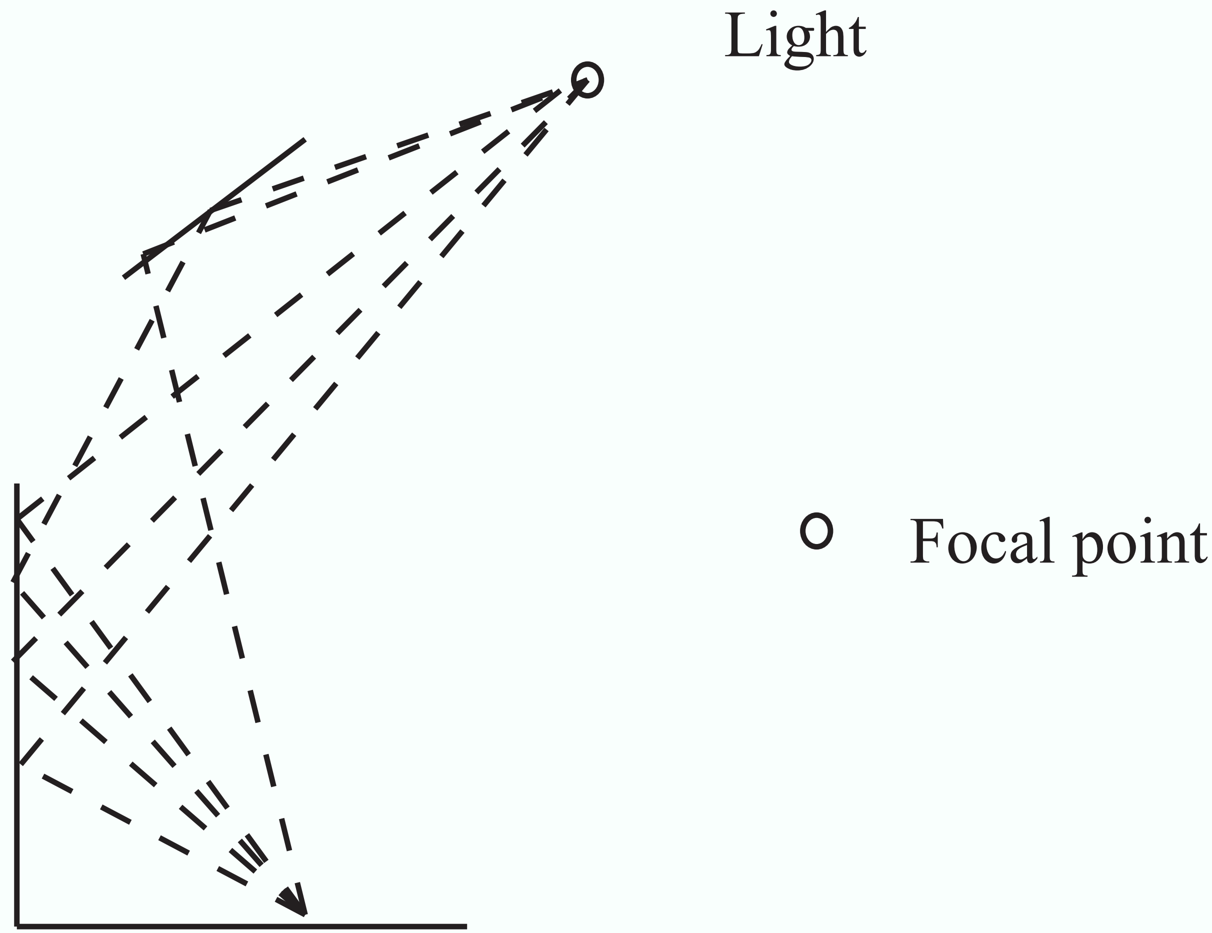
