Paths, diffuse interreflections, caching and radiometry

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How we got here

- We want to render diffuse interreflections
  - strategy: compute approximation B-hat, then gather

$$B = E + (\rho\mathcal{K})E + (\rho\mathcal{K})(\hat{B} - E)$$

Exitance

Source term

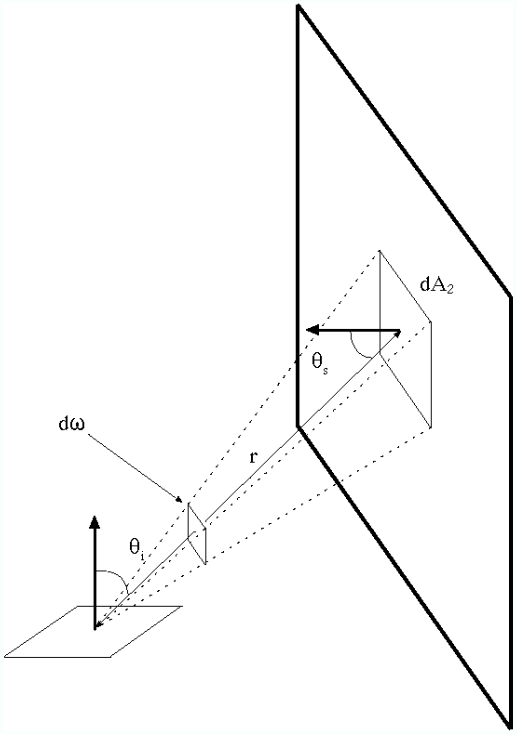
mostly zero

One or more bounces

Can change fast - shadows, etc.

Changes much more slowly, because K smoothes,
so we should approximate this

Gathering



- We gather radiosity from B-hat
  - Here S is all the surfaces in the world

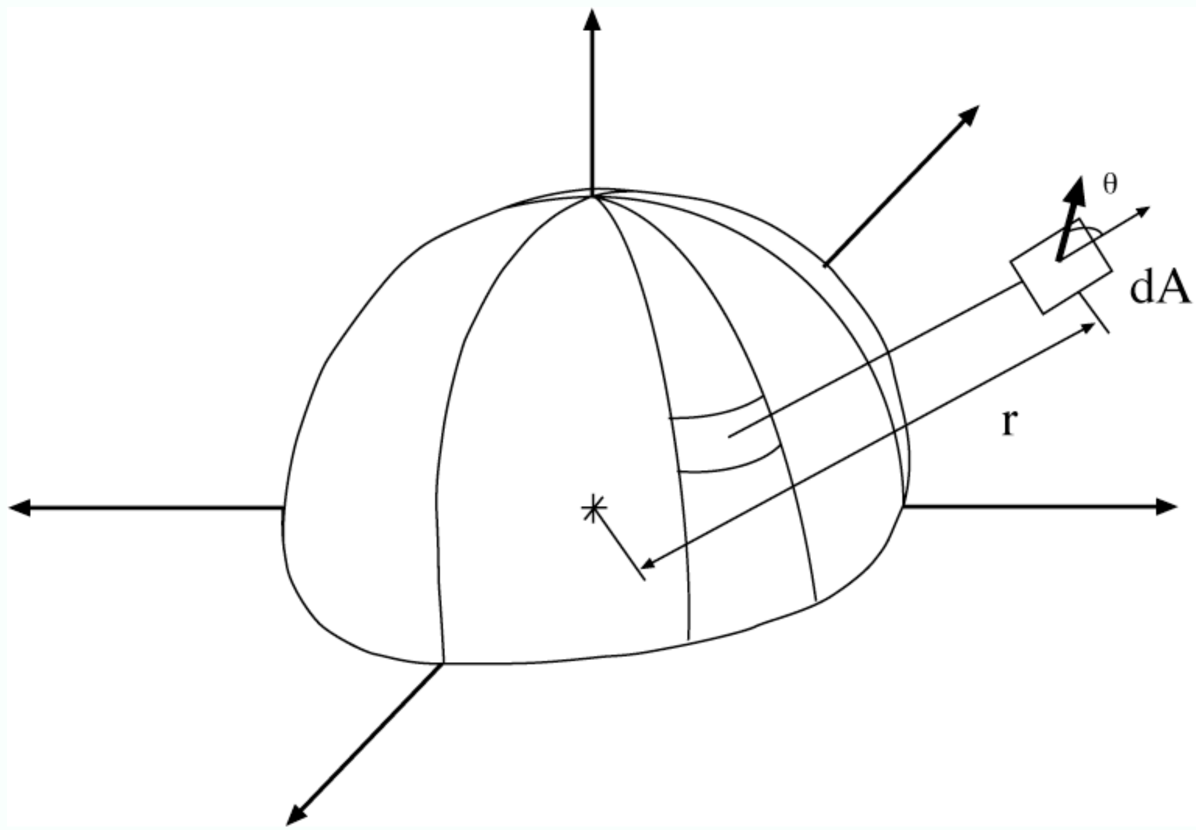
$$(\rho\mathcal{K})(\hat{B} - E) = \rho(\mathbf{x}) \int_S \frac{\cos\theta_i \cos\theta_s}{\pi r^2} Vis(\mathbf{x}, \mathbf{u})(\hat{B}(\mathbf{u}) - E(\mathbf{u}))dA_s$$

- Another integral
  - but not a good idea to integrate over dAs
    - too much area, too many samples
  - instead, integrate over hemisphere

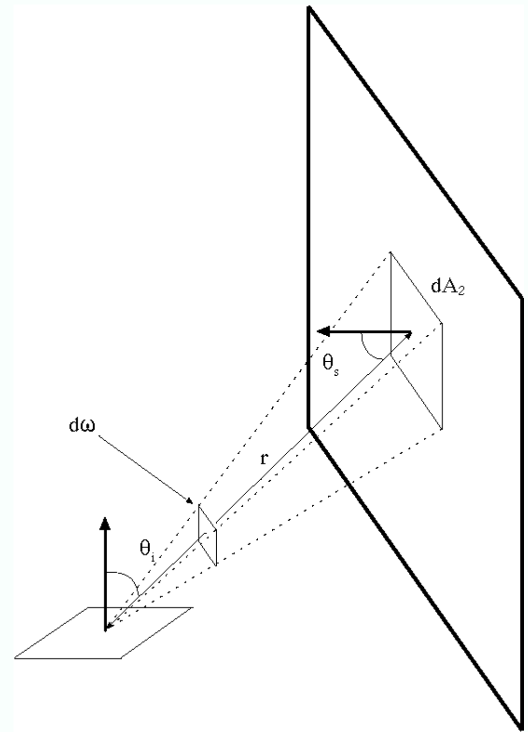
Remember Solid Angle

- By analogy with angle (in radians)
- The solid angle subtended by a patch area dA is given by

$$d\omega = \frac{dA \cos \vartheta}{r^2}$$



Changing variables



- Rather than integrate over all area, integrate over hemisphere
  - equivalently, integrate over solid angle

$$\frac{\cos\theta_s}{r^2} dA_s = d\omega_s$$

Changing variables

- Start with:

$$\rho(\mathbf{x}) \int_S \frac{\cos\theta_i \cos\theta_s}{\pi r^2} Vis(\mathbf{x}, \mathbf{u})(\hat{B}(\mathbf{u}) - E(\mathbf{u}))dA_s$$

- Substitute: $$\frac{\cos\theta_s}{r^2} dA_s = d\omega_s$$

- Get:

Value at far end of ray through angle

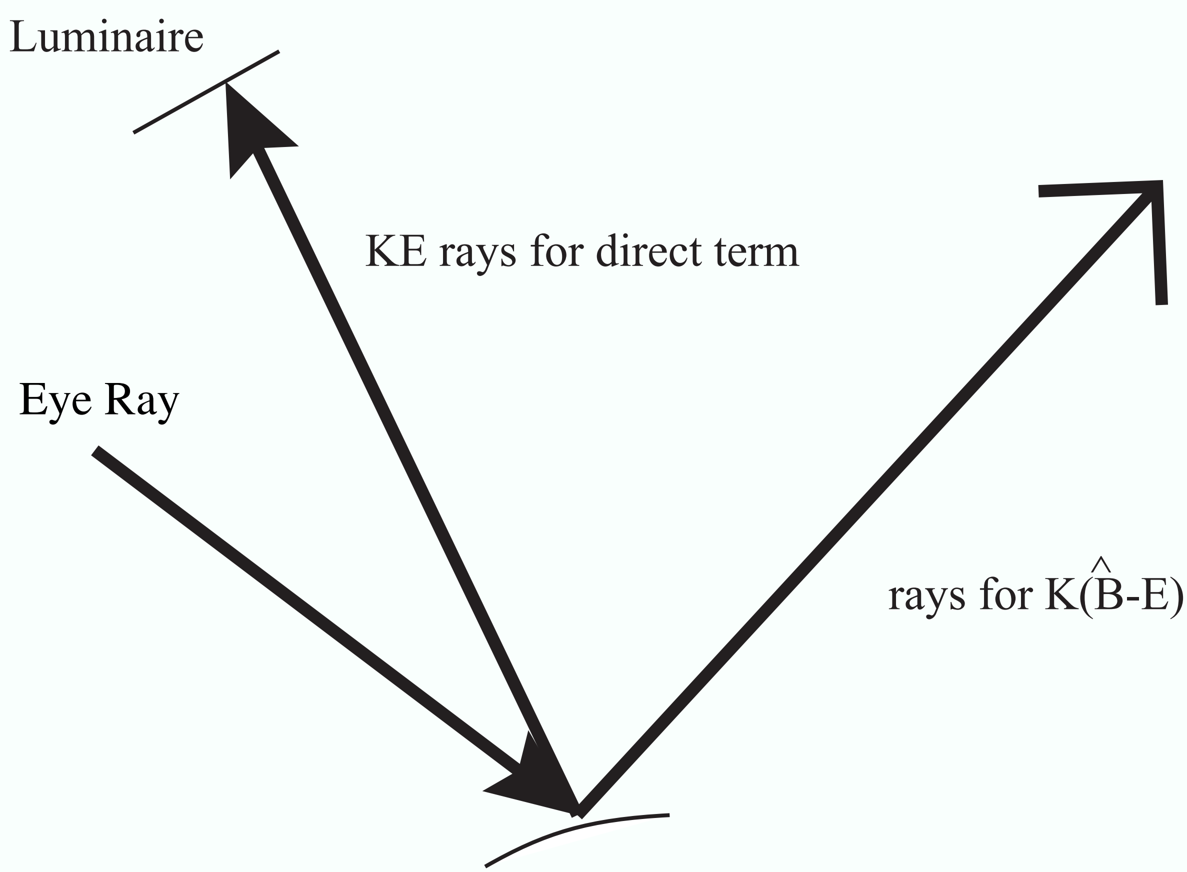
$$\rho(\mathbf{x}) \frac{1}{\pi} \int_\Omega \cos\theta_i (\hat{B}(\omega) - E(\omega))d\omega$$

