Path tracing everything

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The Rendering Equation- 1

- We can now write

$$L_o(\mathbf{x}, \omega_o) = L_e(\mathbf{x}, \omega_o) + \int_\Omega \rho_{bd}(\mathbf{x}, \omega_o, \omega_i) L_i(\mathbf{x}, \omega_i) \cos \theta_i d\omega_i$$

Angle between normal and incoming direction

BRDF

Incoming irradiance

Average over hemisphere

Radiance emitted from surface at that point in that direction

Radiance leaving a point in a direction

The Rendering Equation - II

- This balance works for
  - each wavelength,
  - at any time, so
- So

$$\begin{aligned} L_o(\mathbf{x}, \omega_o, \lambda, t) &= L_e(\mathbf{x}, \omega_o, \lambda, t) + \\ & \int_\Omega \rho_{bd}(\mathbf{x}, \omega_o, \omega_i, \lambda, t) L_i(\mathbf{x}, \omega_i, \lambda, t) \cos\theta_i d\omega_i \end{aligned}$$

Detectors respond to irradiance

- Report

$$\int_D \int_\Omega \int_\lambda \int_T \sigma(\mathbf{x}, \lambda) L(\mathbf{x}, \omega, \lambda, t) \cos\theta d\mathbf{x} d\omega d\lambda dt$$

- sigma is sensitivity to wavelength
- typically:
  - shutter is open for short time (T)
  - particular detector is tiny
    - so integral over D isn't significant
  - Omega is set of directions through lens
    - usually close to normal to device, so cos theta doesn't vary much

Strategy: evaluate this integral

- At detector, average many samples of incoming radiance
  - at different times, directions, wavelengths, perhaps locations
  - value? rendering equation
- Evaluating the rendering equation
  - at each point, compute
    - L_e (usually zero)
    - Integral
      - fire off some sample rays, evaluate at the far end
      - very like diffuse path tracing
      - improvements
        - russian roulette (seen this)
        - importance sampling

Importance sampling - 1

- We have N samples x_i from probability distribution p(x)
- Then

$$\frac{1}{N}\sum_i f(x_i) \rightarrow \int f(x)p(x)dx$$

- Generally, we've assumed p is uniform
  - not a great idea - what if f is large in some places, small in others?
    - variance in the estimate
  - we can shape p to get a better estimate of the integral

Importance sampling - 2

- As long as p(x) is non-zero when f(x) is non-zero

$$\frac{1}{N}\sum_i \frac{f(x_i)}{p(x_i)} \to \int f(x)dx$$

- We could use p(x) to improve our estimate of integral

Path tracing

$L(\mathbf{x}, \mathbf{v}, \lambda, t)$
$\mathbf{u} \rightarrow$ first hit along ray from $\mathbf{x}$in direction $\mathbf{v}$

compute α (which might be a function of BRDF)
compute r uniform in range [0-1]

if $r < \alpha$
return $L_e(\mathbf{u}, -\mathbf{v}, \lambda, t)$
else
choose N directions $\mathbf{w}_i$ independently, at random, from distribution $P(\omega)$ on hemisphere.