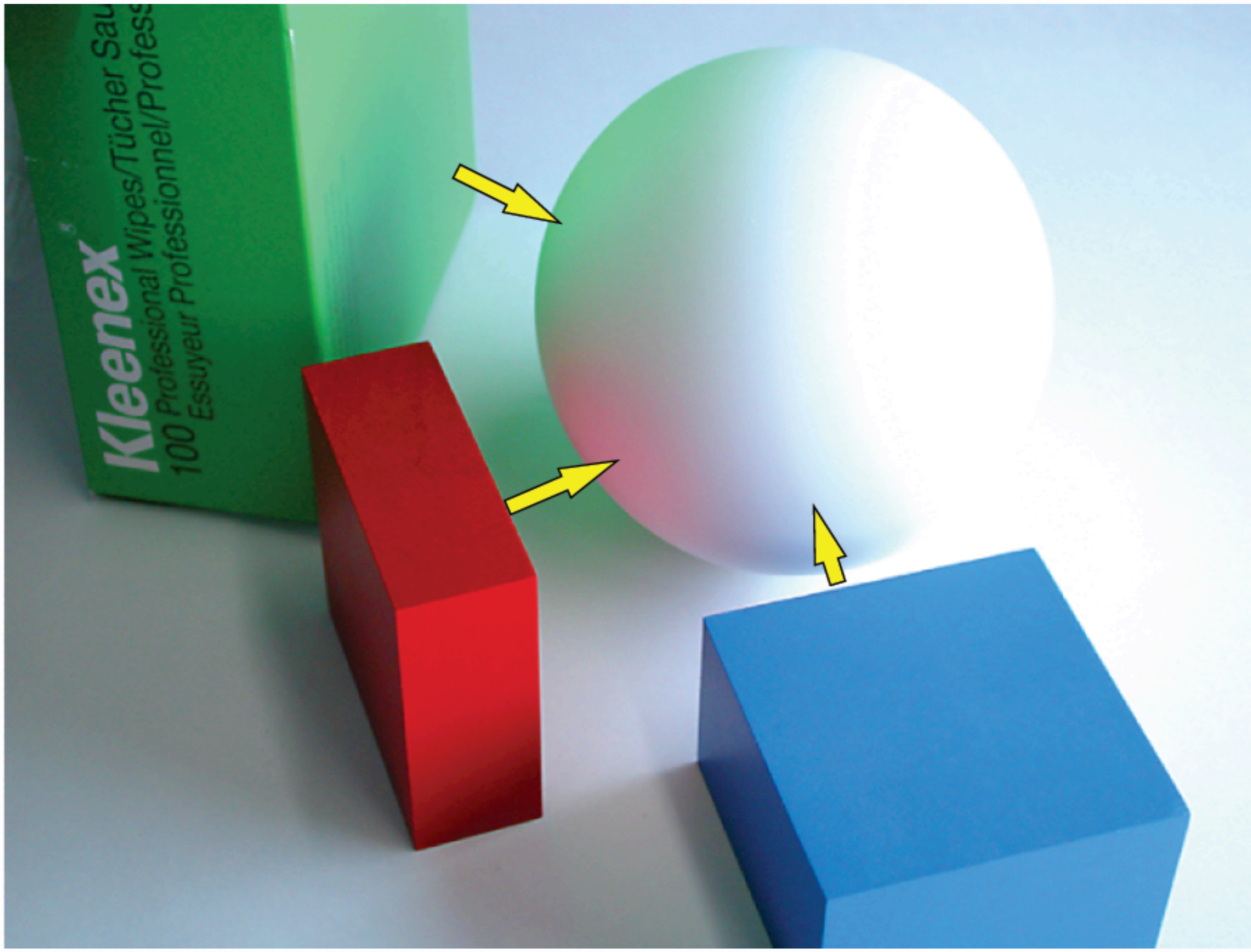
Rendering diffuse interreflections

