Antialiasing

CS 319

Advanced Topics in Computer Graphics

John C. Hart

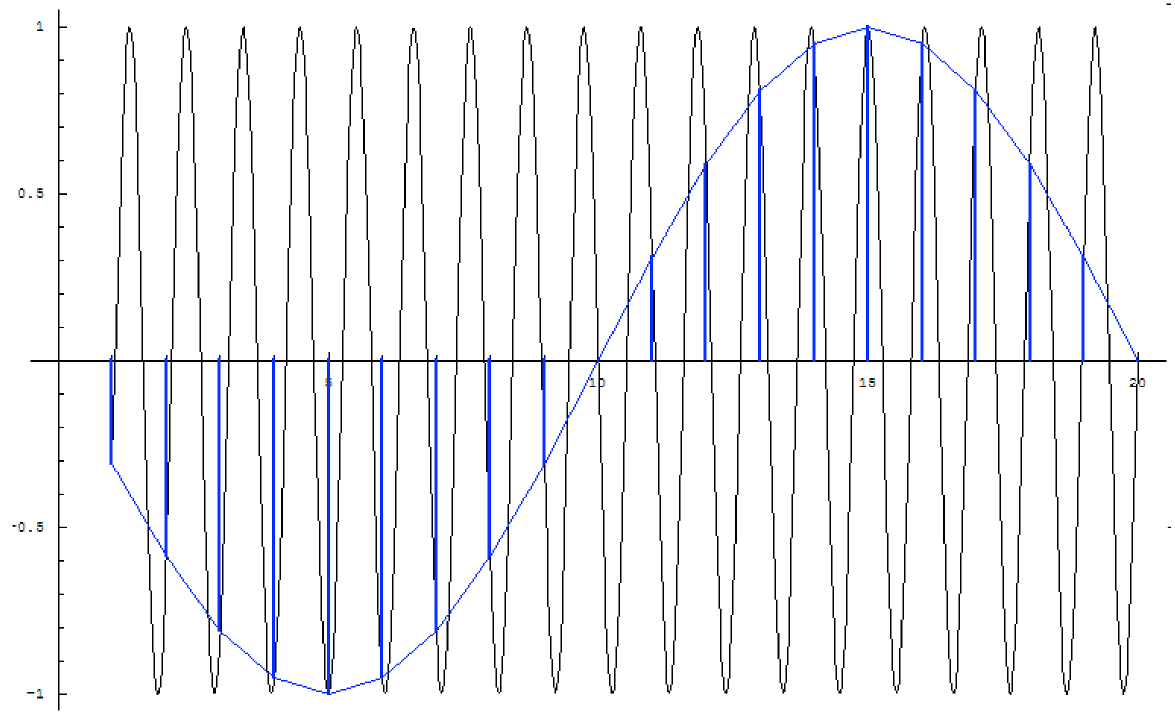
Aliasing

- Aliasing occurs when signals are sampled too infrequently, giving the illusion of a lower frequency signal
- **alias** *noun* (c. 1605) an assumed or additional name

$f(\mathrm{t}) = \sin 1.9\pi t$

- Plotted for $t \in [1,20]$
- Sampled at integer t
- Reconstructed signal appears to be