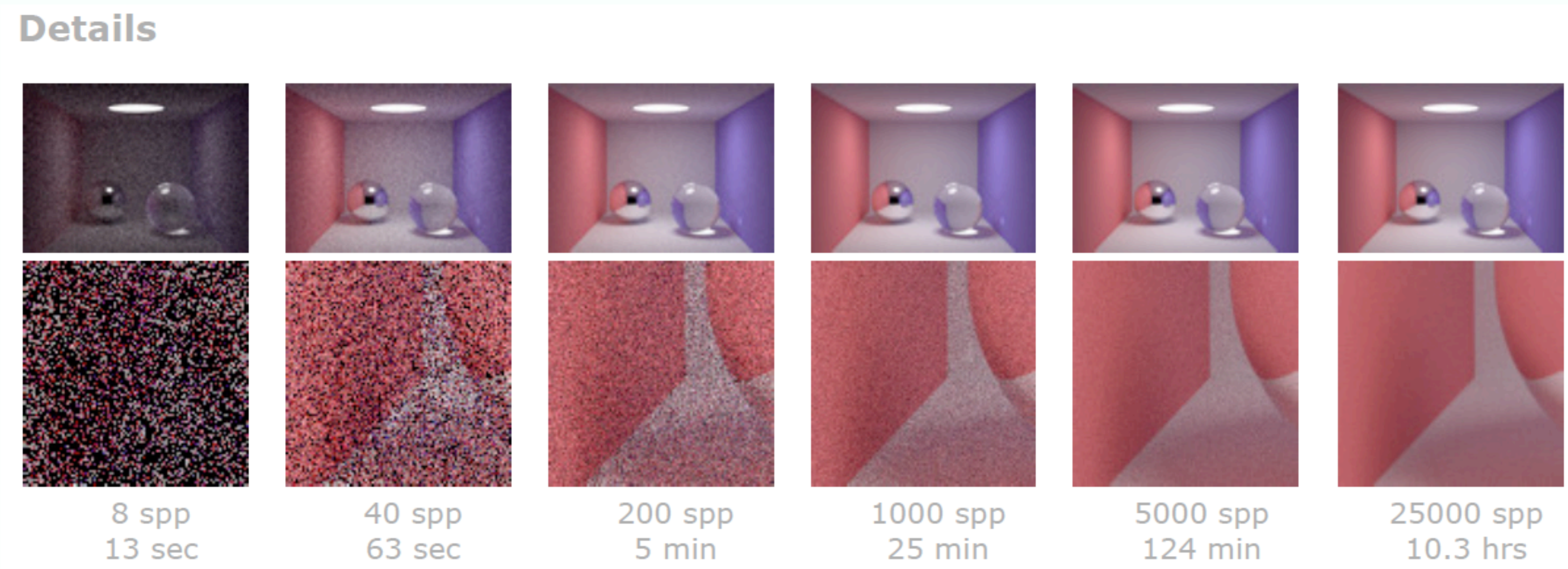
return $L_e(\mathbf{u}, -\mathbf{v}, \lambda, t) + \frac{1}{\alpha}\frac{1}{N}\sum_i \left(\frac{\rho_{bd}(\mathbf{u},\mathbf{v},\mathbf{w}_i,\lambda,t)L(\mathbf{u},\mathbf{w}_i,\lambda,t)\cos\theta_i}{P(\mathbf{w}_i)}\right)$

To avoid variance problems, choose good P
for surfaces with any diffuse component, P should have strong bias to light
for specular surfaces, P should heavily emphasize the specular direction


RENDERED USING DALI - HENRIK WANN JENSEN 2000

Example pathtracer

http://www.kevinbeason.com/smallpt/



Variance problems

- Paths may not find the light often
  - this could be fixed by clever choice of P to heavily emphasize directions toward the source
- Caustics will be poorly rendered, because the path to the source is obscure

Bidirectional path tracing

- Start paths at both eye and light and join them
- Notice:
  - a pair of eye-light paths generates many possible transfer paths
  - we can use each of these, if we compute weights correctly to get integral estimate right