Incoming hemisphere

Evaluating integral

- Procedure
  - Generate N uniform random samples on hemisphere
    - procedure described on whiteboard
  - Find B-hat-E at far end of each ray
  - Average
- How big should N be?
  - Variance
    - estimate is a random variable, so must have variance
    - small N implies high variance, fast
    - large N implies low variance, slow
  - Variance will look like noise
    - but should be small, because the term is small
    - suggests small N is OK

Gathering from B-hat - E



$$B = E + (\rho\mathcal{K})E + (\rho\mathcal{K})(\hat{B} - E)$$

Alternative: B-hat via random paths

- Notice that B-hat is also an integral
  - approximation to B
- Now from $B = E + (\rho\mathcal{K})B$
  - we expect $\hat{B} = E + (\rho\mathcal{K})\hat{B}$
  - so $\hat{B} - E = (\rho\mathcal{K})\hat{B}$
  - expand by substituting to get $\hat{B} = E + (\rho\mathcal{K})(E + (\rho\mathcal{K})\hat{B})$
  - ie $\hat{B} - E = (\rho\mathcal{K})(E + (\rho\mathcal{K})\hat{B})$
  - substitute from above to get $\hat{B} - E = (\rho\mathcal{K})E + (\rho\mathcal{K})(\hat{B} - E)$

Alternative

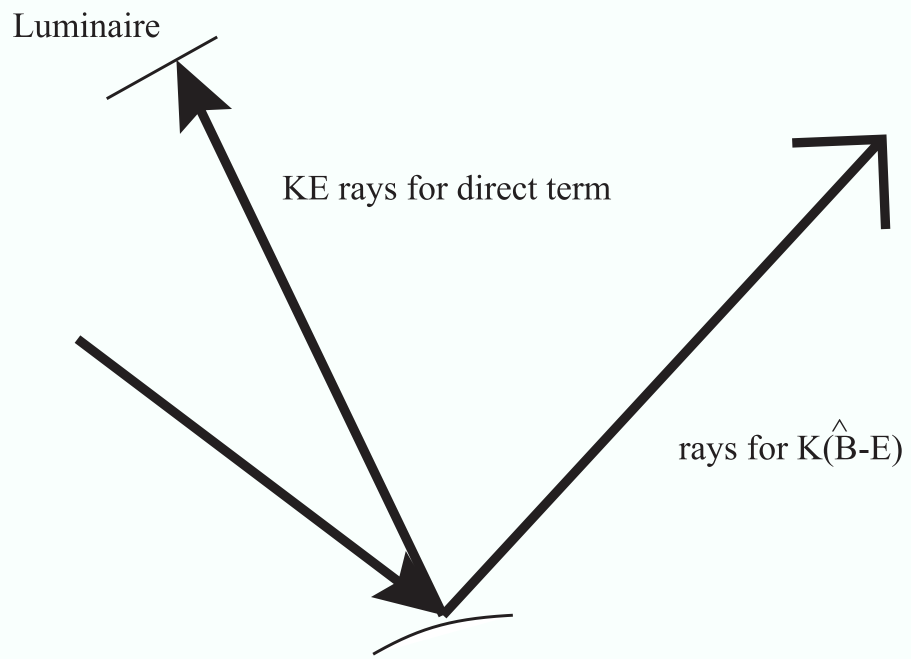
- We could evaluate B-hat - E recursively

$$\hat{B} - E = (\rho\mathcal{K})E + (\rho\mathcal{K})(\hat{B} - E)$$

Direct term — Indirect term

Recursive evaluation

$$\text{shade}(x) = E(x) + \rho(x)\text{direct}(x) + \text{RKBME}(x)$$



Recursive evaluation: direct term

$$\text{direct}(x) = \sum_{l \in \text{luminaires}} \text{directfromL}(x, l)$$

directfromL(x, L)

generate N uniform random samples u_i on luminaire L with area A_l
return $\frac{A_l}{N} \sum_i \frac{\cos\theta_x \cos\theta_u}{\pi r^2} E(u_i)$

We did this when we discussed area luminaires - no big mystery here

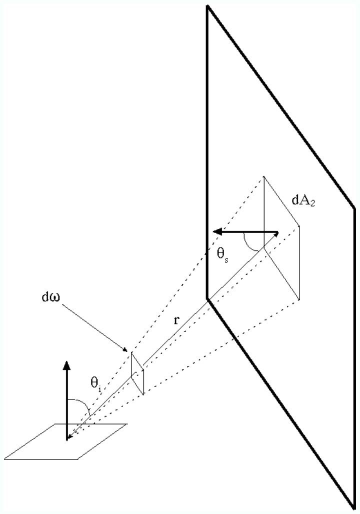
Recursive evaluation: Indirect term

This form isn't yet practical, because the recursion is infinite!

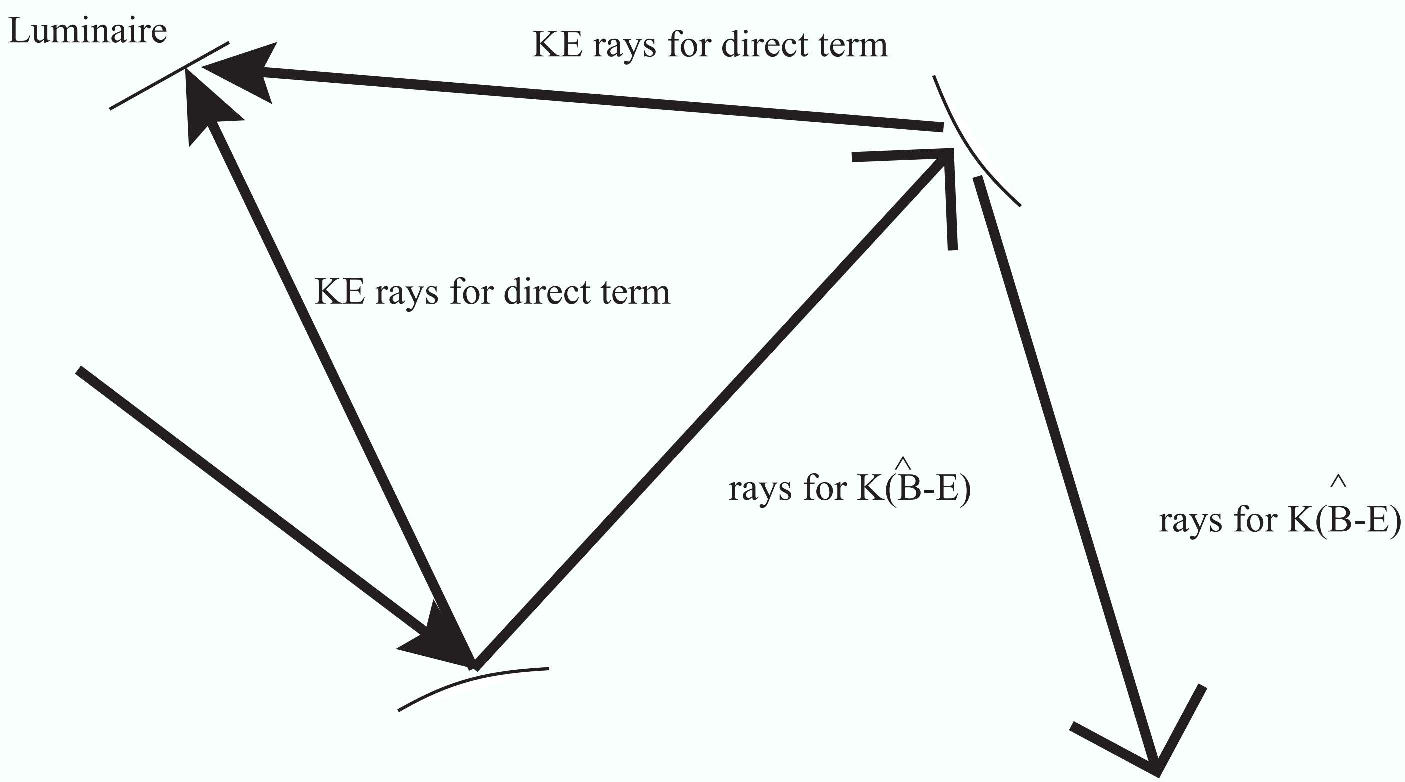
RKBME(x)

Generate M points p_i uniformly at random on unit hemisphere at x
For each point p_i, write u_i for the first hit on the ray from x to p_i
write $\cos\theta_{si}$ for the cosine at x of the i'th direction