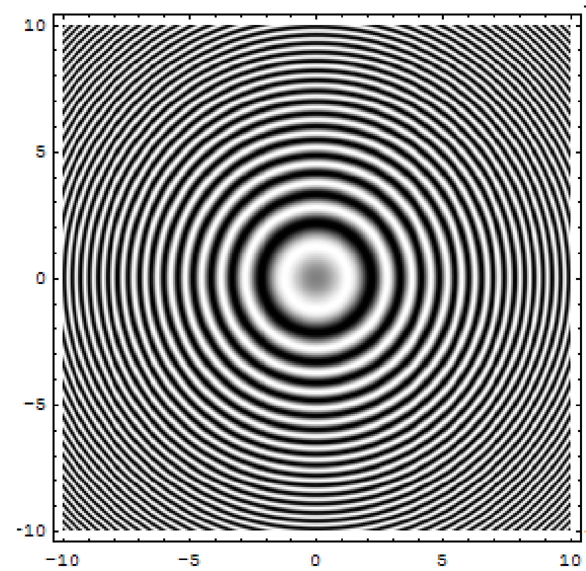
$f(\mathrm{t}) = \sin 0.1\pi t$

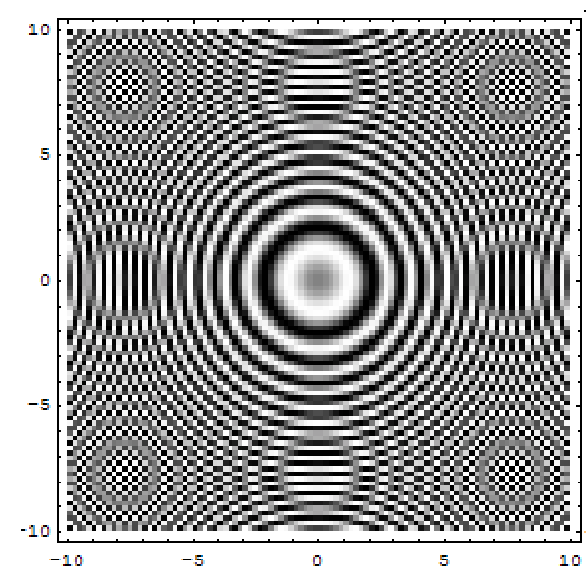


Zone Plate

$$f(x,y) = \sin(x^2 + y^2)$$

- Gray-level plot above
  - Evaluated over $[-10,10]^2$
  - 1000×1000 samples (more than needed)
  - Frequency = $(x^2 + y^2)/2\pi$
  - About 30Hz (cycles per unit length) in the corners
