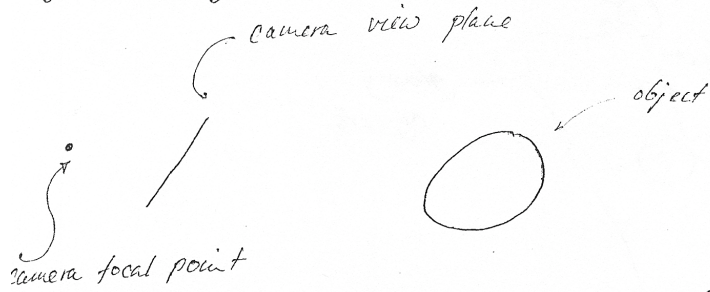


Ray Tracing: - (general ideas)



intensity at pixel  $(i,j)$  = intensity at closest  $\cap$  along ray from fp through  $(i,j)$

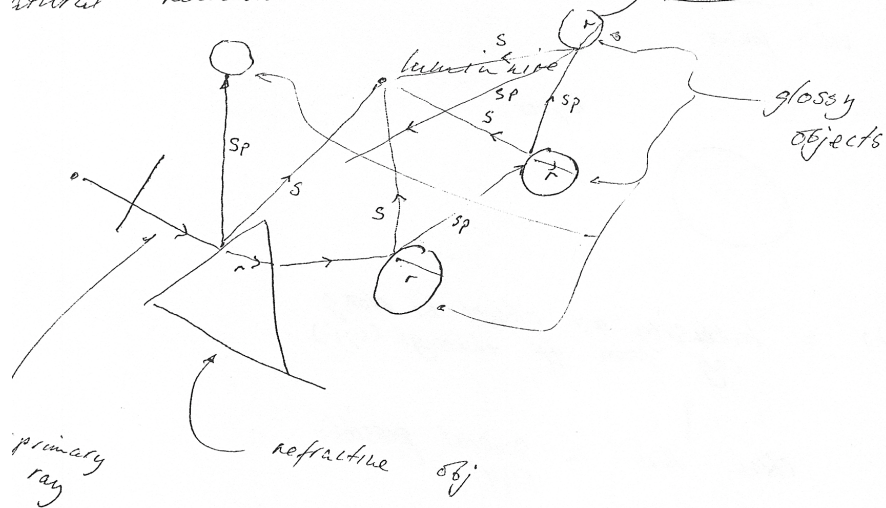
obtain this in several possible ways

- Local Shading:
- only luminaires contribute?
  - can intersection see luminaires?
    - test this by checking line segments for intersections

- Propagation:
- does ray propagate?
    - specular reflection
    - refraction
  - test by spawning new ray.

- Global Shading:
- intensity at distant surfaces contributes as well
  - how much?
    - interrogate by spawning rays

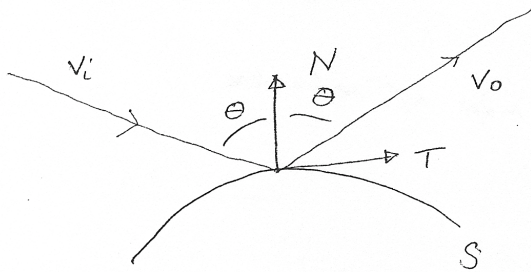
Structural recursion leads to a ray tree



core issues

- Mechanics of reflection, refraction
- Details of shading computation
- Ray intersection
  - Object representation
- Managing the ray tree.

## Specular reflection



- Standard specular surface
- $N$  is unit normal
- $T$  is tangent vector in plane of  $(N, V_i)$

$$V_o \cdot N = -V_i \cdot N$$

$$V_o \cdot T = V_i \cdot T$$

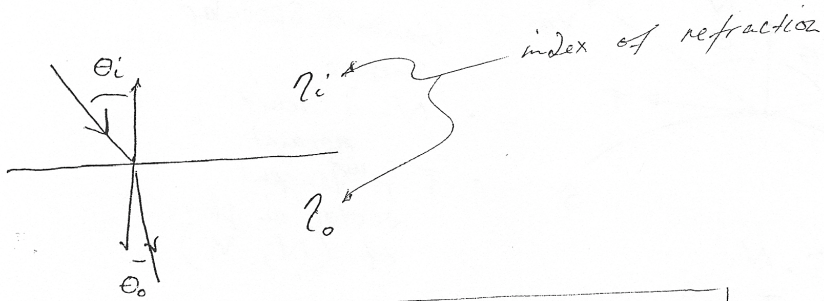
$$(V_o \cdot T)T = V_i - (V_i \cdot N)N$$

$$\begin{aligned} \therefore V_o &= (V_o \cdot N)N + (V_o \cdot T)T \\ &= \boxed{V_i - 2(V_i \cdot N)N} \end{aligned}$$