D.A. Forsyth
with slides from John Hart

Diffuse-diffuse transfer



Interreflections are significant



From Koenderink slides on image texture and the flow of light

Radiosity and diffuse interreflections

- Assume we’re in a world of diffuse surfaces
- Rendering
  - cast eye rays
  - evaluate radiosity at first hit
  - average, stick into pixel
- Not practical --- we don’t know radiosity
- Model

Interreflection model

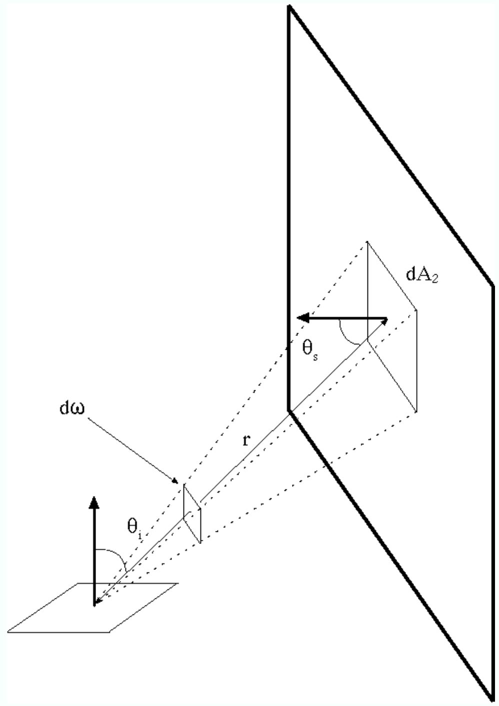
Integral over all incoming directions

$$B(x) = E(x) + \int_{\Omega} \left\{ \begin{array}{l} \text{radiosity due to} \\ \text{incoming radiance} \end{array} \right\} d\omega$$

For the moment, read this as incoming light

All diffuse surfaces are area sources!

- Receiver can't tell whether light is created or reflected at source
- If receiver is luminaire (makes light), that just adds
- Diffuse interreflection equation
  - vis(x, u)=1 if they can see each other, 0 otherwise
  - Notice nasty property
    - B (unknown) is inside the integral!
    - Fredholm equation of the second kind



$$B(\mathbf{x}) = E(\mathbf{x}) + \rho(\mathbf{x}) \int_S \frac{\cos\theta_i \cos\theta_s}{\pi r^2} Vis(\mathbf{x}, \mathbf{u}) B(\mathbf{u}) dA_s$$

Evaluating the radiosity

- cast eye rays
- evaluate radiosity at first hit
- average, stick into pixel

- Not practical --- we don't know radiosity
- But

$$B(\mathbf{x}) = E(\mathbf{x}) + \rho(\mathbf{x}) \int_S \frac{\cos\theta_i \cos\theta_s}{\pi r^2} Vis(\mathbf{x}, \mathbf{u}) B(\mathbf{u}) dA_s$$

This is an average over all light
coming from somewhere else; this average smoothes

Rewrite model

$$B(x) = E(x) + \int_{\Omega} \begin{Bmatrix} \text{radiosity due to} \\ \text{incoming radiance} \end{Bmatrix} d\omega$$

$$\mathrm{B(x)=E(x)}+\rho(x) \int_{\mathrm{D}} \begin{Bmatrix} \text{power due to} \\ \text{radiosity at u} \end{Bmatrix} du$$

Here D is every point that can be seen from x

$$\mathrm{B(x)=E(x)}+\rho(x) \int_{\mathrm{D}} \{\text{power arriving due to } B(u)\}\, du$$

Rewrite model

$$\mathrm{B(x)}=\mathrm{E(x)}+\rho(x)\int_{\mathrm{D}}\{\text{power arriving due to } B(u)\}\,du$$

We know an expression for this

$$\mathrm{B(x)}=\mathrm{E(x)}+\rho(x)\int_{\mathrm{D}_x}\left\{\text{power due to }\left[E(u)+\rho(u)\int_{\mathrm{D}_u}\{\text{power due to } B(v)dv\}\right]\right\}du$$

Here D_x is every point that can be seen from x,
D_u is every point that can be seen from u

Notation

- We know form of “Power arriving due to B(u)”
  - but it’s tediously long
  - rewrite