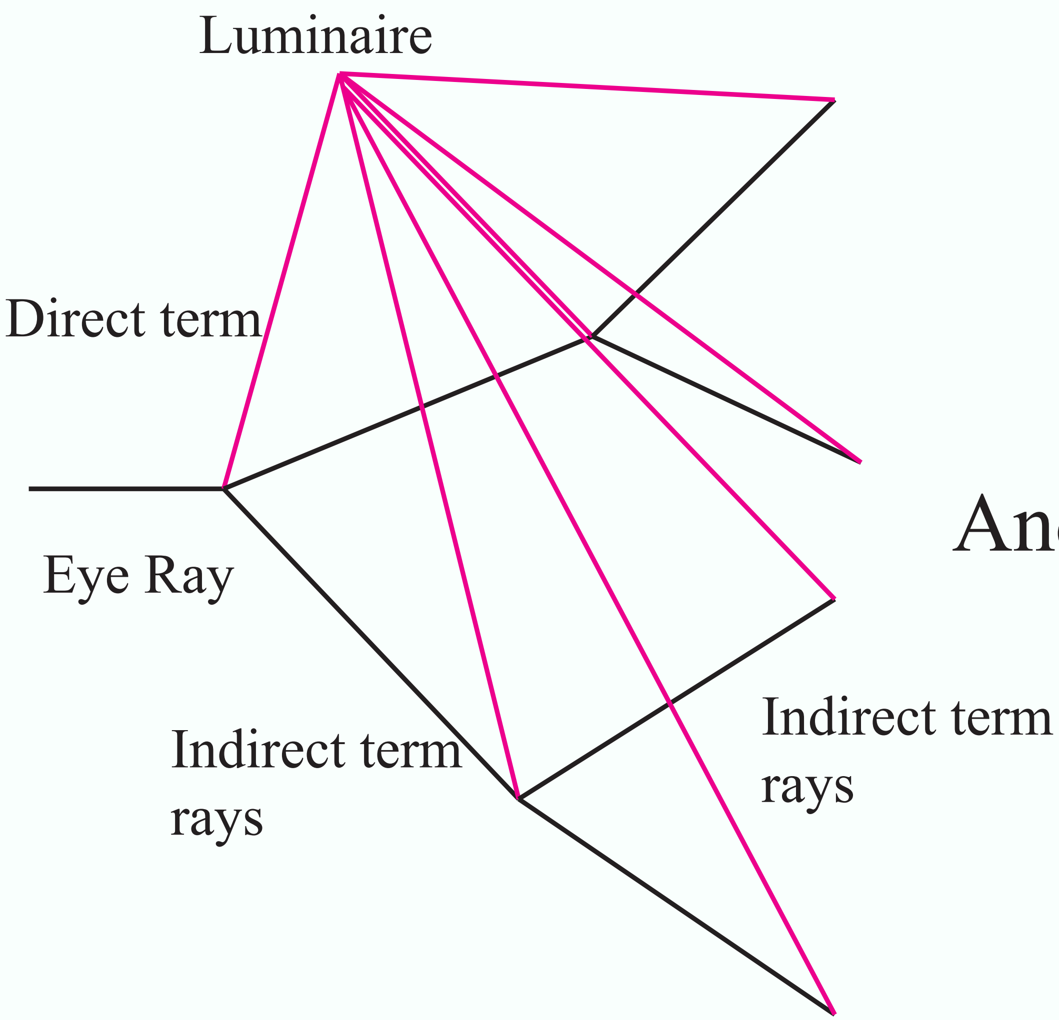
return $\rho(x)2\pi\frac{1}{\pi}\frac{1}{M}\sum_i\left(\rho(u_i)\text{direct}(u_i)+\text{RKBME}(u_i)\right)\cos\theta_{si}$



B-hat via random paths becomes a tree



B-hat via random paths becomes a tree



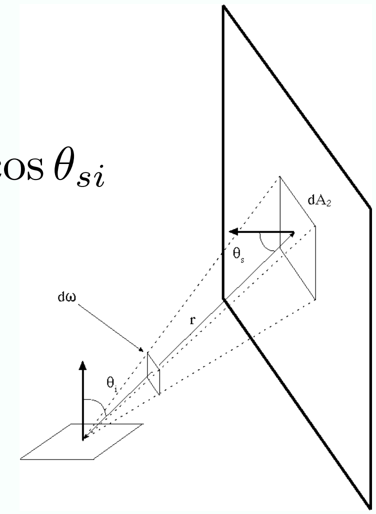
And so on...

Recursive evaluation: Indirect term

Recursion no longer infinite, but estimate must be (very slightly) too small

RKBME(x, depth)

Generate M points p_i uniformly at random on unit hemisphere at x
For each point p_i, write u_i for the first hit on the ray from x to p_i
write $\cos\theta_{si}$ for the cosine at x of the i'th direction
if depth==0
return 0
else
return $\rho(x) 2\pi \frac{1}{\pi} \frac{1}{M} \sum_i \left(\rho(u_i)\text{direct}(u_i) + \text{RKBME}(u_i, depth - 1)\right) \cos\theta_{si}$

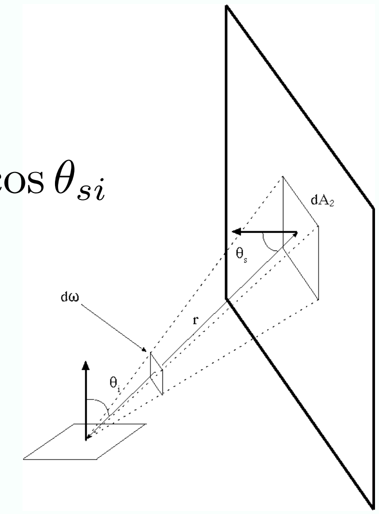


Recursive evaluation: Indirect term

Recursion no longer infinite, not as deep as previous,
but estimate must still be (very slightly) too small

RKBME(x, ρ_{acc})

Generate M points p_i uniformly at random on unit hemisphere at x
For each point p_i, write u_i for the first hit on the ray from x to p_i
write $\cos\theta_{si}$ for the cosine at x of the i'th direction
if $\rho_{acc} <$ smallthresh
return 0
else
return $\rho(x) 2\pi \frac{1}{\pi} \frac{1}{M} \sum_i \left(\rho(u_i)\text{direct}(u_i) + \text{RKBME}(u_i, \rho(x) * \rho_{acc})\right) \cos\theta_{si}$



Russian roulette

- Consider a random process:
  - with probability p, return S
  - with probability 1-p, return 0
- Expected value:
  - p*S

- We can use this to prune paths at random, mainly pruning when albedo is low

Russian roulette

Notice what's happened to the albedo term. When a path gets to low albedo surface, it has little chance of continuing. This is unbiased!

RKBME(x)

Generate v uniform random variable, $v \in [0, 1]$

if $v > \rho(x)$
return 0
else

Generate M points p_i uniformly at random on unit hemisphere at x
For each point p_i, write u_i for the first hit on the ray from x to p_i
write $\cos\theta_{si}$ for the cosine at x of the i'th direction

return $2\pi \frac{1}{\pi} \frac{1}{M} \sum_i \left(\rho(u_i)\text{direct}(u_i) + \text{RKBME}(u_i)\right) \cos\theta_{si}$

Light paths

- Light starts at the luminaire, ends at the eye
  - Rendering involves accounting for these paths
    - pixel value= Sum over paths (light contributed by path)
  - When we ray trace, we are tracking a path that light followed
    - we could go forward or backward along the path
      - either way involves easy geometry we know how to do
  - Label the path with L (bounces) E
- Bounce labels are D (diffuse), S (specular/transmissive)

- Big distinction:
  - S we know the next dir, D we don't

Light paths

- Example paths
  - e.g. LDE
    - luminaire to diffuse surface to eye
      - already done these; trace eye ray then
        - shadow ray+dot product (point light source)
        - area source integral (area luminaire)
  - LDSE
    - luminaire to diffuse to specular to eye
    - already done these; trace eye ray, one specular/transmissive ray then
        - shadow ray+dot product (point light source)
        - area source integral (area luminaire)

Light paths

- Example paths:
  - LDS*E
    - already done this, multiple specular/transmissive bounces
  - LSDE
    - sketched this; fire light out of luminaire, stick it in a map, pick up later
  - LDD+E
    - i.e. more than one diffuse bounce
      - have not yet talked about this, next topic
      - these paths can contribute a lot of light, but are hard to evaluate

Main points

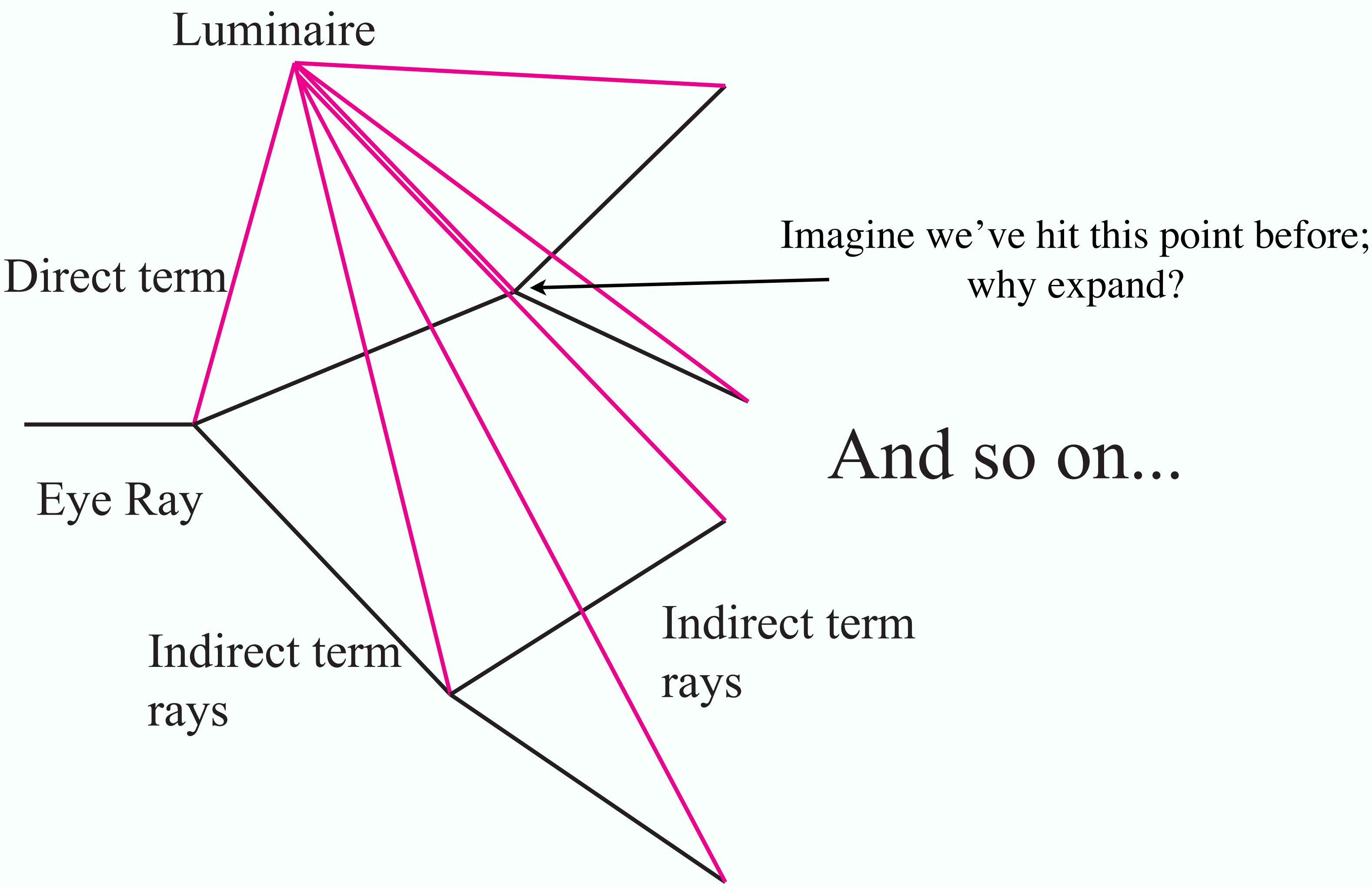
- When a light path arrives at/leaves from S
  - we know where it's going/came from
- When a light path arrives at/leaves from D
  - we don't know where it's going/came from
- Rendering ALWAYS answers "how bright is this"

Brightness = Diffuse term + term from far end of specular+ term from far end of transmitted

Light path analysis

- We've now done LD*E
  - russian roulette cleverly explores paths; if there's lots of albedo, paths tend to be long; else short.
  - russian roulette is a random process
    - random choice of directions; random choice to prune
    - unbiased
      - Expected value is the right answer
    - variance
      - because it's random
      - looks like image noise
      - seen this before in lenses, motion blur
      - control by
        - more rays (!)
        - caching
        - importance sampling (later)

Caching



And so on...