- Poorly sampled version below
  - Only 100×100 samples
  - Moire patterns in higher frequency areas
  - Moire patterns resemble center of zone plate
  - Low frequency features replicated in under-sampled high frequency regions





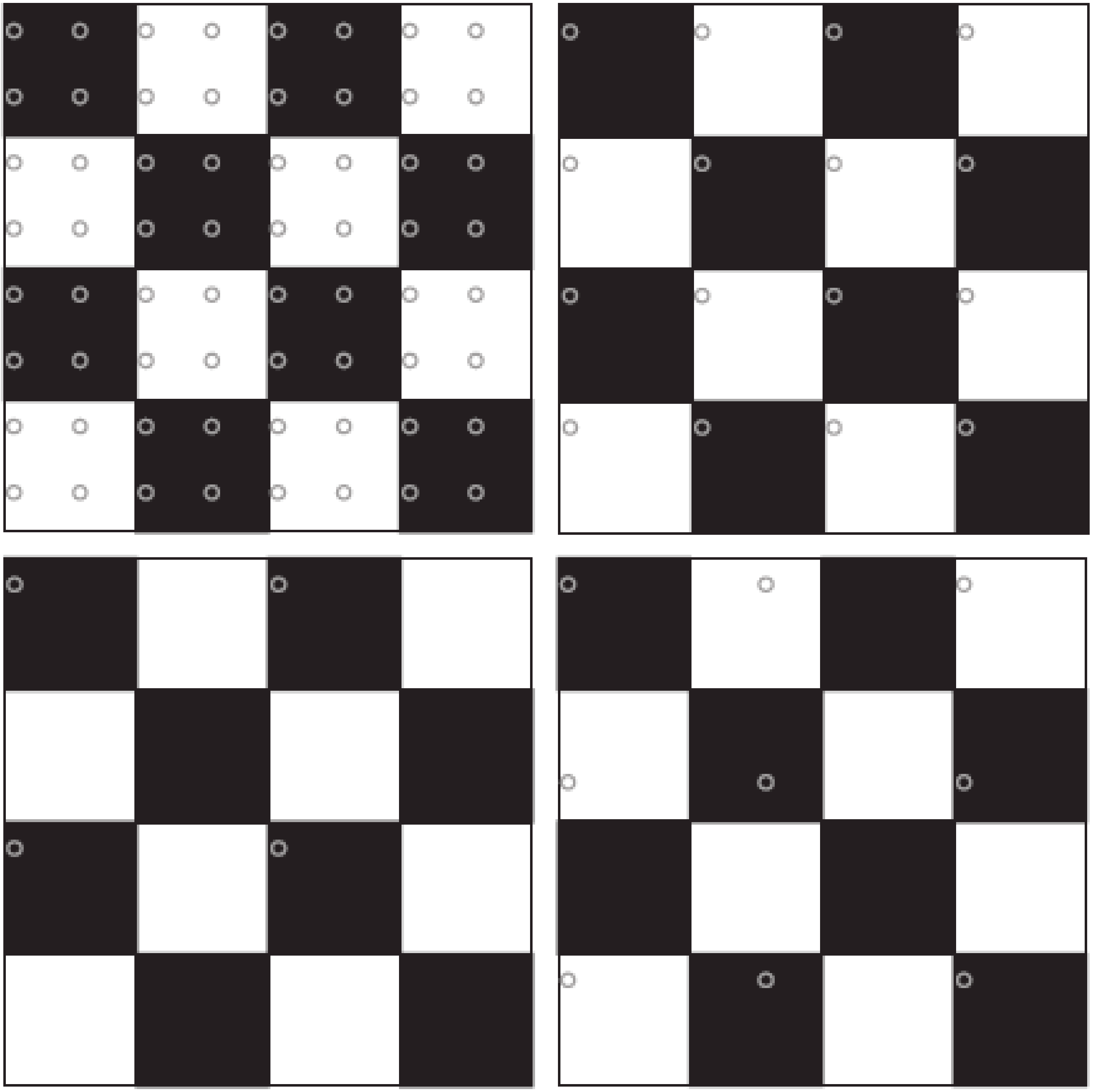
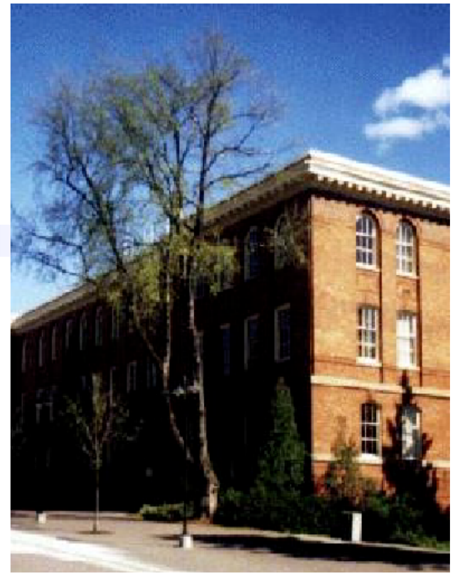