From our work on area sources

↓

$$\rho(\mathbf{x}) \int_S \frac{\cos\theta_i \cos\theta_s}{\pi r^2} Vis(\mathbf{x}, \mathbf{u}) B(\mathbf{u}) d\mathbf{u}_s = \rho(\mathbf{x}) \int_S K(\mathbf{x}, \mathbf{u}) B(\mathbf{u}) d\mathbf{u}_s$$

Notation

- Think of
  - functions as very long vectors
  - K(x, u) as a matrix
  - write

$$\rho(\mathbf{x}) \int_S K(\mathbf{x}, \mathbf{u}) B(\mathbf{u}) d\mathbf{u}_s = \rho \mathcal{K} B$$

Core idea: Neumann series

- We have

$$B(\mathbf{x}) = E(\mathbf{x}) + \rho(\mathbf{x}) \int_S \frac{\cos\theta_i \cos\theta_s}{\pi r^2} Vis(\mathbf{x}, \mathbf{u}) B(\mathbf{u}) dA$$

- Can write:

$$B = E + \rho \mathcal{K} B$$

- Which gives

$$B = E + (\rho\mathcal{K})E + (\rho\mathcal{K})(\rho\mathcal{K})E + (\rho\mathcal{K})^3 E + \dots$$

Exitance

Source term

One bounce

Two bounces

The terms

$$B = E + (\rho\mathcal{K})E + (\rho\mathcal{K})(\rho\mathcal{K})E + (\rho\mathcal{K})^3 E + \ldots$$

Exitance

Source term

One bounce

Two bounces

mostly zero

Can change fast - shadows, etc.

Changes much more slowly, because K smoothes

Changes even more slowly, because K smoothes

Using an estimate

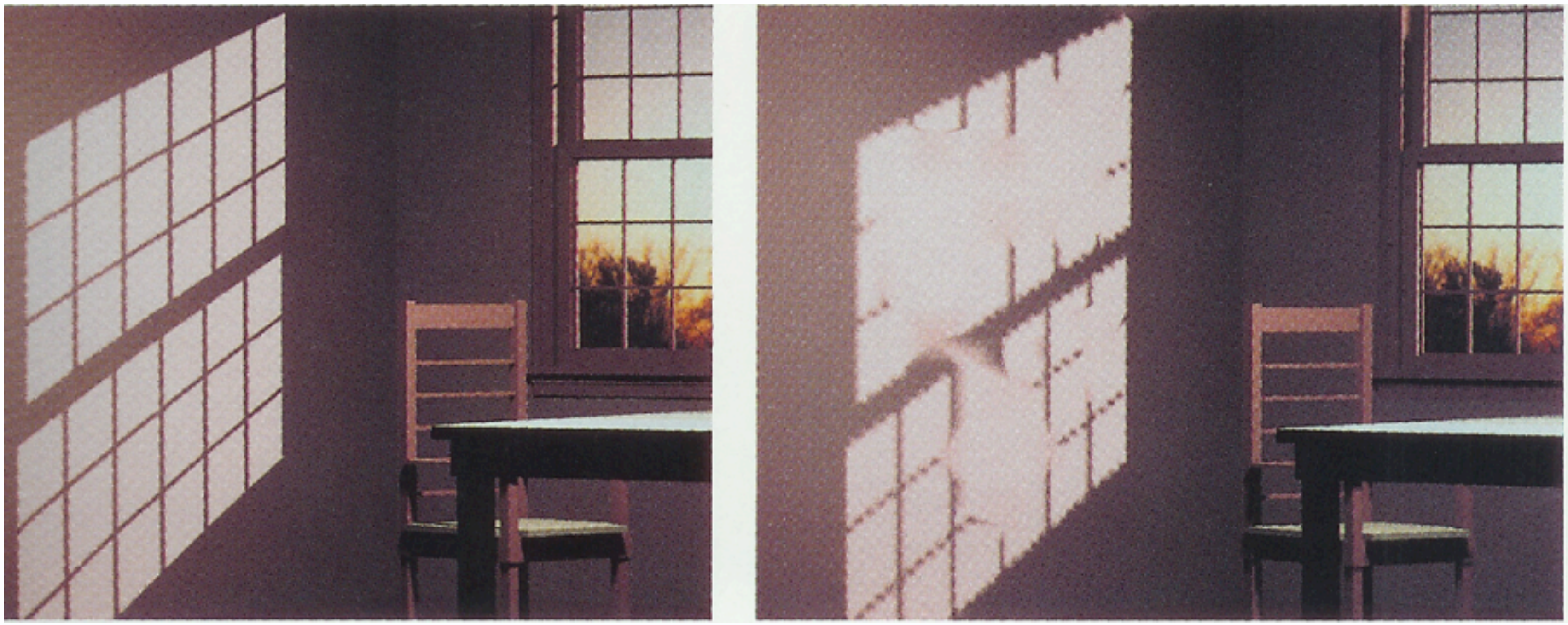
- Notice:

