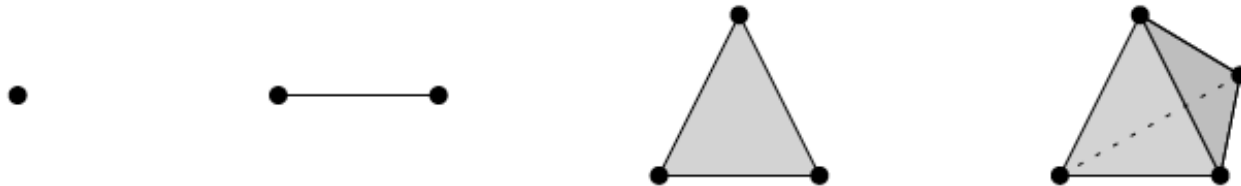


# Figures, etc. for Simplicial Homology

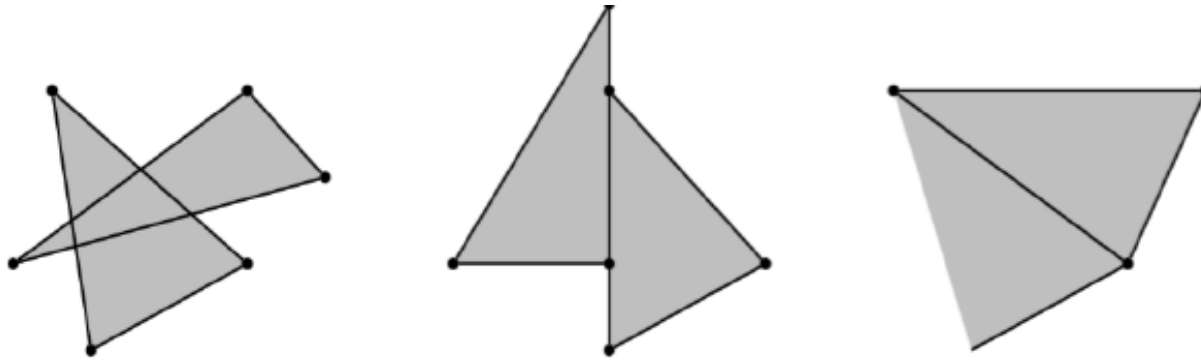
D.A. Forsyth, UIUC

# Simplex

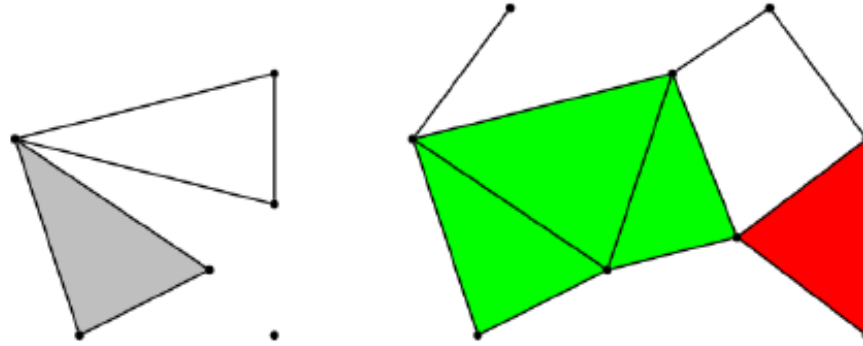
**Definition 1.5 (Simplex).** A  $k$ -simplex is the convex hull of  $k + 1$  affinely independent points,  $\sigma = \text{conv}\{u_0, u_1, \dots, u_k\}$ . We sometimes say the  $u_i$  span  $\sigma$ . Its dimension is  $\dim \sigma = k$ . We use special names for the first few dimensions, *vertex* for 0-simplex, *edge* for 1-simplex, *triangle* for 2-simplex, and *tetrahedron* for 3-simplex.