Caching

RKBME(x)

Generate v uniform random variable, $v \in [0, 1]$

if $v > \rho(x)$
return 0
else

Interrogate cache - do we have an RKBME value close to x?
if yes
return cache value
else

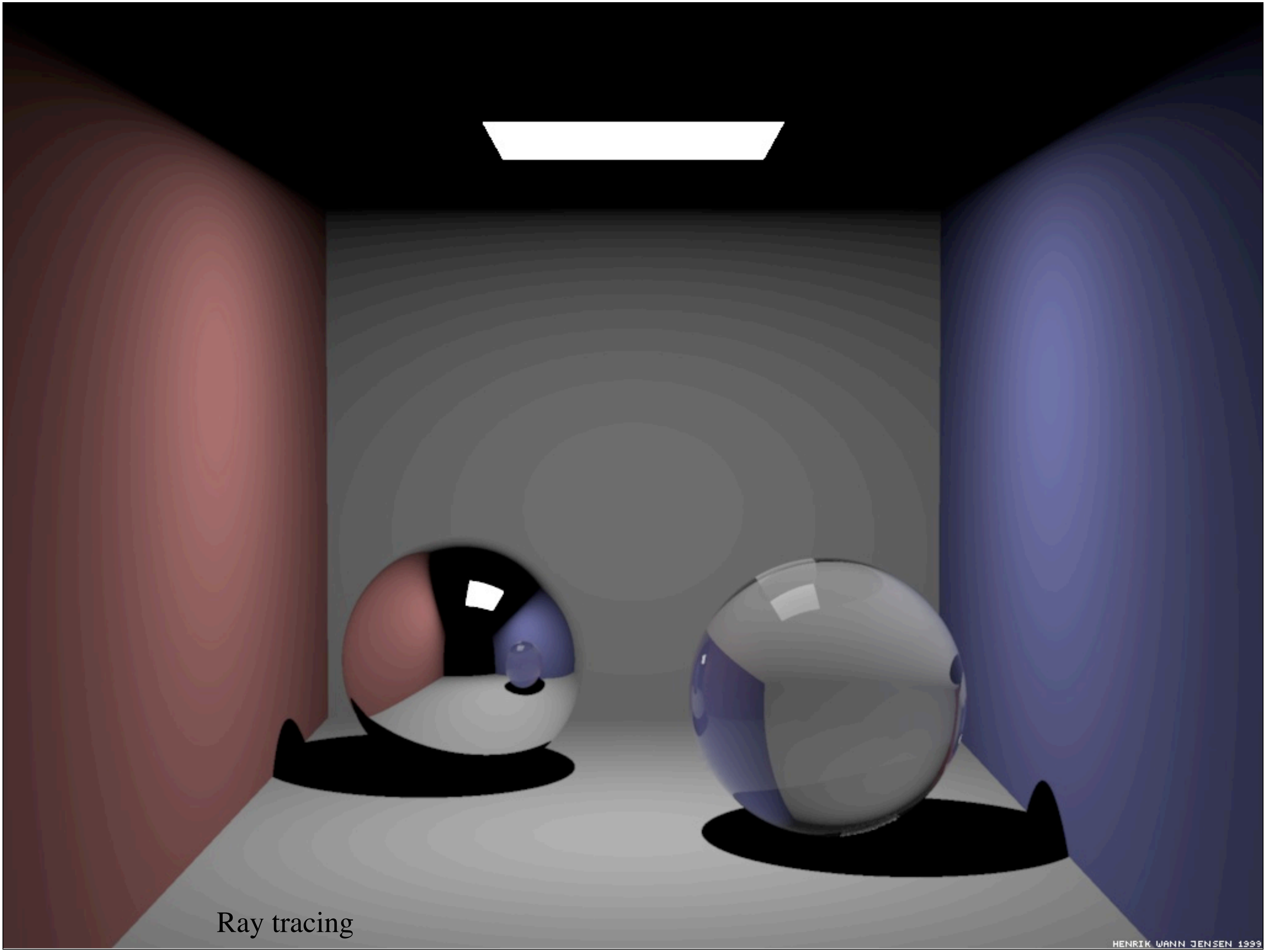
Generate M points p_i uniformly at random on unit hemisphere at x
For each point p_i, write u_i for the first hit on the ray from x to p_i
write $\cos\theta_{si}$ for the cosine at x of the i'th direction

return $2\pi \frac{1}{\pi} \frac{1}{M} \sum_i (\rho(u_i)\mathrm{direct}(u_i) + \mathrm{RKBME}(u_i)) \cos\theta_{si}$

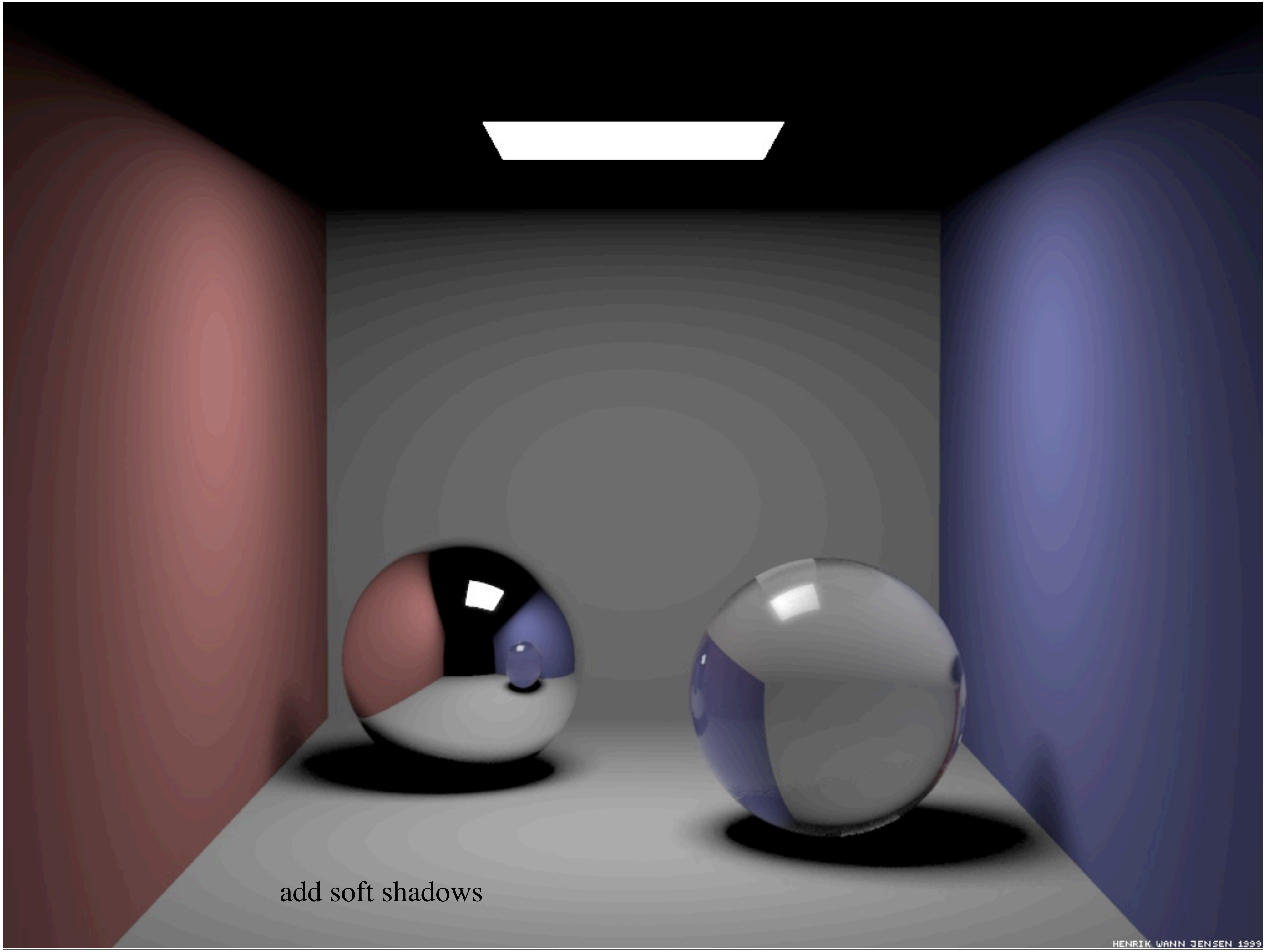
Irradiance cache vs path tracing, from Pharr + Humphreys, for the same amount of cpu