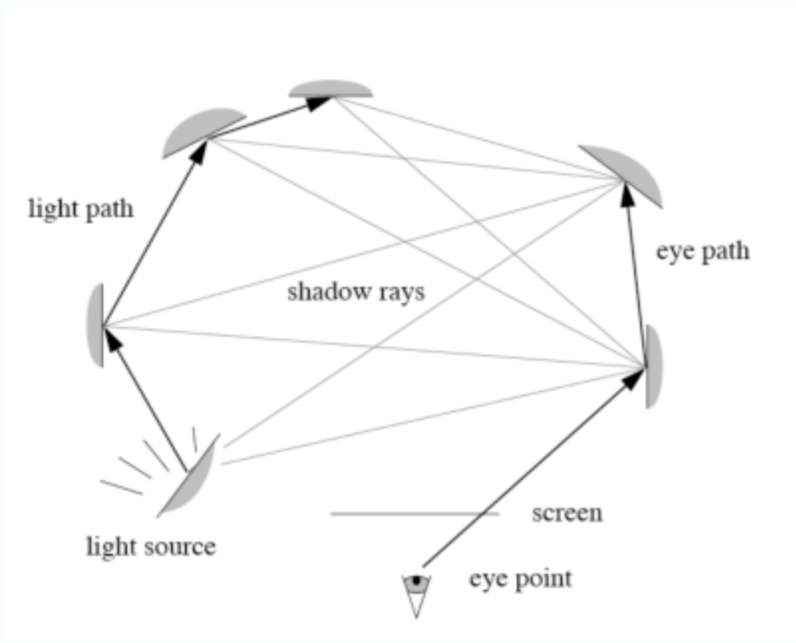


Figure from "Bidirectional path tracing." Lafortune and Willems, 1993

From the eye | From the source | Bidirectional

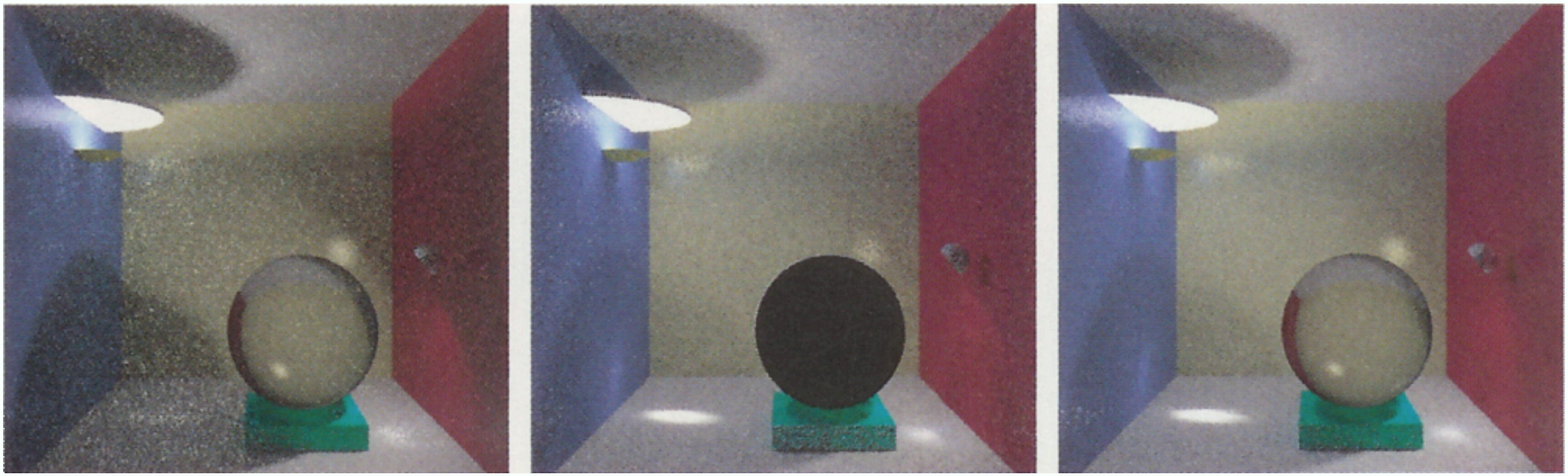


Figure from Dutre, Bekaert, Bala 03; rendered by Suykens-De Laet

Photon maps

- Drop the requirement of an unbiased estimate of illumination
  - accept some bias for better variance properties
- Propagate photons from source, cache when they arrive at surfaces
- Interpolate illumination value by averaging over k-nearest neighbours
- Caustic variance
  - use two classes of photon: sample specular, refractive directions separately
- How many photons?
  - keep trying till it looks good

Photon propagation

- Photons carry Power
  - scale photons from source by number emitted
- reflected
  - diffuse
    - store in map when it arrives, propagate
      - prob proportional to cos theta
      - power scaled by albedo
        - or use russian roulette
  - specular
    - do not store in map, propagate
      - along specular direction
      - power scaled by reflectivity
        - or use russian roulette
  - arbitrary BRDF
    - inportance sample outgoing direction