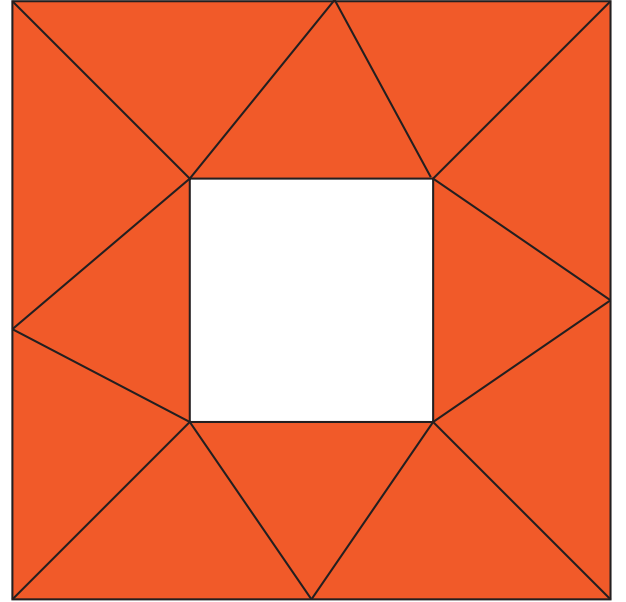
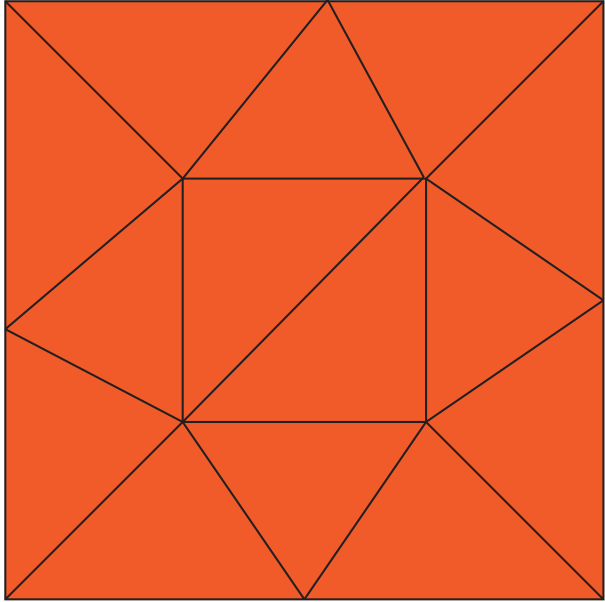