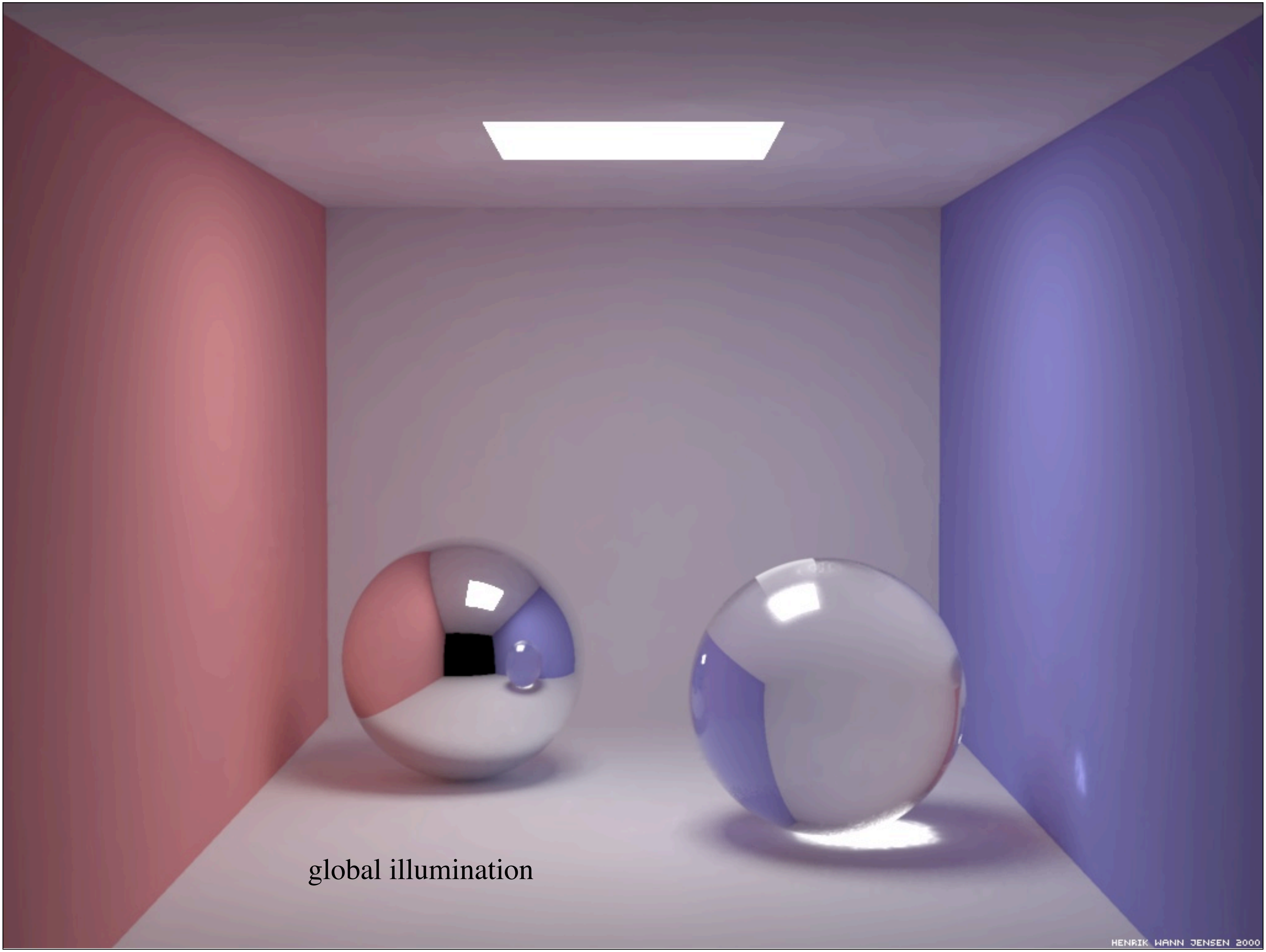
Light path analysis

- Main strategy
  - build and evaluate light paths
- We can do other kinds of path like this, too
  - requires extra radiometry

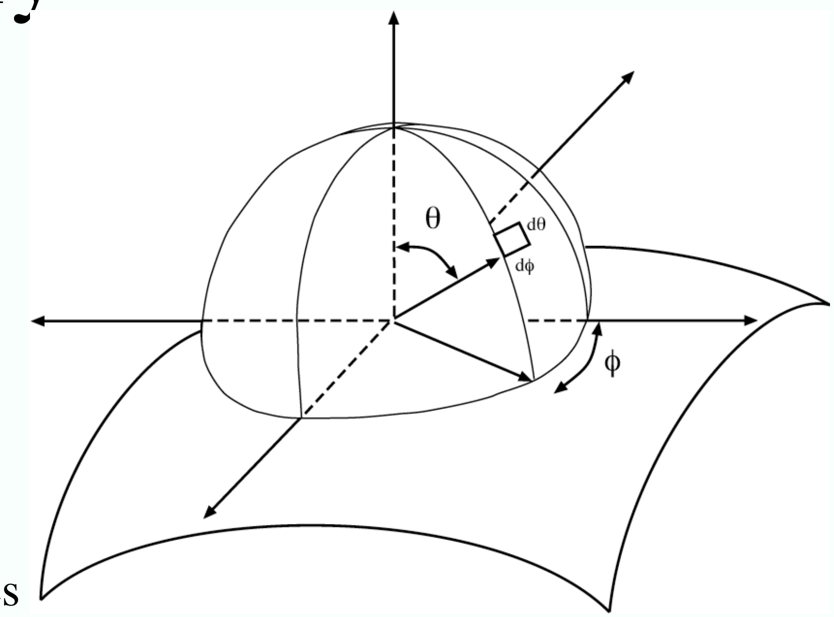


Ray tracing


add soft shadows
HENRIK WANN JENSEN 1999

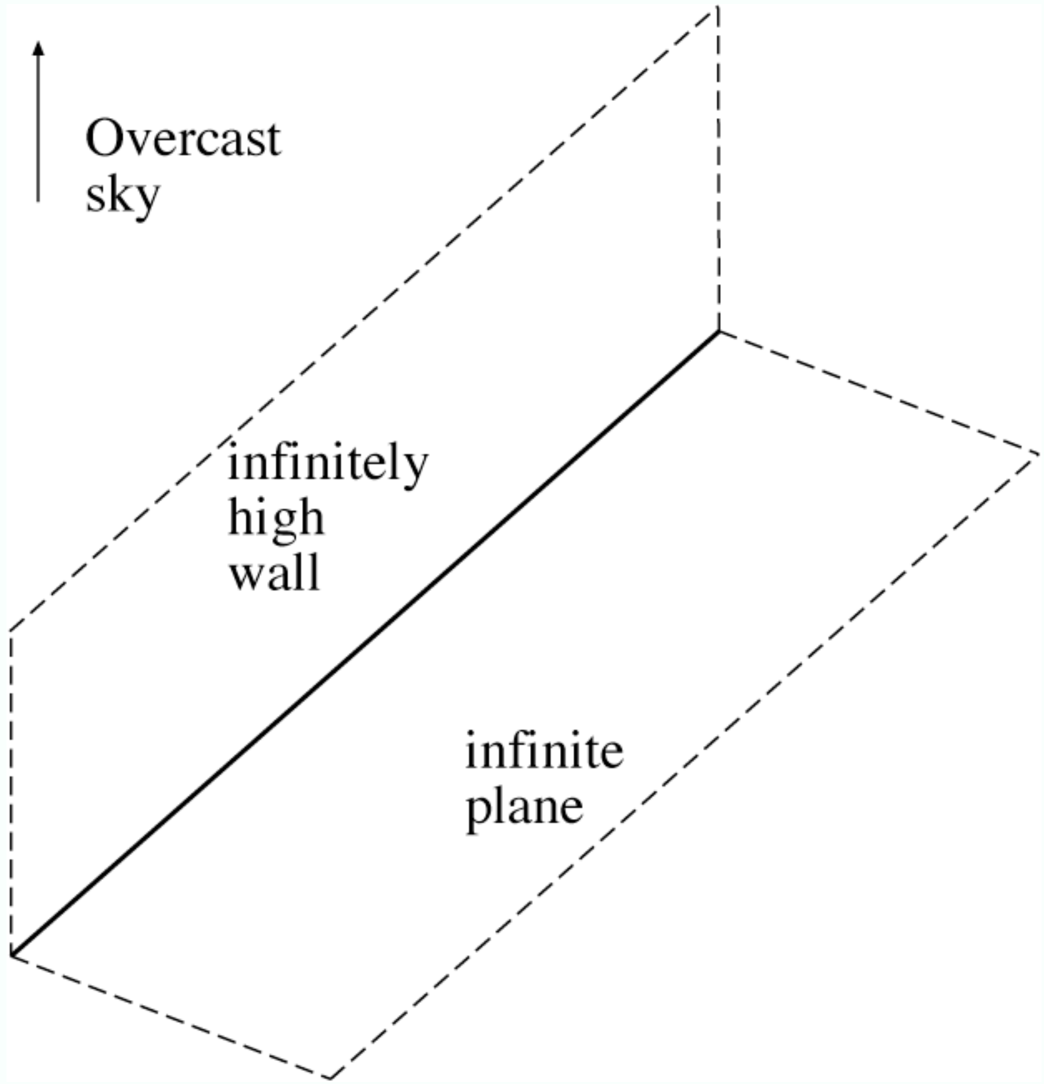

global illumination
HENRIK WANN JENSEN 2000

Radiometry

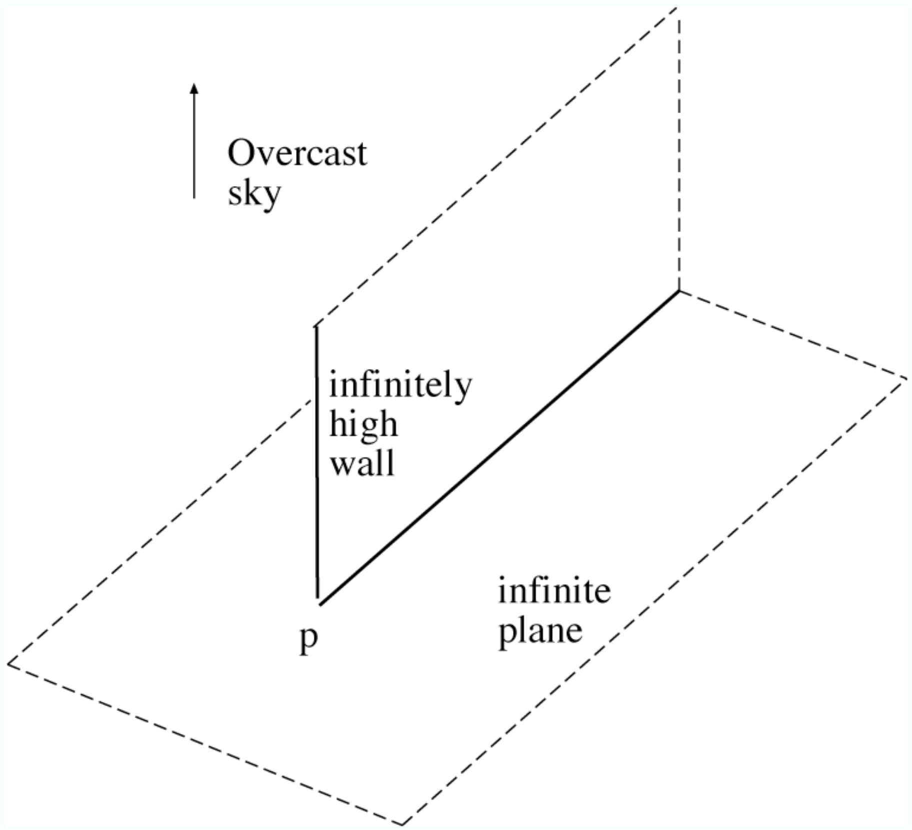


- Questions:
  - how “bright” will surfaces be?
  - what is “brightness”?
    - measuring light
    - interactions between light and surfaces
- Core idea - think about light arriving at a surface
- around any point is a hemisphere of directions
- Simplest problems can be dealt with by reasoning about this hemisphere