Notice that some special surfaces have off-specular glints  
- different calc of spawned rays, otherwise no change.

refraction

change in ray direction due to change in speed of light

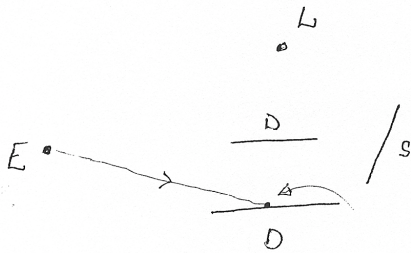


SNELLS LAW

$$n_i \sin \theta_i = n_o \sin \theta_o$$

rendering details:

- Watch this space!
- for the moment, use what is familiar to compute shading values
- But this isn't good enough (lots of reasons)



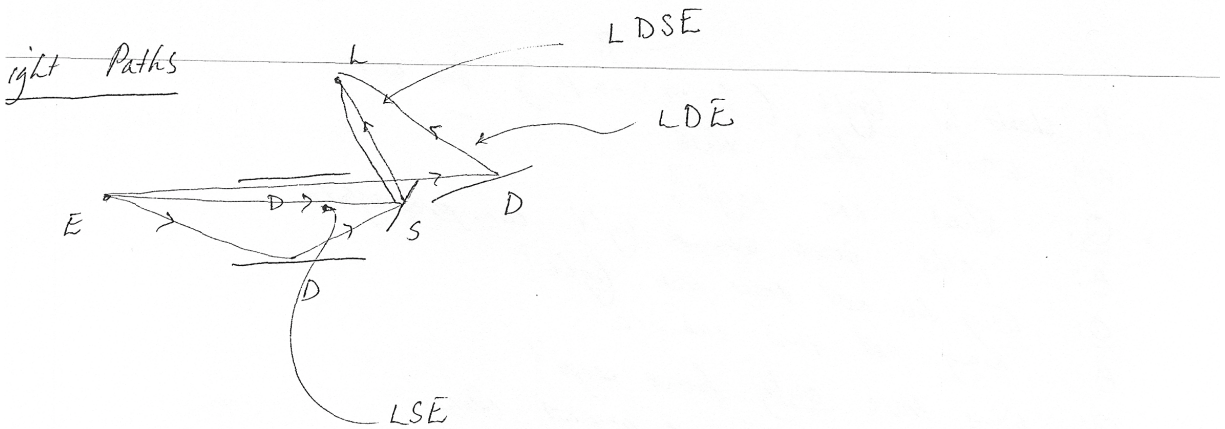
- 
- $P_i$  should be bright ( $L \rightarrow S \rightarrow P_i$ ) but our alg won't find this
  - Q: What are rays?
  - A: paths down which light can propagate
  - Q: Why ~~tra~~ not trace from light?
  - A: may not find camera
  - Q: Why trace only from eye?
  - A: actually it isn't a great idea.
- Specular to Diffuse transfer

strategy:

- ① trace rays from light  $\rightarrow$  first diffuse  
(i.e. L-S-S-S... D)
- 2 and make a record
- ② trace rays from eye  
- do what has been described  
- also sample record.

tips:

- once we've discussed aliasing



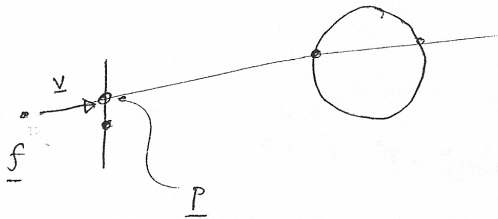
etc:

<u>general path</u>	$L \{S \{D \{S^* \}^* \} E$	} these three don't cover all alternatives !
<u>eyerays</u>	$L D S^* E$	
<u>light rays</u>	$L S^* D E$	
<u>bidirectional</u>	$L S^* D S^* E$	

## Ray Intersection

• Sphere.  $(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 - r^2 = 0$

(Implicit Surface)



- Point on ray  $\underline{f} + t\underline{v}$ ,  $t \geq 0$
- this point is on the sphere if eqn above vanishes
- eqn  $(\underline{x} - \underline{x}_c) \cdot (\underline{x} - \underline{x}_c) - r^2 = 0$

Substitute

$$(\underline{f} + t\underline{v} - \underline{x}_c) \cdot (\underline{f} + t\underline{v} - \underline{x}_c) - r^2 = 0$$

$$t^2 (\underline{v} \cdot \underline{v}) + t (2\underline{v} \cdot (\underline{f} - \underline{x}_c)) + [(\underline{f} - \underline{x}_c) \cdot (\underline{f} - \underline{x}_c) - r^2] = 0$$

quadratic eqn.