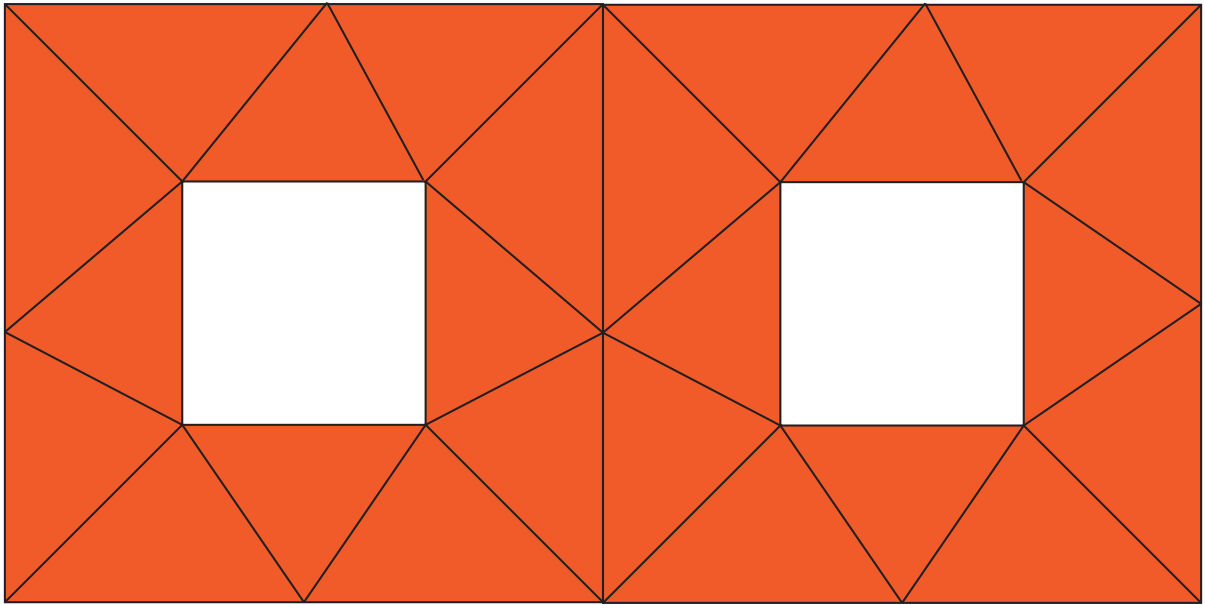
# NOT Simplicial complexes

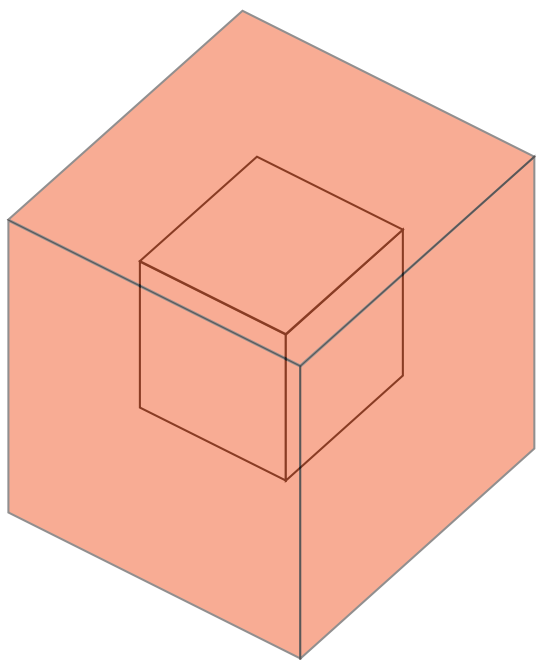


# Simplicial complexes

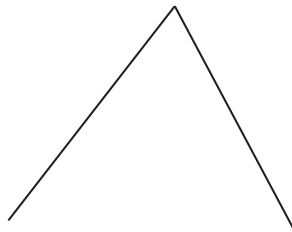
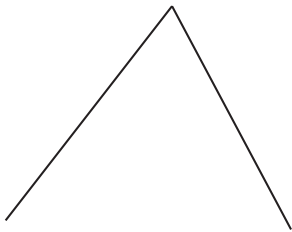
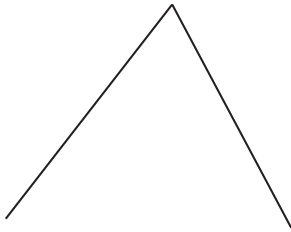
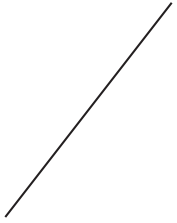




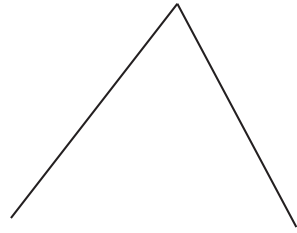




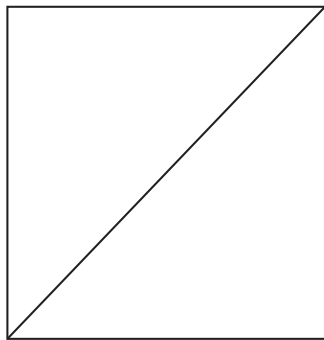
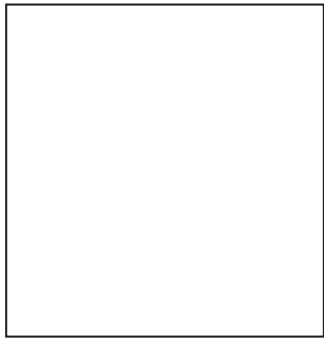
Compute  $B_0$