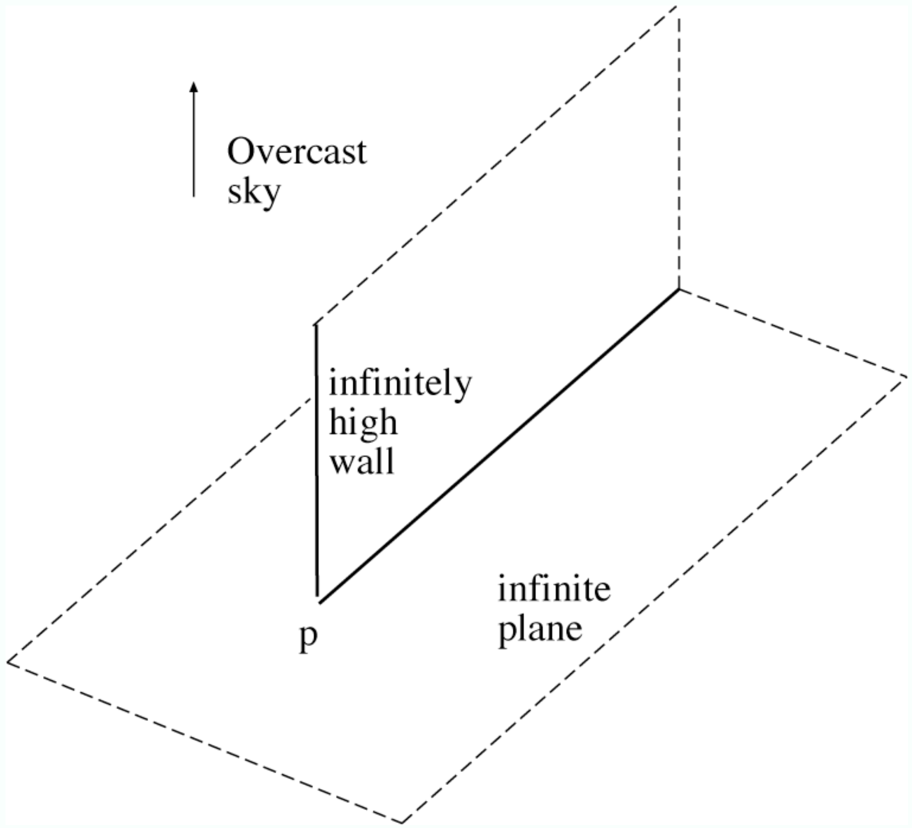
Lambert’s wall

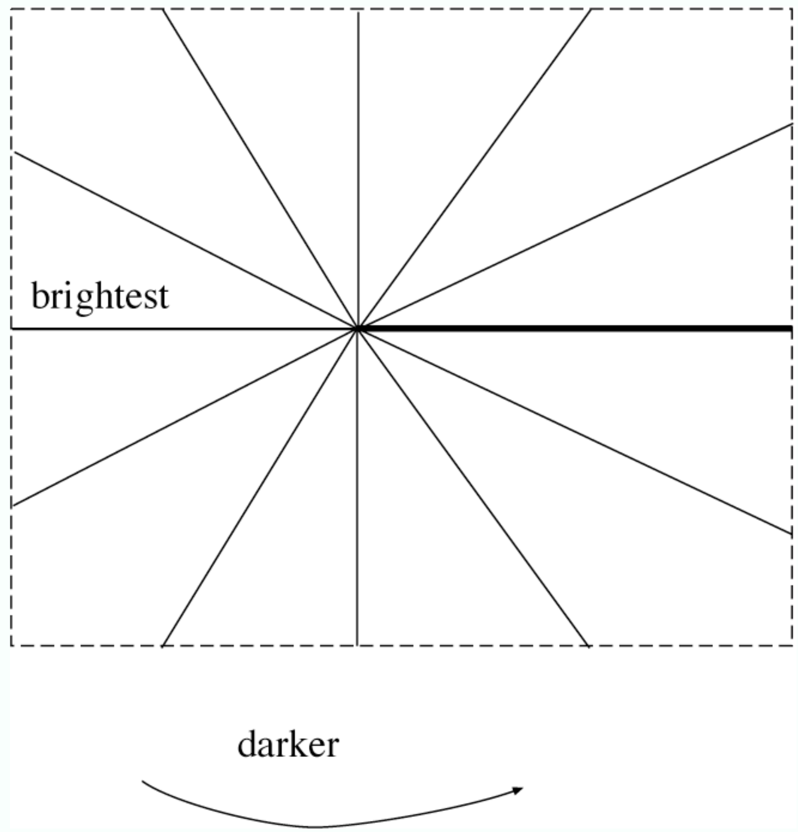


More complex wall



More complex wall

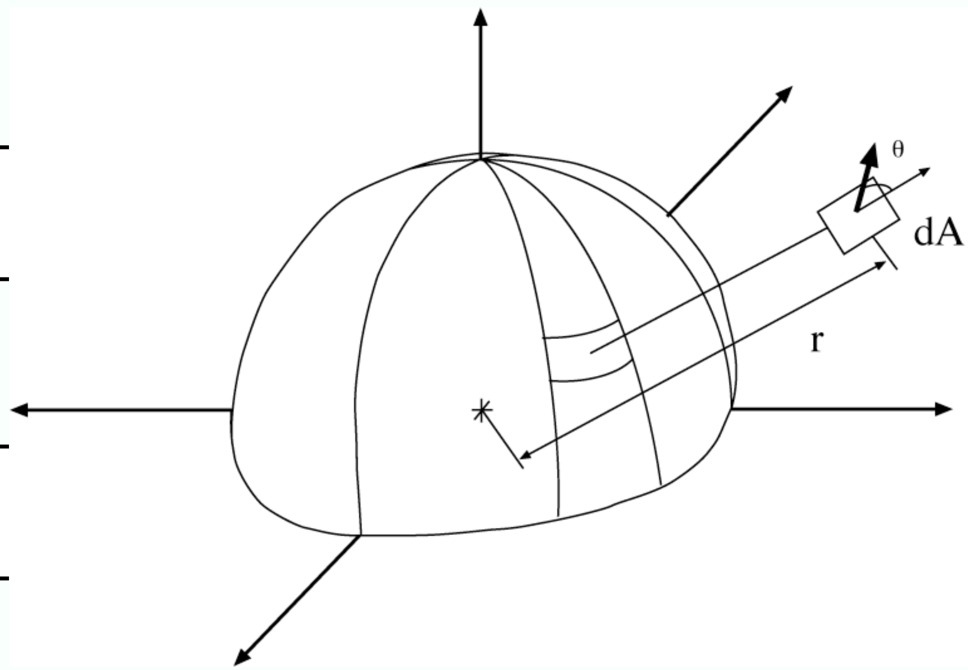




Solid Angle

- By analogy with angle (in radians)
- The solid angle subtended by a patch area dA is given by

$$d\omega = \frac{dA \cos\vartheta}{r^2}$$



- Another useful expression:

$$d\omega = \sin\vartheta (d\vartheta)(d\phi)$$

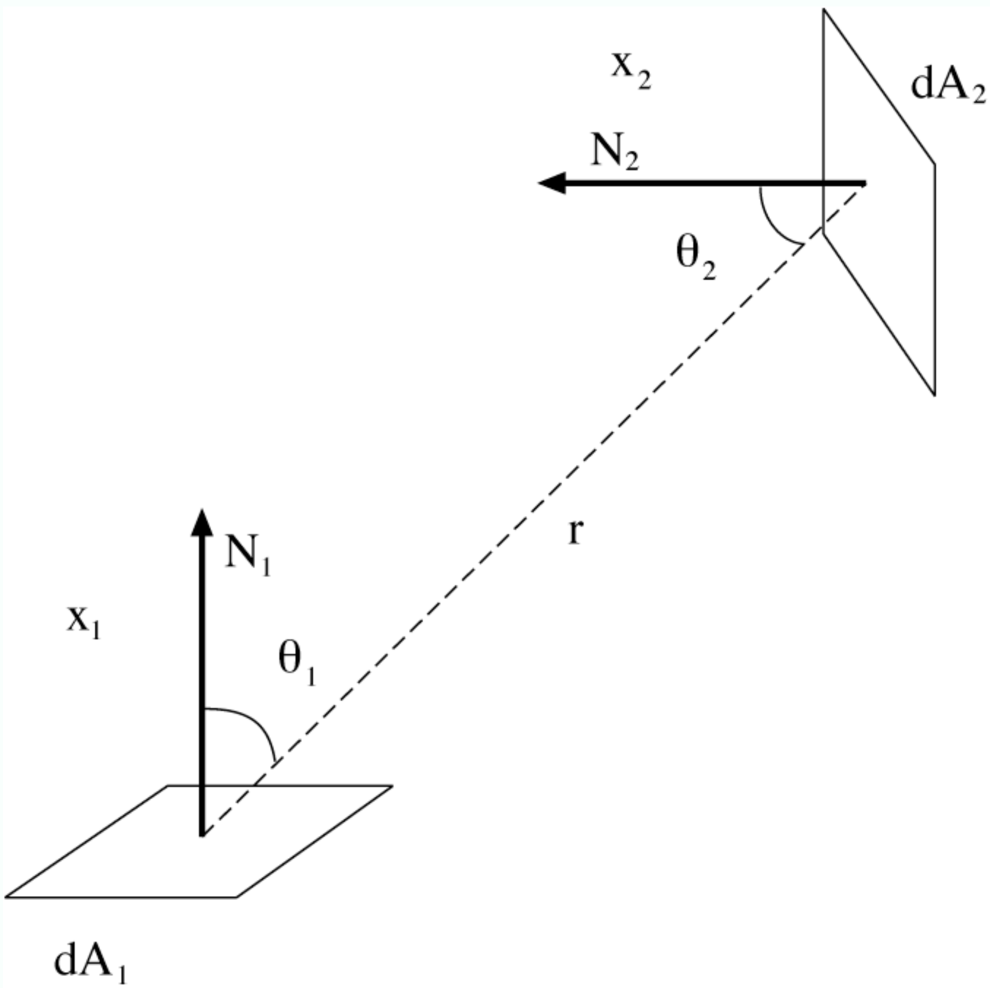
Radiance

- Measure the "amount of light" at a point, in a direction
- Property is:
  Radiant power per unit foreshortened area per unit solid angle
- Units: watts per square meter per steradian (wm-2sr-1)
- Usually written as:

$$L(\underline{x}, \vartheta, \varphi)$$

• Crucial property:
In a vacuum, radiance leaving p in the direction of q is the same as radiance arriving at q from p
– hence the units