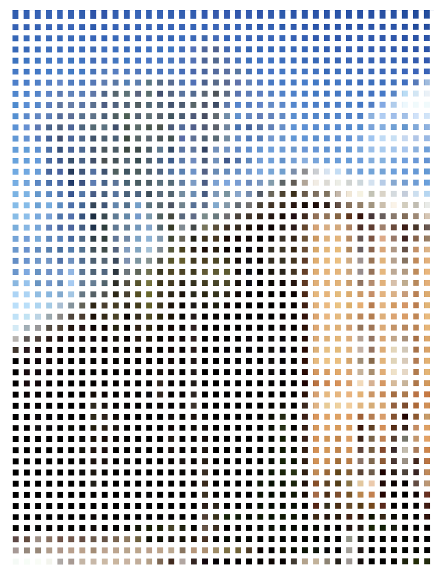
Image Functions

- Analog image
  - 2-D region of varying color
  - Continuous
  - e.g. optical image

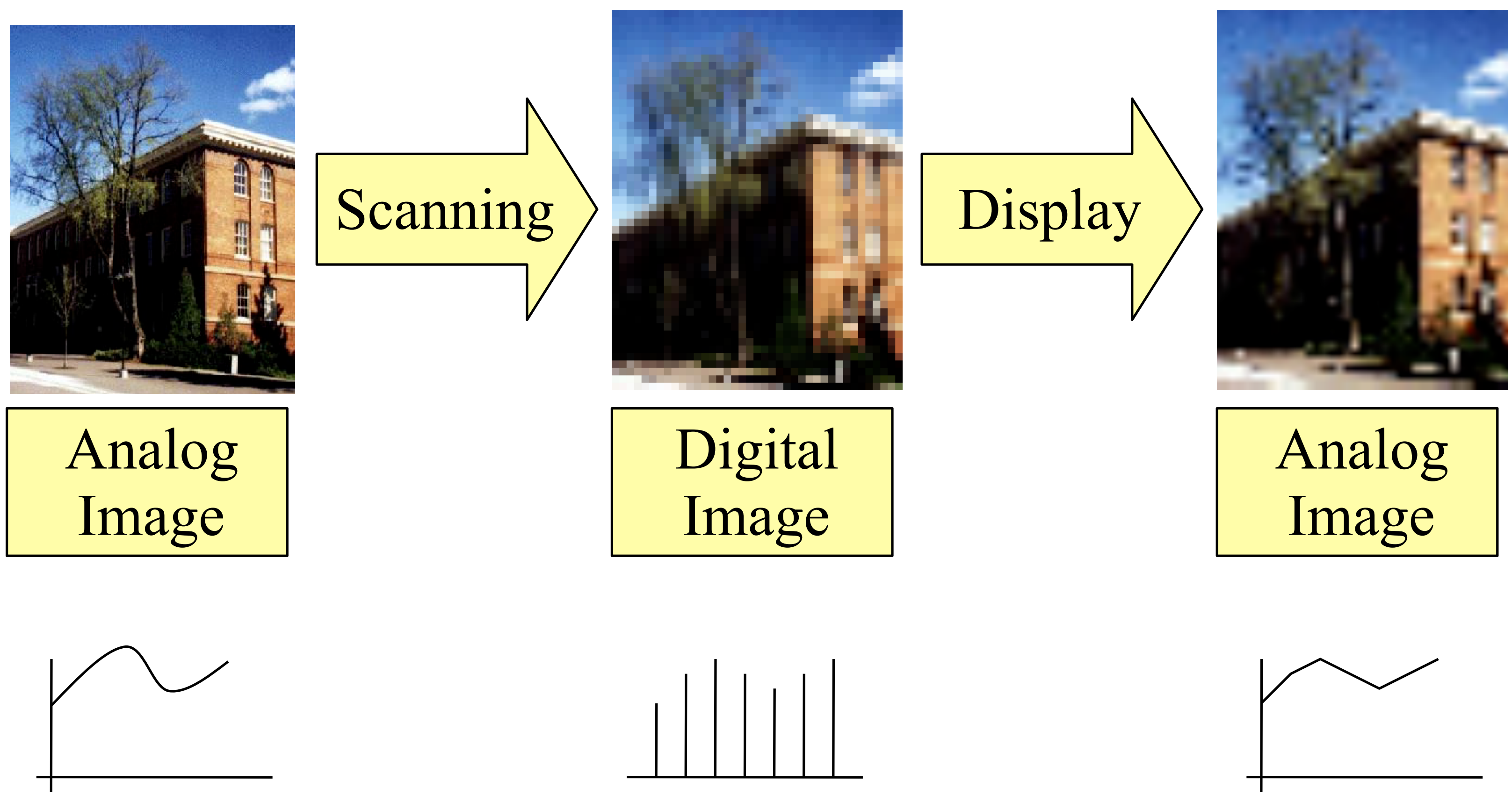
- Symbolic image
  - Any function of two real variables
  - Continuous
  - e.g. $\sin(x^2 + y^2)$, theoretical rendering