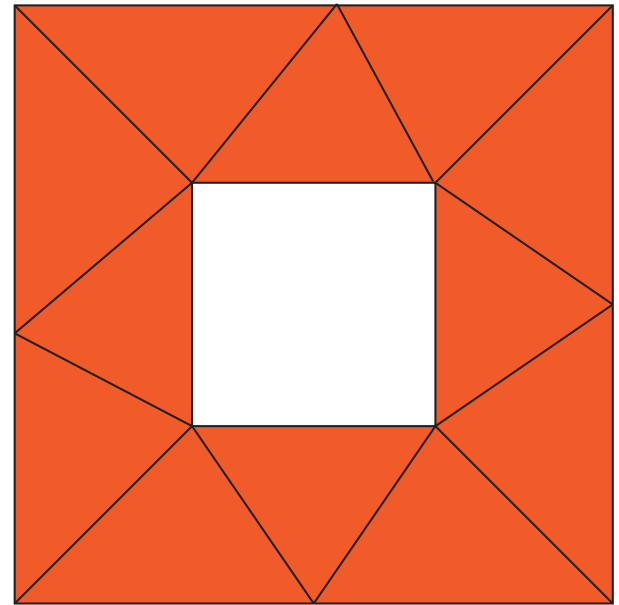
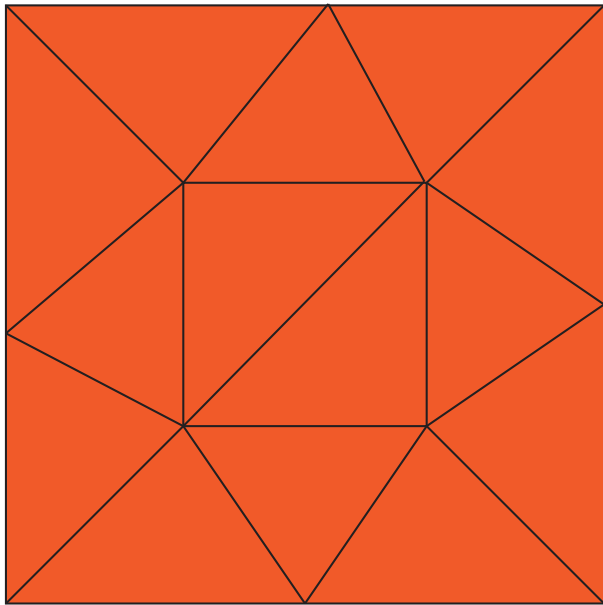