$$B = E + (\rho\mathcal{K})B$$

- Assume that I have a very rough estimate of B
  - I could render this using

$$B = E + (\rho\mathcal{K})\hat{B}$$

  - This isn't such a good idea, because our shadows will be mangled



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The right way

$$B = E + (\rho\mathcal{K})E + (\rho\mathcal{K})(\hat{B} - E)$$

Exitance

Source term

mostly zero

One or more bounces

Can change fast - shadows, etc.

Changes much more slowly, because K smoothes,
so we should approximate this

Computing the integrals

$$\rho(\mathbf{x}) \int K(\mathbf{x}, \mathbf{u}) E(\mathbf{u}) d\mathbf{u}$$

- Two terms
  - source term
    - we expect to need multiple samples, some large values, large changes over space
    - large variance will be ugly - should compute this term carefully at each point to render
  - indirect term
    - this term should change slowly over space, and should be smaller in value
    - large variance less ugly - we can use fewer samples and pool samples

$$\rho(\mathbf{x}) \int K(\mathbf{x}, \mathbf{u}) (\hat{B}(\mathbf{u}) - E(\mathbf{u})) d\mathbf{u}$$

Integrals with importance sampling

- Recall definition: $$\rho(\mathbf{x})\mathcal{K}F = \rho(\mathbf{x}) \int K(\mathbf{x}, \mathbf{u})F(\mathbf{u})d\mathbf{u}$$
- How to evaluate this integral at a point?
  - obtain $$\mathbf{u}_i \sim p(\mathbf{u})$$
  - Form: $$\frac{1}{N} \sum_{i=1}^{N} \frac{K(\mathbf{x}, \mathbf{u}_i)F(\mathbf{u}_i)}{p(\mathbf{u}_i)}$$
- Similar to evaluating illumination from area source

Importance sampling

- What is a good p(u)?
  - p(u) should be big when K(x, u) F(u) is big
  - this helps to control variance
  - known as importance sampling
  - Significant considerations:
    - fast variation in F(u)
    - fast variation in K
      - usually due to visibility
- How many samples?
  - fixed number
    - may be expensive, ineffective
  - by estimate of variance
    - this goes down as 1/N, which is very bad news

Computing the direct term

- We know where E is non-zero
  - luminaires
  - zero at most points
- Treat these as area sources
  - ie samples randomly distributed across area
    - number of samples prop to intensity, total energy
  - or stratified sampling
  - use visibility considerations to choose which sources are sampled

$$\rho(\mathbf{x}) \int K(\mathbf{x}, \mathbf{u}) E(\mathbf{u}) d\mathbf{u}$$

Computing the indirect term

- Small (ish)
- Varies relatively slowly across space
- Non-zero at most points
- Don't really know where it will be large
- Strategies
  - choose directions on the input hemisphere uniformly at random
  - make an importance map for input hemisphere, reuse

$$\rho(\mathbf{x}) \int K(\mathbf{x}, \mathbf{u})(\hat{B}(\mathbf{u}) - E(\mathbf{u}))d\mathbf{u}$$