## Cameras

CS-543, D.A. Forsyth

## Cameras

- First photograph due to Niepce
- First on record, 1822
- Key abstraction
- Pinhole camera



## Pinhole camera




Freestanding room-sized camera obscura outside Hanes Art Center at the University of North Carolina at Chapel Hill. Picture taken by User:Seth llys on 23 April 2005 and released into the public domain.


A photo of the Camera Obscura in San Francisco. This Camera Obscura is located at the Cliff House on the Pacific ocean. Credit to Jacob Appelbaum of http://www.appelbaum.net.

## Distant objects are smaller in a pinhole camera



## Parallel lines meet in a pinhole camera

П

## Vanishing points

- Each set of parallel lines meets at a different point
- The vanishing point for this direction
- Coplanar sets of parallel lines have a horizon
- The vanishing points lie on a line
- Good way to spot faked images






Camera obscura - aus einer franz. "Encyclopédie, ou dictionnaire raisonné des sciences, des arts et des métiers" von 1772
Public Domain

## Projection in Coordinates

- From the drawing, we have $\mathrm{X} / \mathrm{Z}=-\mathrm{x} / \mathrm{f}$
- Generally



## Homogeneous coordinates

- Add an extra coordinate and use an equivalence relation
- for 2D
- three coordinates for point
- equivalence relation $\mathrm{k}^{*}(\mathrm{X}, \mathrm{Y}, \mathrm{Z})$ is the same as $(\mathrm{X}, \mathrm{Y}, \mathrm{Z})$
- for 3D
- four coordinates for point
- equivalence relation $\mathrm{k}^{*}(\mathrm{X}, \mathrm{Y}, \mathrm{Z}, \mathrm{T})$ is the same as $(\mathrm{X}, \mathrm{Y}, \mathrm{Z}, \mathrm{T})$
- Canonical representation
- by dividing by one coordinate (if it isn't zero).


## Homogeneous coordinates

- Why?
- Possible to represent points "at infinity"
- Where parallel lines intersect (vanishing points)
- Where parallel planes intersect (horizons)
- Possible to write the action of a perspective camera as a matrix


## A perspective camera as a matrix

- Turn previous expression into HC's
- HC's for 3D point are (X,Y,Z,T)
- HC's for point in image are (U,V,W)

$$
\left(\begin{array}{c}
U \\
V \\
W
\end{array}\right)=\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \frac{1}{f} & 0
\end{array}\right)\left(\begin{array}{c}
X \\
Y \\
Z \\
T
\end{array}\right)
$$

## A general perspective camera - I

- Can place a perspective camera at the origin, then rotate and translate coordinate system
- In homogeneous coordinates, rotation, translation are:

$$
\mathcal{E}=\left(\begin{array}{cc}
\mathcal{R} & \mathrm{t} \\
\mathbf{0} & 1
\end{array}\right)
$$

- So rotated, translated camera is:


## A general perspective camera - II

- In the camera plane, there can be a change of coordinates
- choice of origin
- there is a "natural" origin --- the camera center
- where the perpendicular passing through the focal point hits the image plane
- rotation
- pixels may not be square
- scale
- Camera becomes

MCE

Intrinsics - typically come with the camera

Extrinsics - change when you move around

## What are the transforms?

$$
\left(\begin{array}{c}
U \\
V \\
W
\end{array}\right)=\left(\begin{array}{c}
\text { Transform } \\
\text { representing } \\
\text { intrinsic parameters }
\end{array}\right)\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{array}\right)\left(\begin{array}{c}
\text { Transform } \\
\text { representing } \\
\text { extrinsic parameters }
\end{array}\right)\left(\begin{array}{c}
X \\
Y \\
Z \\
T
\end{array}\right)
$$

## Camera Calibration

- Issues:
- what is the camera matrix? (including intrinsic and extrinsic)
- what are intrinsic parameters of the camera?
- General strategy:
- view calibration object
- identify image points
- obtain camera matrix by minimizing error
- obtain intrinsic parameters from camera matrix
- Error minimization:
- Linear least squares
- easy problem numerically, solution can be rather bad
- Minimize image distance
- more difficult numerical problem, solution is better


## Problem: Vanishing points

- Lines in world coordinates: $\mathbf{u}+t \mathbf{v}$
- Camera: MCE
- Vanishing point in camera coordinates?


## Weak perspective

- Issue
- perspective effects, but not over the scale of individual objects
- For example, texture elements in picture below
- collect points into a group at about the same depth, then divide each point by the depth of its group
- Adv: easy, useful when depth range is small
- Disadv: wrong when depth range is large



## Orthographic projection

- Perspective effects are often not significant
- eg
- pictures of people
- all objects at the same distance



## Orthographic projection in HC's

- In conventional coordinates, we just drop z
- In Homogeneous coordinates, can write a matrix

$$
\left(\begin{array}{c}
U \\
V \\
W
\end{array}\right)=\left(\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right)\left(\begin{array}{l}
X \\
Y \\
Z \\
T
\end{array}\right)
$$

## Calibration and orthographic cameras

- Some parameters can't be estimated
- translation of camera perpendicular to image plane
- Intrinsics slightly different:
- no "natural" origin in the image plane


## Pinhole Problems

Pinhole too big: brighter, but blurred

Pinhole right size: crisp, but dark


## Lens Systems

- Collect light from a large range of directions



## Lens Systems

- Collect light from a large range of directions



## A lens model - the thin lens



$$
\frac{1}{z^{\prime}}-\frac{1}{z}=\frac{1}{f}
$$

## Lens Problems

- Chromatic aberration
- Light at different wavelengths follows different paths; hence, some wavelengths are defocussed
- Machines: coat the lens
- Humans: live with it
- Scattering at the lens surface
- Some light entering the lens system is reflected off each surface it encounters (Fresnel's law gives details)
- Machines: coat the lens, interior
- Humans: live with it (various scattering phenomena are visible in the human eye)
- Geometric phenomena (Barrel distortion, etc.)


## Lens Problems - Spherical Aberration



## Lens Systems

## $((8)),) \cdot(\theta) \| \cdot \theta \pi^{\prime} \theta$

## Vignetting



## Geometric properties of projection

- Points -> points
- Lines -> lines
- Polyhedra -> polyhedra
- Degeneracies
- line through focal point (pinhole) $->$ point
- plane through focal point (pinhole) $->$ line
- Curved surfaces are complicated



## Polyhedra project to polygons

- because lines project to lines, etc



## Junctions are constrained

- Which leads to a process called line labelling
- look for consistent junction, edge labels
- BUT can't get real lines, junctions from real images



## Curved surfaces are more interesting

- Outline
- set of points where view direction is tangent to surface
- projection of a space curve which varies from view to view of a surface



Panagis Alexatos, by Jim Childs