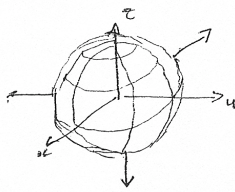
Q: What is geometric meaning of

$$\text{disc} \begin{cases} < \\ = \\ > \end{cases} 0 \quad ?$$

• General Quadratic form

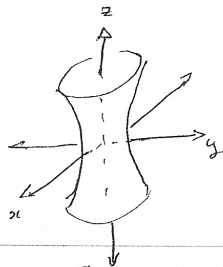
• includes spheres, ellipsoids, hyperboloids of revolution, etc

• eg.



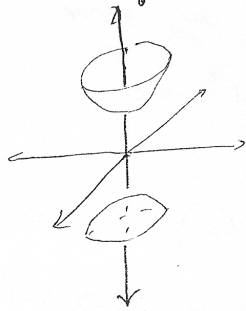
$$x^2 + y^2 + z^2 - r^2 = 0$$

• eg



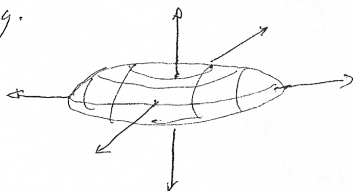
$$x^2 + y^2 - z^2 - r^2 = 0$$

• eg



$$z^2 - (x^2 + y^2) = r^2$$

• eg.



$$\lambda_1 x^2 + \lambda_2 y^2 + \lambda_3 z^2 - 1 = 0$$



we can compare via of maximum ... of locate roots

File:

- coefficients can grow fast (try some eq.'s w/ Mathematica)
- One SS. per fixel?

numerical root finding

- to polish root
- to exploit coherence

Interval halving

- if we know there is one <sup>isolated</sup> root in  $[a, b]$ , we can use sign of  $Q(t)$ , split intervals

- very robust
- relatively slow

Newton's method

$$Q(t_i + \delta t) \approx Q(t_i) + \delta t \cdot \left. \frac{dQ}{dt} \right|_{t=t_i}$$

now if  $t_i + \delta t$  is a root,

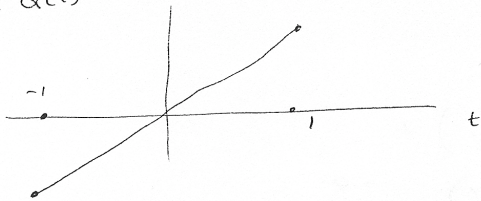
$$\text{then } \delta t = \frac{-Q(t_i)}{\left. \frac{dQ}{dt} \right|_{t=t_i}}$$

Iteration

$$t_{i+1} = t_i + \left[ \frac{-Q(t_i)}{\left. \frac{dQ}{dt} \right|_{t=t_i}} \right]$$

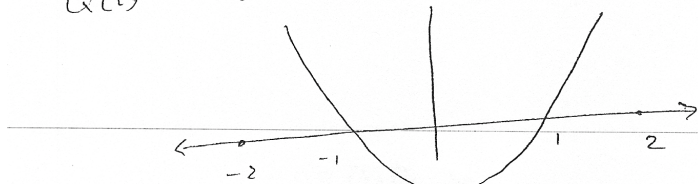
Examples:

~~Q(t) = t~~  $Q(t) = t$



$Q(-1) = -1$  ;  $Q(1) = 1$   
 $\Rightarrow$  must be a root  $\in [-1, 1]$

$Q(t) = t^2 - 1$



$Q(-2) = 3$   
 $Q(2) = 3$

is there a root?

look at  $\frac{dQ}{dt} = 2t$

$\frac{dQ}{dt}(-2) = -4 \therefore$  falling

$\frac{dQ}{dt}(2) = 4 \therefore$  rising

Q: does it pass through 0?  
A:  $\frac{d^2Q}{dt^2}$  could tell us.

$$\underline{x}' A \underline{x} + \underline{b}' \underline{x} + c = 0$$

↑ symmetric

root finding on

$$t^2 (\underline{v}' A \underline{v}) + t (2 \underline{f}' A \underline{v} + \underline{b}' \underline{v}) + (\underline{f}' A \underline{f} + \underline{b}' \underline{f} + c) = 0$$

again, straight forward.

More general algebraic surface:

$$p(x, y, z) = 0$$

↑ polynomial.

$Q(t) = 0$ , by substitution

Problem 1: find smallest  $t$ ,  $t > 0$ , st.  $Q(t) = 0$   
(primary ray)

Problem 2: is there a  $0 < t < 1$  such that  $Q(t) = 0$   
(shadow cast).

Root finding techniques:

Exact (Sturm seqs, etc)

Numerical (Newton's method).