Photon propagation

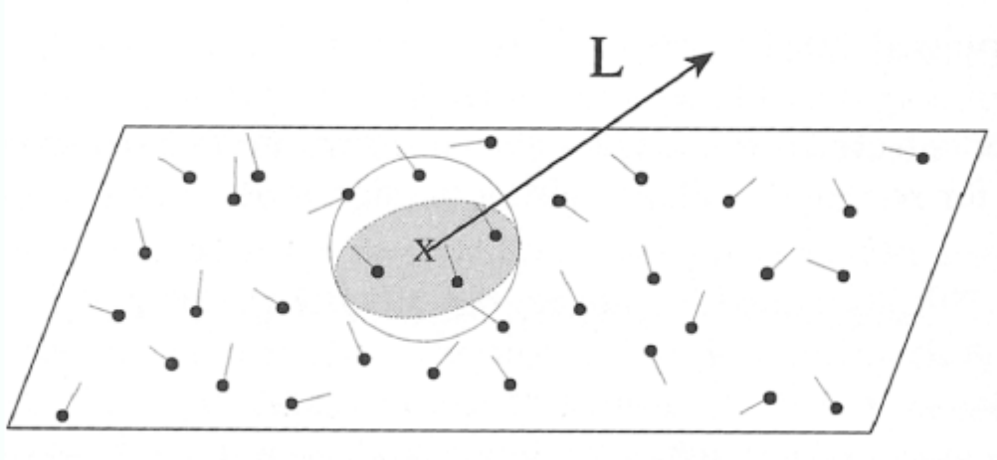
- When a photon arrives at complex surface, multiple photons could be generated
  - eg specular + diffuse
    - russian roulette to decide whether
      - specular
        - reflected/absorbed
      - diffuse
        - reflected/absorbed
- Photons are stored at diffuse (non-specular!) surfaces only
- Stored as:
  - Power, Location, Normal

Photon storage and querying

- Store in k-d tree
  - to look up r closest photons
  - tree represents free space close to surfaces

Evaluating Radiance

- Reflected radiance is: $$L_r(\mathbf{x},\omega) = \int_\Omega \rho_{bd}(\omega,\omega_i) L_i(\mathbf{x},\omega_i) \cos\theta d\omega_i$$
- Each photon carries known power, in known direction
  - assume the relevant photons all arrive at x
  - each contributes radiance (power/dA)
  - assume surface is flat around x, build a circle
    - photons in this circle contribute
    - area is known



$$L_r(\mathbf{x},\omega) = \frac{1}{\pi r^2} \sum \rho_{bd}(\omega,\omega_j) P_j(x,\omega_j)$$

Figure from Jensen's book

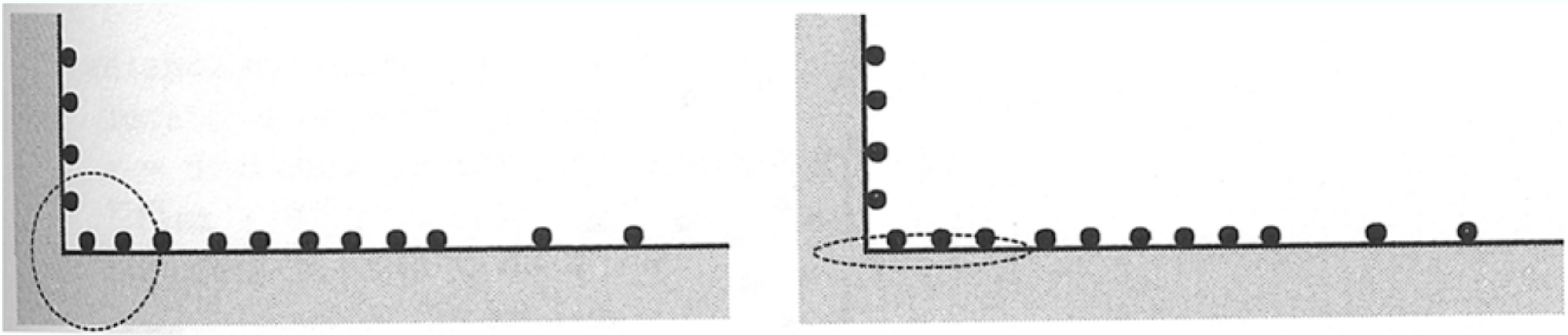


Figure from Jensen's book

A two pass renderer

- Propagate photons
  - two classes
    - caustic photons toward specular/glossy, refractive objects
      - large numbers
      - caustic map
    - global illumination photons toward diffuse objects
      - small numbers
- Gather
  - render using
    - direct term by area source sampling
    - specular term by ray-tracing
    - caustic term by direct query to photon map
    - global illumination term by gathering the photon map