Radiance is constant along straight lines



- Power 1->2, leaving 1:

$$L(\underline{x}_1,\vartheta,\varphi)(dA_1\cos\vartheta_1)\left(\frac{dA_2\cos\vartheta_2}{r^2}\right)$$

- Power 1->2, arriving at 2:

$$L(\underline{x}_2,\vartheta,\varphi)(dA_2\cos\vartheta_2)\left(\frac{dA_1\cos\vartheta_1}{r^2}\right)$$

Irradiance

- How much light is arriving at a surface?
- Sensible unit is Irradiance
- Incident power per unit area not foreshortened
- This is a function of incoming angle.
- A surface experiencing radiance $L(x,\theta,\phi)$ coming in from $d\omega$ experiences irradiance

$$L(\underline{x},\vartheta,\varphi)\cos\vartheta d\omega$$

$$\int_{\Omega} L(\underline{x},\vartheta,\varphi)\cos\vartheta\sin\vartheta d\vartheta d\varphi$$

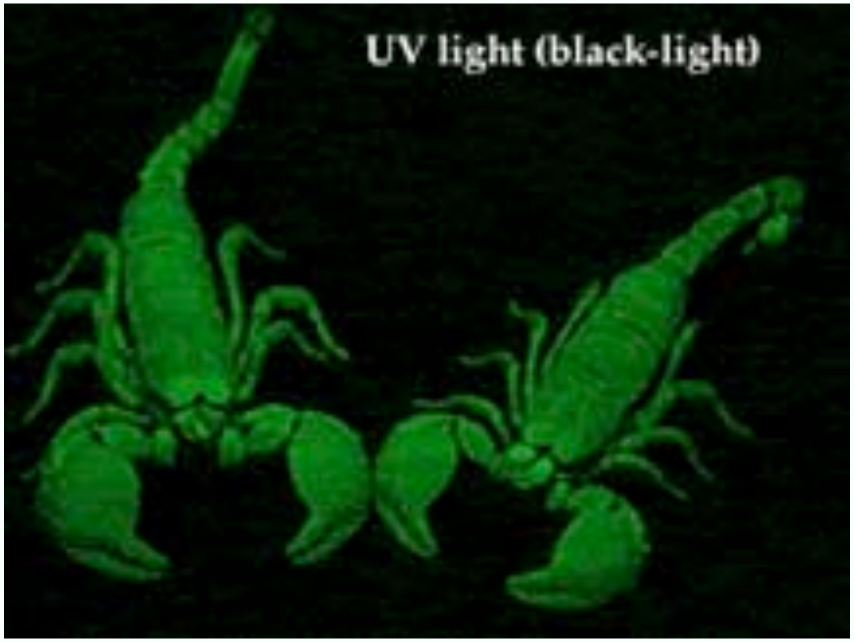
- Crucial property: Total power arriving at the surface is given by adding irradiance over all incoming angles --- this is why it's a natural unit

Surfaces and the BRDF

- Many effects when light strikes a surface -- could be:
  - absorbed; transmitted. reflected; scattered
- Assume that
  - surfaces don't fluoresce
  - surfaces don't emit light (i.e. are cool)
  - all the light leaving a point is due to that arriving at that point
- Can model this situation with the Bidirectional Reflectance Distribution Function (BRDF)
- the ratio of the radiance in the outgoing direction to the incident irradiance

$$\rho_{bd}(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i,) = \frac{L_o(\underline{x},\vartheta_o,\varphi_o)}{L_i(\underline{x},\vartheta_i,\varphi_i)\cos\vartheta_i d\omega}$$


White light (indoor)


UV light (black-light)

BRDF

- Units: inverse steradians (sr-1)
- Symmetric in incoming and outgoing directions
- Radiance leaving in a particular direction:
  - add contributions from every incoming direction

$$\int_{\Omega} \rho_{bd}\left(\underline{x}, \vartheta_o, \varphi_o, \vartheta_i, \varphi_i,\right) L_i\left(\underline{x}, \vartheta_i, \varphi_i\right) \cos\vartheta_i d\omega_i$$

Suppressing Angles - Radiosity

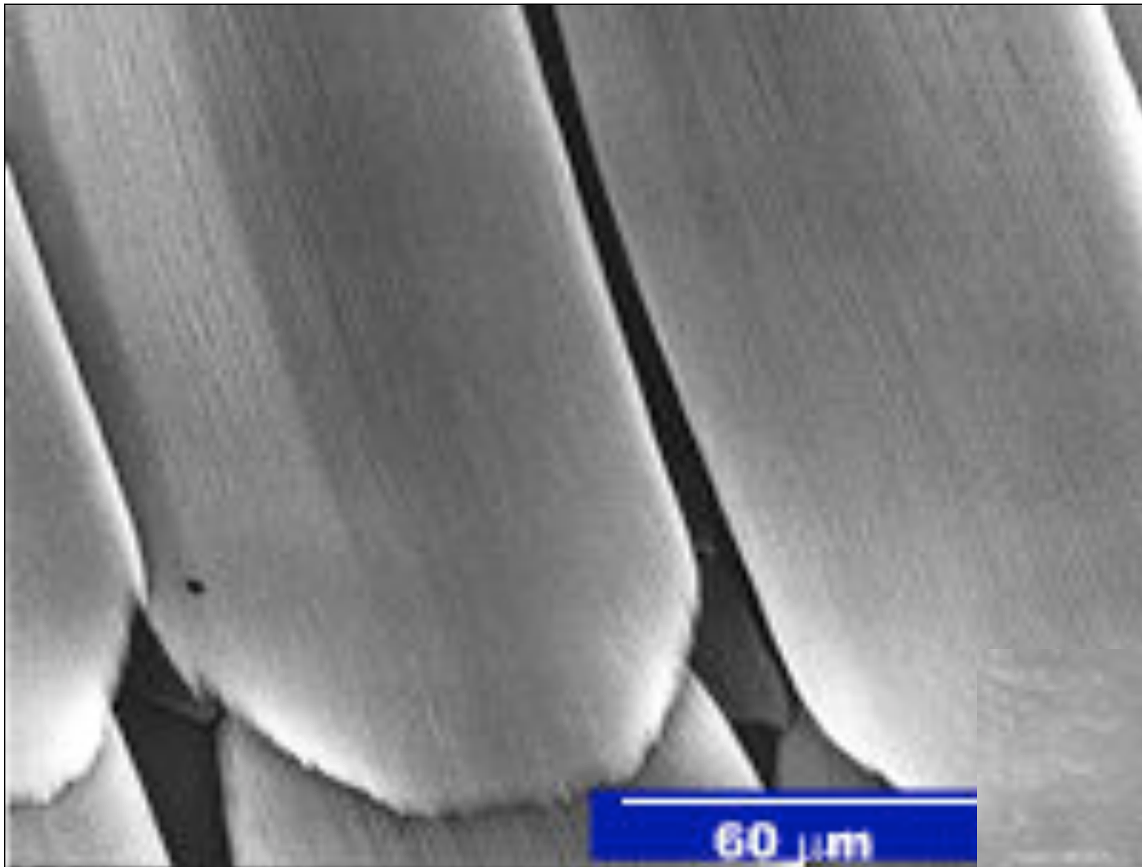
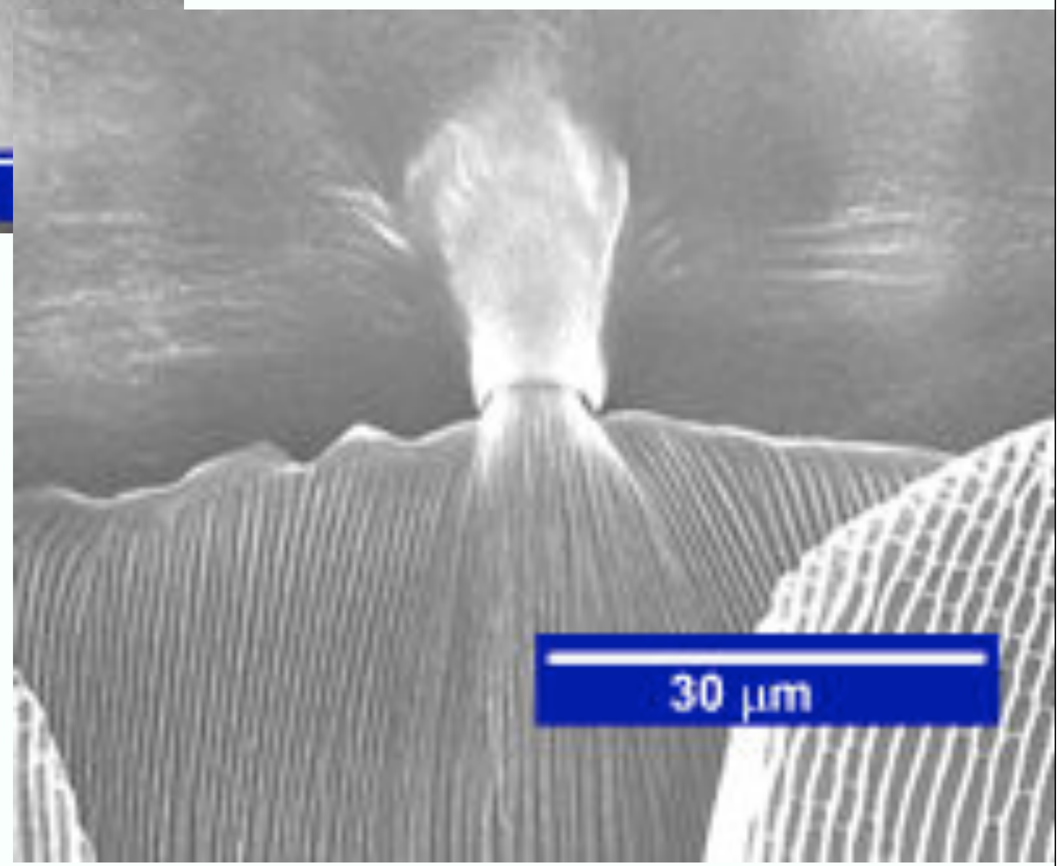
- In many situations, we do not really need angle coordinates
  - e.g. cotton cloth, where the reflected light is not dependent on angle
- Appropriate radiometric unit is radiosity
  - total power leaving a point on the surface, per unit area on the surface (Wm-2)
- Radiosity from radiance?
  - sum radiance leaving surface over all exit directions