- Digital image
  - 2-D array of uniformly spaced color "pixel" values
  - Discrete
  - e.g. framebuffer

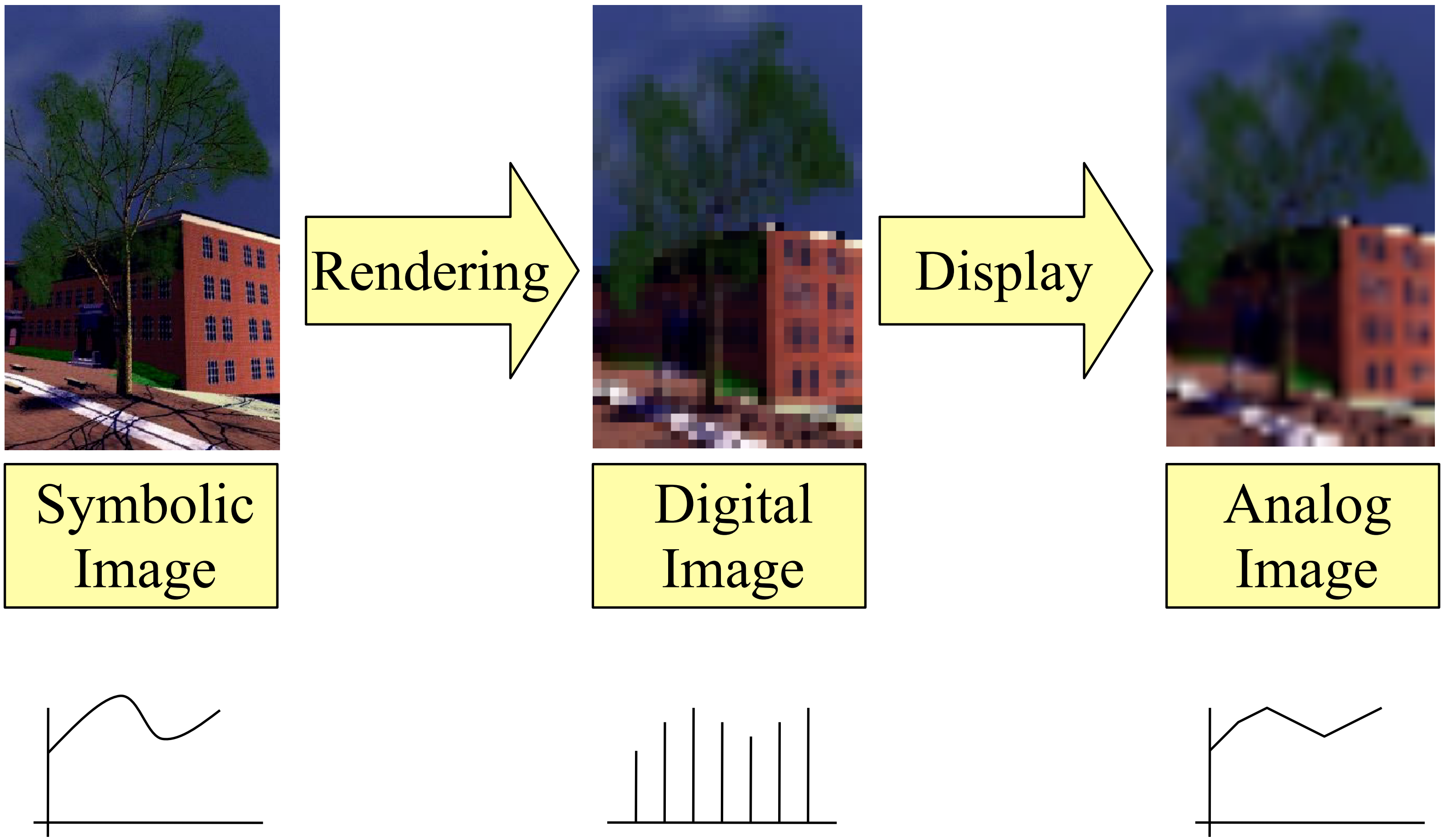




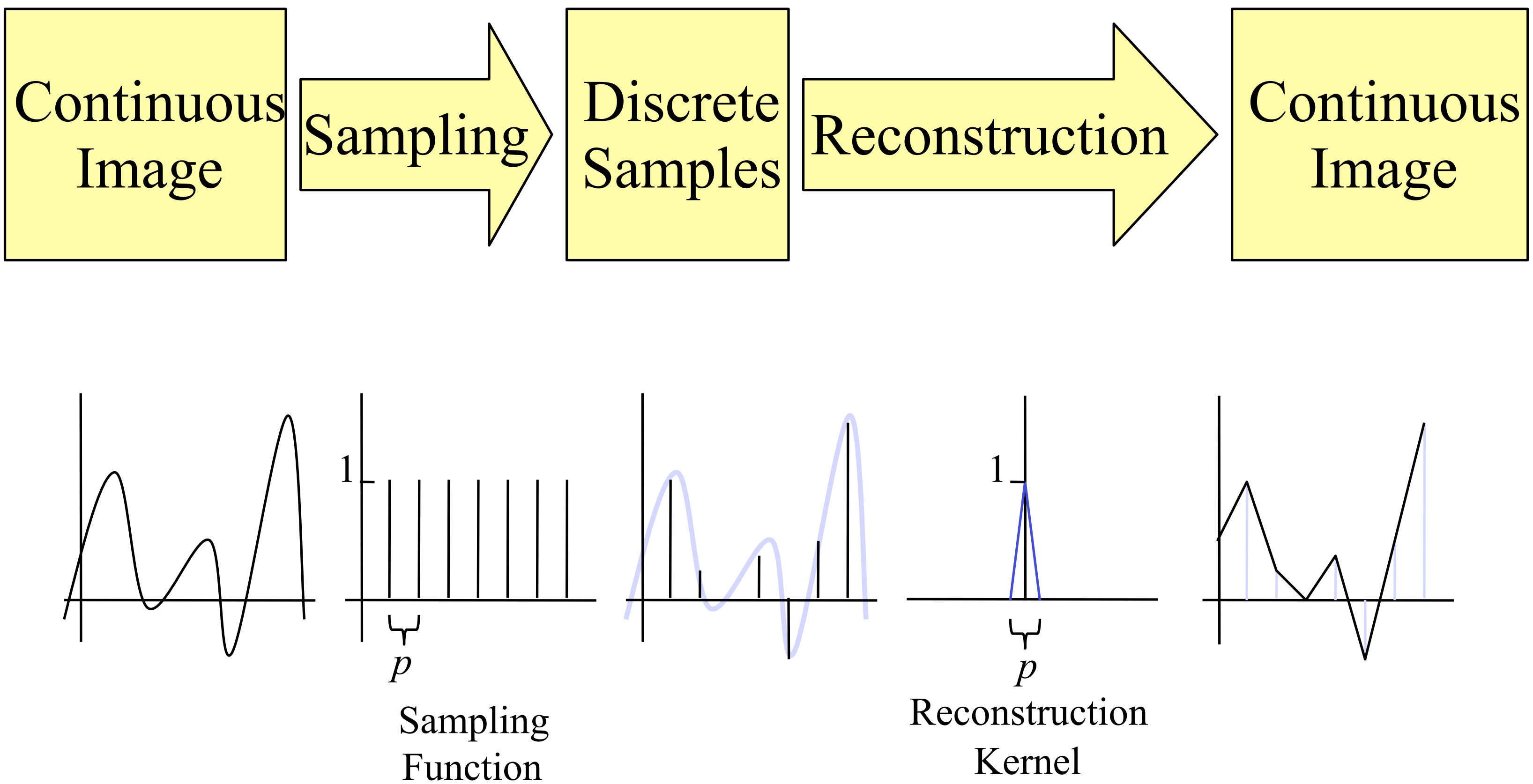
Photography



Graphics


Rendering
Display
Symbolic Image
Digital Image
Analog Image

Sampling and Reconstruction



1-D Fourier Transform

- Makes any signal $I(x)$ out of sine waves
- Converts spatial domain into frequency domain
- Yields spectrum $F(u)$ of frequencies u
  - u is actually complex
  - Only worried about amplitude: $|u|$
- DC term: $F(0)$ = mean $I(x)$
- Symmetry: $F(-u) = F(u)$

$$F(u) = \frac{1}{2\pi}\int I(x)\exp(-jux)\,dx$$