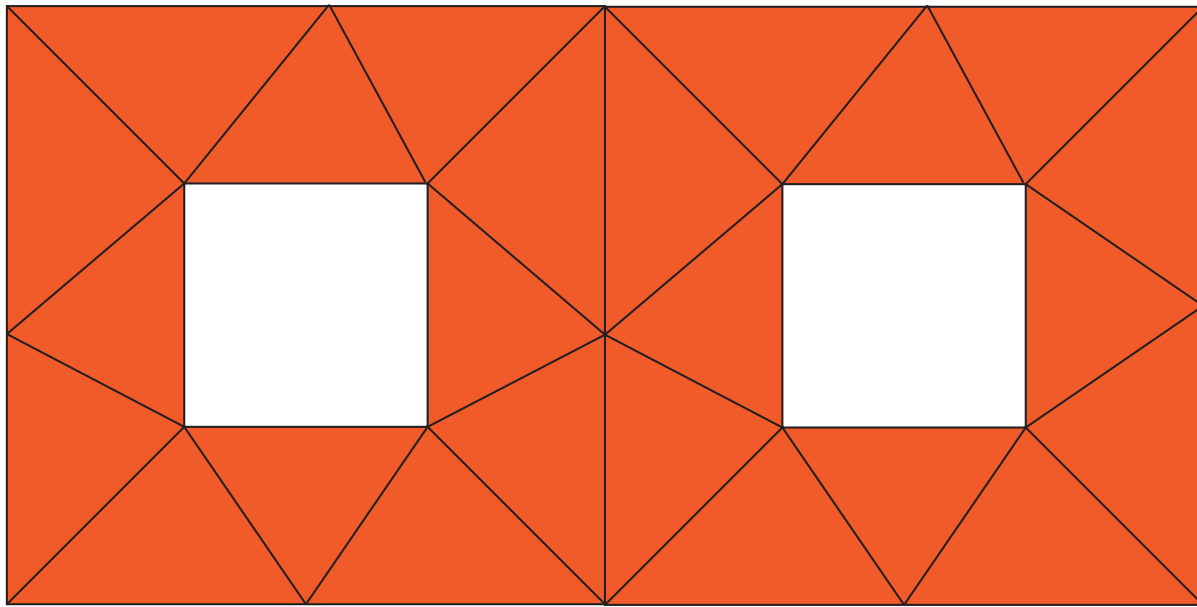
Compute B1



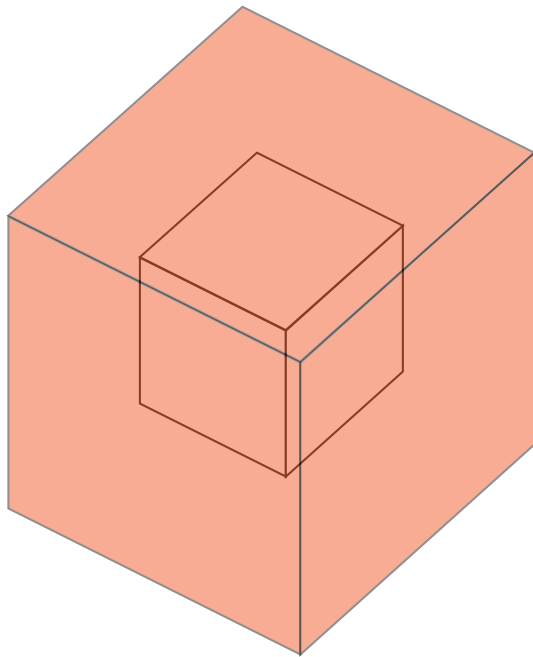
Compute  $B_0, B_1, B_2$



Compute  $B_0, B_1, B_2$



Compute  $B_0, B_1, B_2$



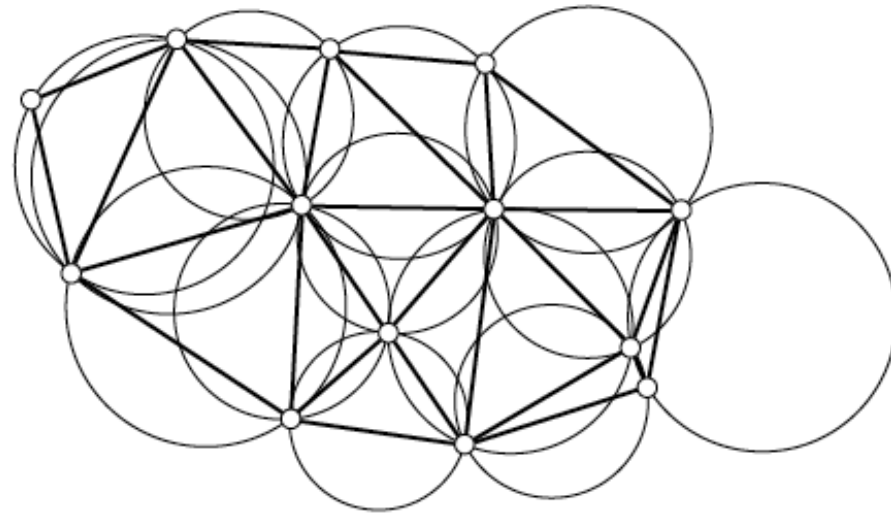


Figure 8: Every triangle in a Delaunay triangulation has an empty open circumdisk.

Dey, qv

# Witness Complex

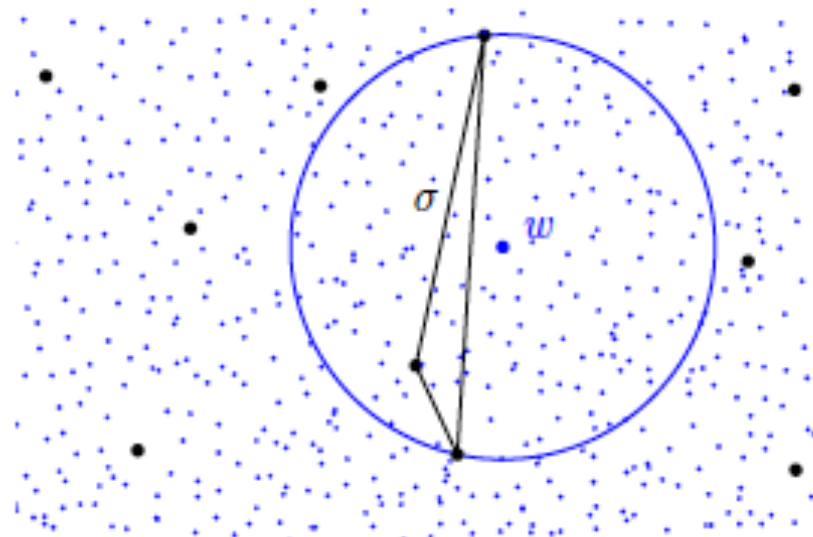
[de Silva]