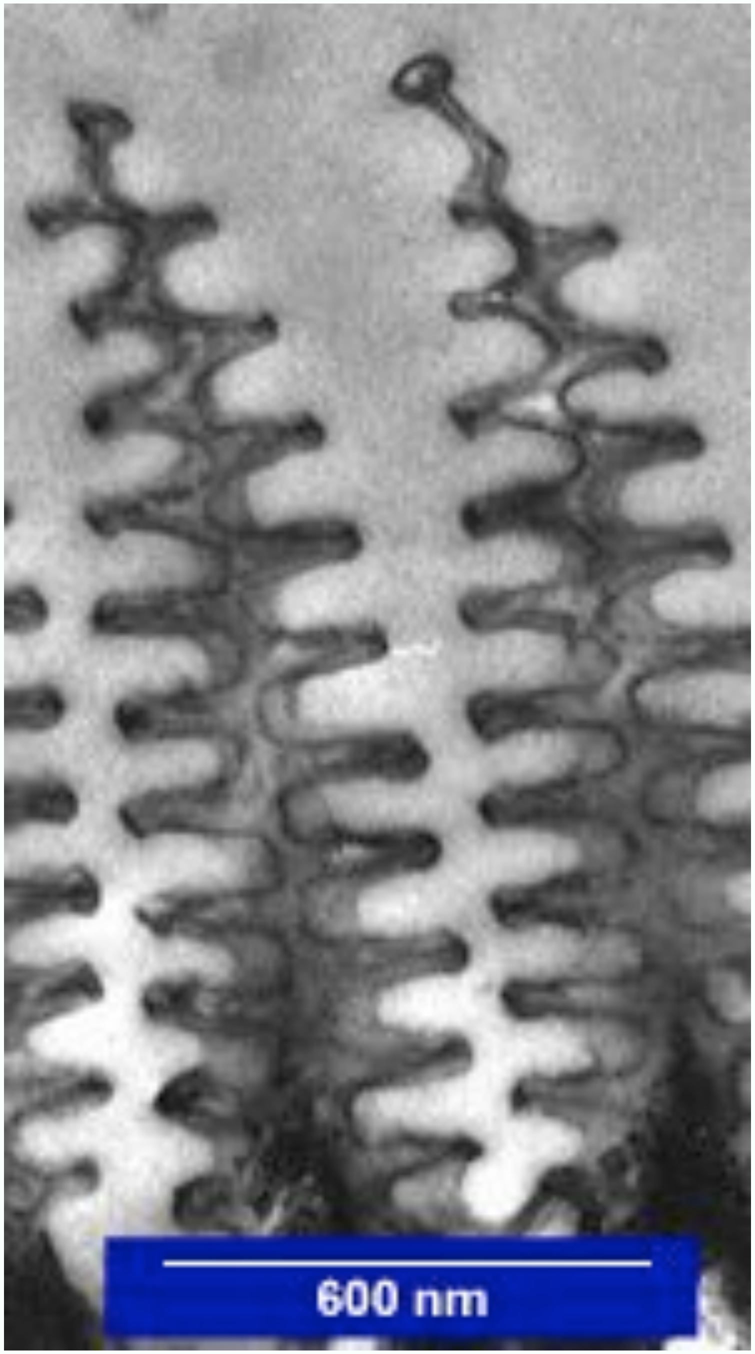
$$B(\underline{x}) = \int_{\Omega} L_o(\underline{x}, \vartheta, \varphi) \cos\vartheta d\omega$$

Radiosity

- Important relationship:
  - radiosity of a surface whose radiance is independent of angle (e.g. that cotton cloth)

$$
\begin{aligned}
B(\underline{x}) &= \int_{\Omega} L_o(\underline{x}, \vartheta, \varphi) \cos\vartheta \, d\omega \\
&= L_o(\underline{x}) \int_{\Omega} \cos\vartheta \, d\omega \\
&= L_o(\underline{x}) \int_0^{\pi/2} \int_0^{2\pi} \cos\vartheta \sin\vartheta \, d\varphi \, d\vartheta \\
&= \pi L_o(\underline{x})
\end{aligned}
$$


60 µm

30 µm


600 nm

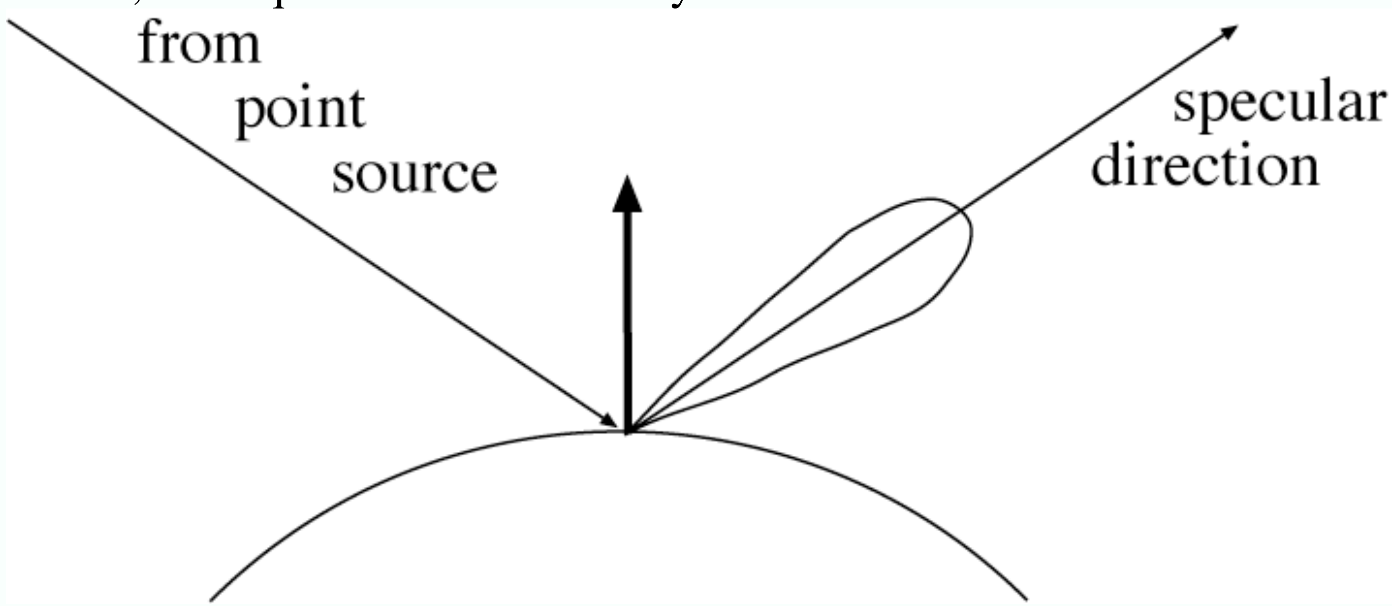
Lambertian surfaces and albedo

- For some surfaces, the BRDF is independent of direction
  - cotton cloth, carpets, matte paper, matte paints, etc.
  - radiance leaving the surface is independent of angle
  - Lambertian surfaces (same Lambert) or ideal diffuse surfaces
  - Use radiosity as a unit to describe light leaving the surface
  - percentage of incident light reflected is diffuse reflectance or albedo
- Useful fact:

$$\rho_{brdf} = \frac{\rho_d}{\pi}$$

Specular surfaces

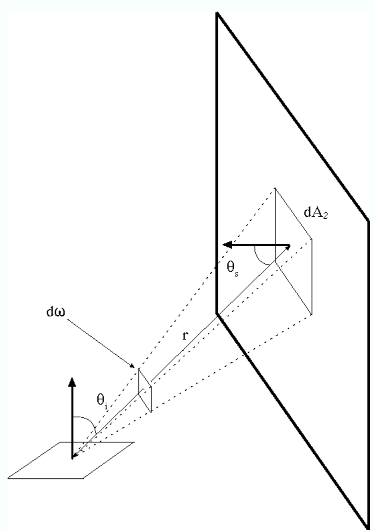
- Another important class of surfaces is specular, or mirror-like.
  - radiation arriving along a direction leaves along the specular direction
  - reflect about normal
  - some fraction is absorbed, some reflected
  - on real surfaces, energy usually goes into a lobe of directions
  - can write a BRDF, but requires the use of funny functions



Radiosity due to an area source

- rho is albedo
- E is exitance
- r(x, u) is distance between points
- u is a coordinate on the source

$$
\begin{aligned}
B(x) &= \rho_d(x)\int_{\Omega} L_i(x, u \rightarrow x)\cos\theta_i d\omega \\
&= \rho_d(x)\int_{\Omega} L_e(x, u \rightarrow x)\cos\theta_i d\omega \\
&= \rho_d(x)\int_{\Omega} \left(\frac{E(u)}{\pi}\right)\cos\theta_i d\omega \\
&= \rho_d(x)\int_{source} \left(\frac{E(u)}{\pi}\right)\cos\theta_i \left(\cos\theta_s \frac{dA_u}{r(x,u)^2}\right) \\
&= \rho_d(x)\int_{source} E(u)\frac{\cos\theta_i \cos\theta_s}{\pi r(x,u)^2} dA_u
\end{aligned}
$$



The Rendering Equation- 1

- We can now write

$$L_o(\mathbf{x}, \omega_o) = L_e(\mathbf{x}, \omega_o) + \int_\Omega \rho_{bd}(\mathbf{x}, \omega_o, \omega_i) L_i(\mathbf{x}, \omega_i) \cos\theta_i d\omega_i$$

Angle between normal and incoming direction

BRDF

Incoming irradiance

Average over hemisphere

Radiance emitted from surface at that point in that direction

Radiance leaving a point in a direction

The Rendering Equation - II

- This balance works for
  - each wavelength,
  - at any time, so
- So

$$
\begin{aligned}
L_o(\mathbf{x}, \omega_o, \lambda, t) \quad = \quad & L_e(\mathbf{x}, \omega_o, \lambda, t) + \\
& \int_\Omega \rho_{bd}(\mathbf{x}, \omega_o, \omega_i, \lambda, t) L_i(\mathbf{x}, \omega_i, \lambda, t) \cos\theta_i d\omega_i
\end{aligned}
$$