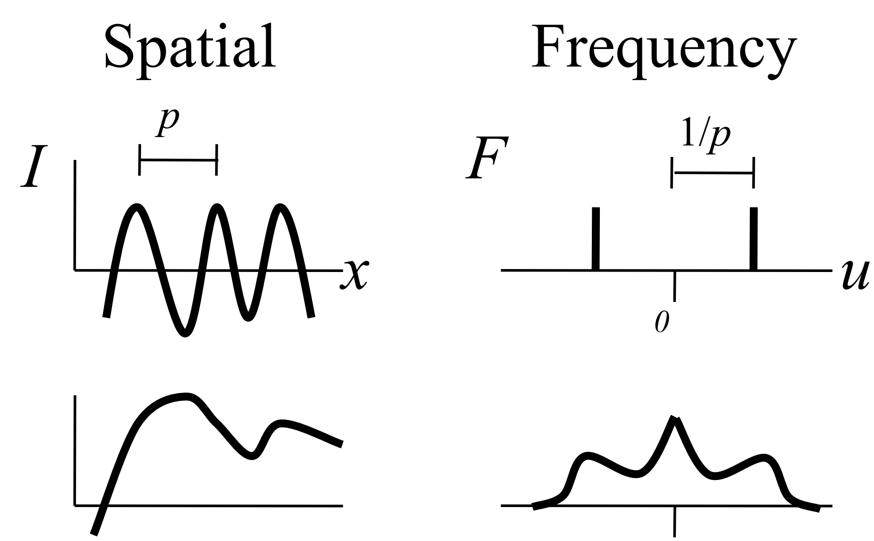
$$I(x) = \frac{1}{2\pi}\int F(u)\exp(jux)\,du$$

$$j^2 = -1$$

$$\exp(-jux) = \cos ux - j\sin ux$$

$$|a + bj| = \sqrt{a^2 + b^2}$$

$$\arg a + bj = \arctan(b/a)$$

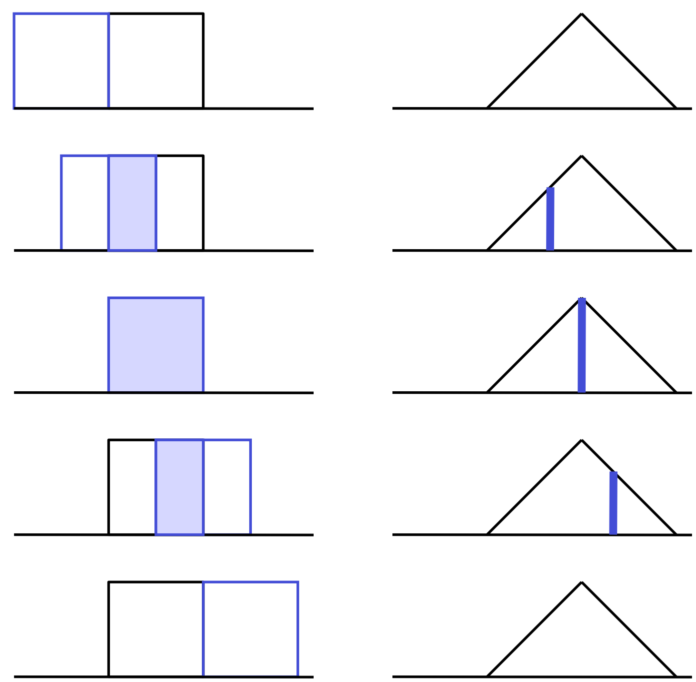


Product and Convolution

- Product of two functions is just their product at each point
- Convolution is the sum of products of one function at a point and the other function at all other points
- E.g. Convolution of square wave with square wave yields triangle wave
- Convolution in spatial domain is product in frequency domain, and vice versa
  - f*g ⇔ FG
  - fg ⇔ F*G