$L$  a finite set of points (**landmarks**)

vertices of the complex

$W$  a dense sample (**witnesses**)

pseudo circumcenters



Let  $\sigma$  be a (abstract) simplex with vertices in  $L$ , and let  $w \in W$ . We say that  $w$  is a **witness** of  $\sigma$  if

$$\|w - p\| \leq \|w - q\| \quad \forall p \in \sigma \text{ and } \forall q \in L \setminus \sigma$$

The **witness complex**  $\text{Wit}(L, W)$  is the complex consisting of all simplexes  $\sigma$  such that for any simplex  $\tau \subseteq \sigma$ ,  $\tau$  has a witness in  $W$

# Construction of witness complexes

Time-complexity :  $O((|WC| + |W|) d^2 \log |L|)$  [B., Maria]

Algebraic complexity : comparisons of (squared) distances : degree 2

Implementation and experimental results : see the Gudhi library !

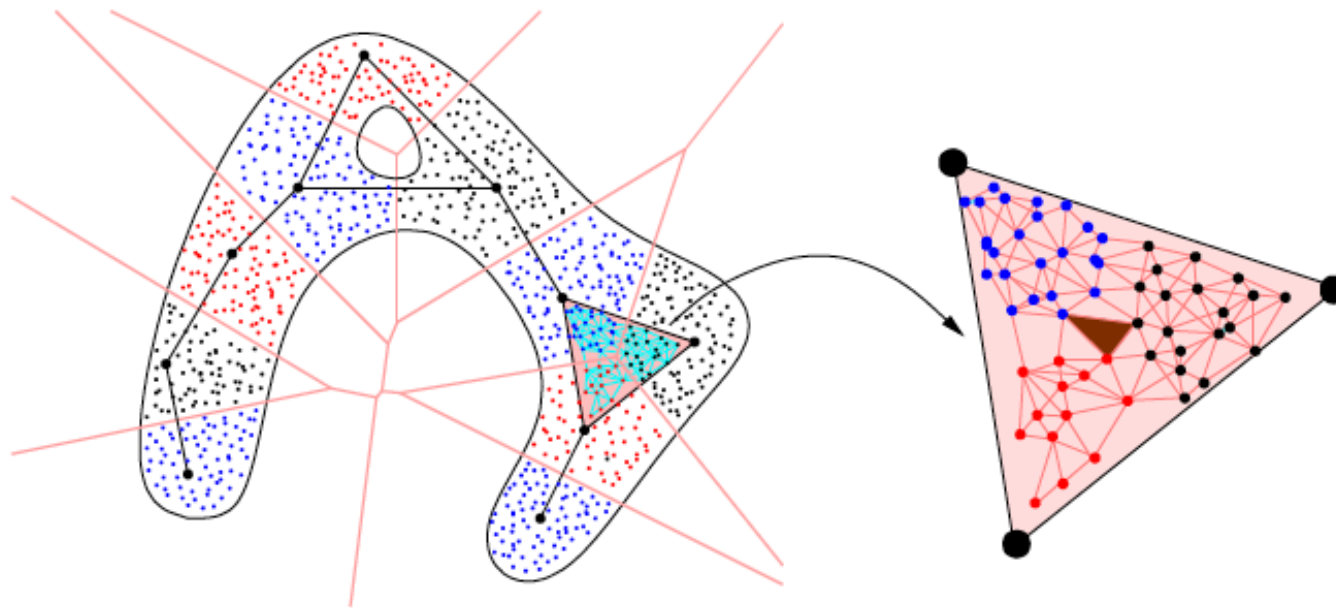


Figure 10: A graph induced complex shown with bold vertices, edges, and a shaded triangle on left. The input graph within the shaded triangle is shown on right. The three differently colored vertices of the input graph (shown inside the shaded triangle on right) cause the shaded triangle to be in the graph induced complex.

Dey, qv