Coherence

$$\frac{t^2-1}{f_0: t^2-1}$$

$$f_1: 2t$$

$$f_2: 2$$

(because  $tf_1 - 2 = 2f_0$ )

$$\therefore \begin{matrix} + \\ - \\ + \end{matrix}$$

$$V(-2) = 2$$

$$2: \begin{matrix} + \\ + \\ + \end{matrix}$$

$$V(2) = 0$$

$\therefore$  2 distinct roots

$$\frac{t^3-t}{f_0: t^3-t}$$

$$f_1: 3t^2-1$$

$$f_2: 2t$$

$$f_3: 2$$

(because  $tf_1 - 2t = 3f_0$ )

(because  $3tf_2 - 2 = 2f_1$ )

$$\therefore \begin{matrix} - \\ + \\ - \\ + \end{matrix}$$

$$V(-2) = 3$$

$$2: \begin{matrix} + \\ + \\ + \\ + \end{matrix}$$

$$V(2) = 0$$

$\therefore$  3 distinct roots

$$\frac{t^4-t^2}{f_0: t^4-t^2}$$

$$f_1: 4t^3-2t$$

$$f_2: 2t^2$$

$$f_3: 2t$$

$$f_4: 0$$

(because  $tf_1 - 2t^2 = 4f_0$ )

(because  $2tf_2 - 2t = f_1$ )

(because  $tf_3 = f_2$ )

$$2: \begin{matrix} - \\ + \\ - \\ + \end{matrix}$$

$$V(-2) = 3$$

$$2: \begin{matrix} + \\ + \\ + \\ + \end{matrix}$$

$$V(2) = 0$$

$\therefore$  3 distinct roots.

turn 27.

$$f_0(t) = Q(t)$$

$$f_1(t) = \frac{dQ}{dt}$$

$$f_{i+1}(t) = -f_{i-1} \text{ mod } f_i$$

↳ remainder from division alg,  
~~keeping root~~

$$f_{s+1} = 0$$

Define  $V(t_0, f_1 \dots f_s; a)$  to be the number of changes in  $\uparrow$  sign of:

$$f_0(a)$$

$$f_1(a)$$

$$f_2(a)$$

⋮

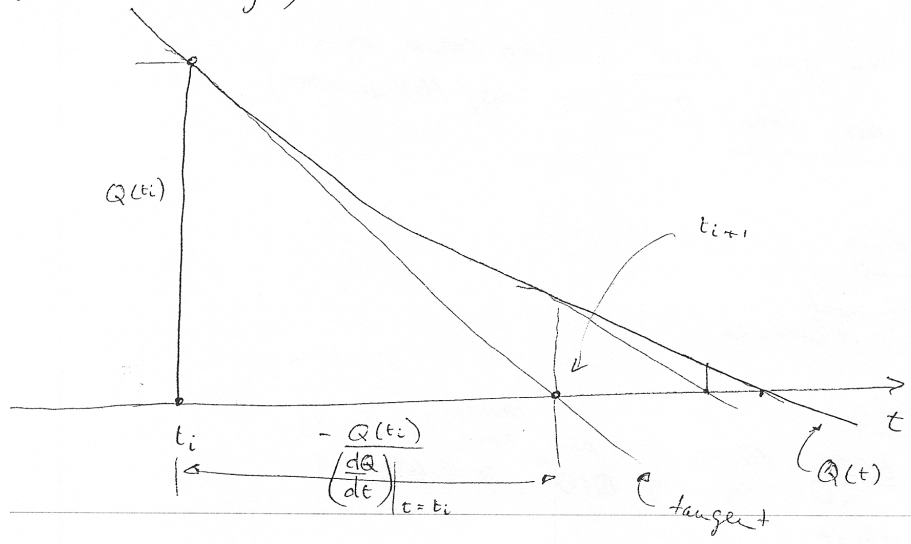
$$f_s(a)$$

(See example)

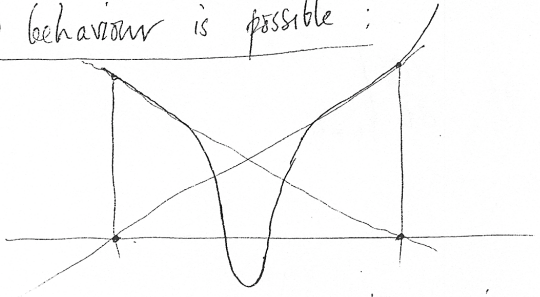
$$\# \text{ of } \wedge \text{ DISTINCT roots in } [a, b] = V(t_0, f_1, f_2 \dots f_s; a) - V(t_0, f_1, f_2, f_s; b)$$

(as long as  $a, b$  are NOT roots)

is method is (but can diverge)



weird behaviour is possible:



- but not common if t is close to root.

we can compare via of maximum ... of locate roots

File:

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