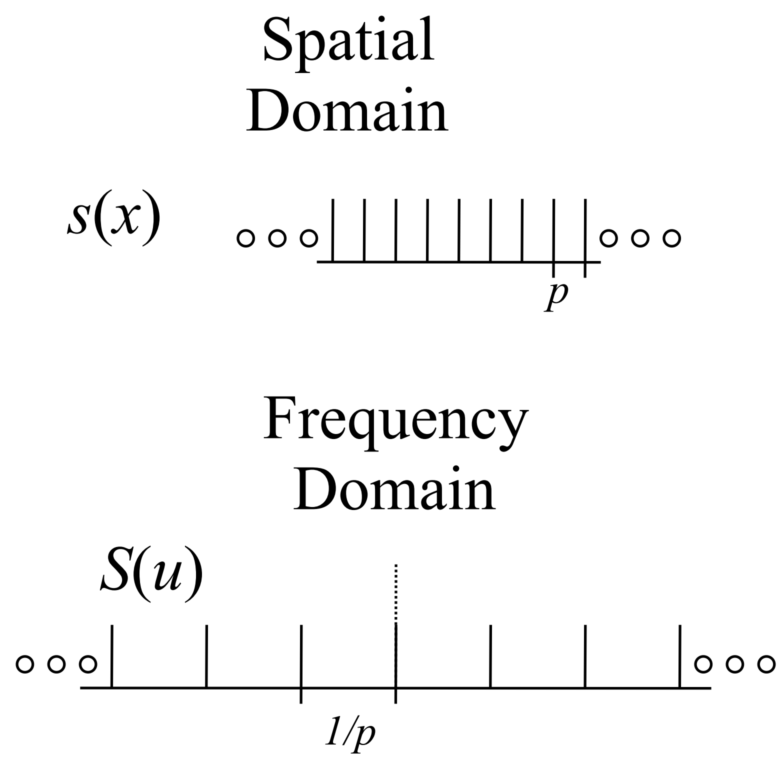
$$(gh)(x) = g(x)h(x)$$

$$(g * h)(x) = \int g(s)h(x-s)ds$$

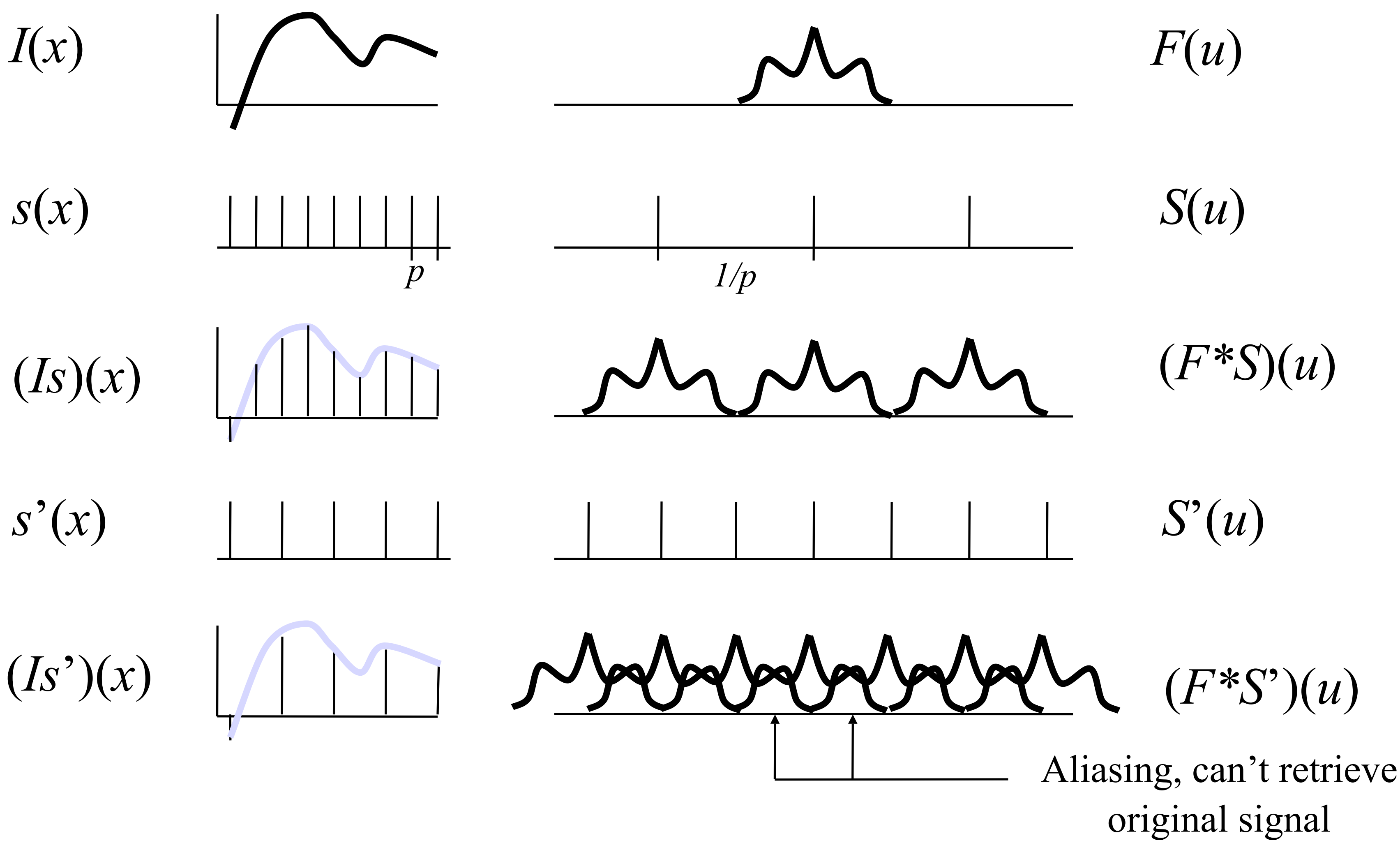


Sampling Functions

- Sampling takes measurements of a continuous function at discrete points
- Equivalent to product of continuous function and sampling function
- Uses a sampling function $s(x)$
- Sampling function is a collection of spikes
- Frequency of spikes corresponds to their resolution
- Frequency is inversely proportional to the distance between spikes
- Fourier domain also spikes
- Distance between spikes is the frequency

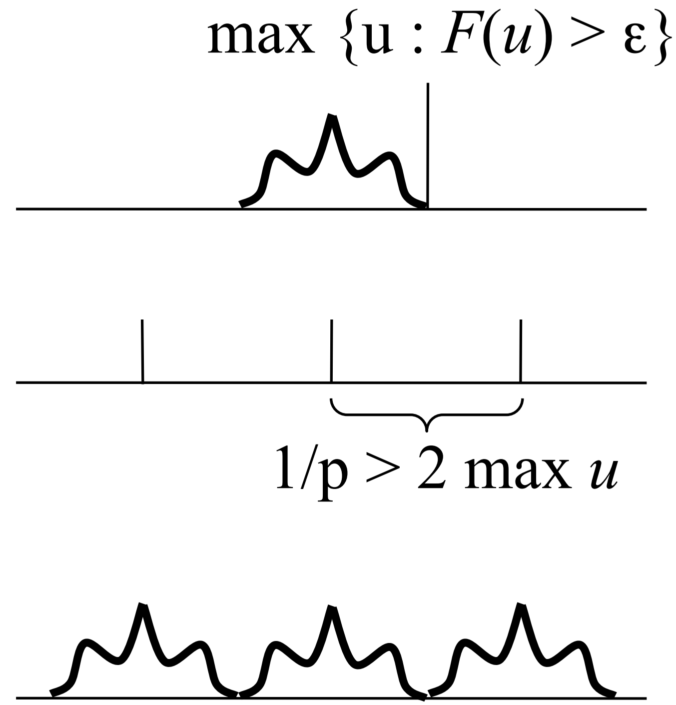


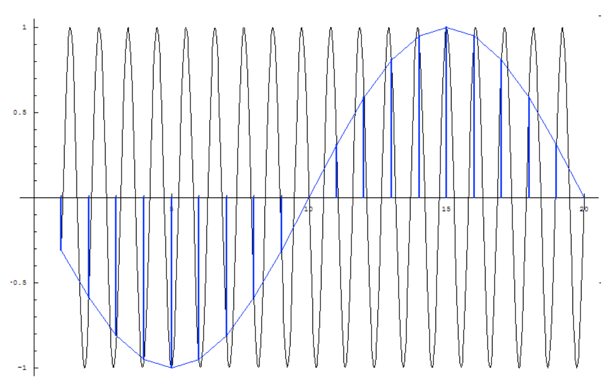
Sampling


I(x)
F(u)
s(x)
p
1/p
S(u)
(Is)(x)
(F*S)(u)
s’(x)
S’(u)
(Is’)(x)
(F*S’)(u)
Aliasing, can’t retrieve original signal

Shannon's Sampling Theorem

- Sampling frequency needs to be at least twice the highest signal frequency
- Otherwise the first replica interferes with the original spectrum
- Sampling below this *Nyquist limit* leads to aliasing
- Conceptually, need one sample for each peak and another for each valley



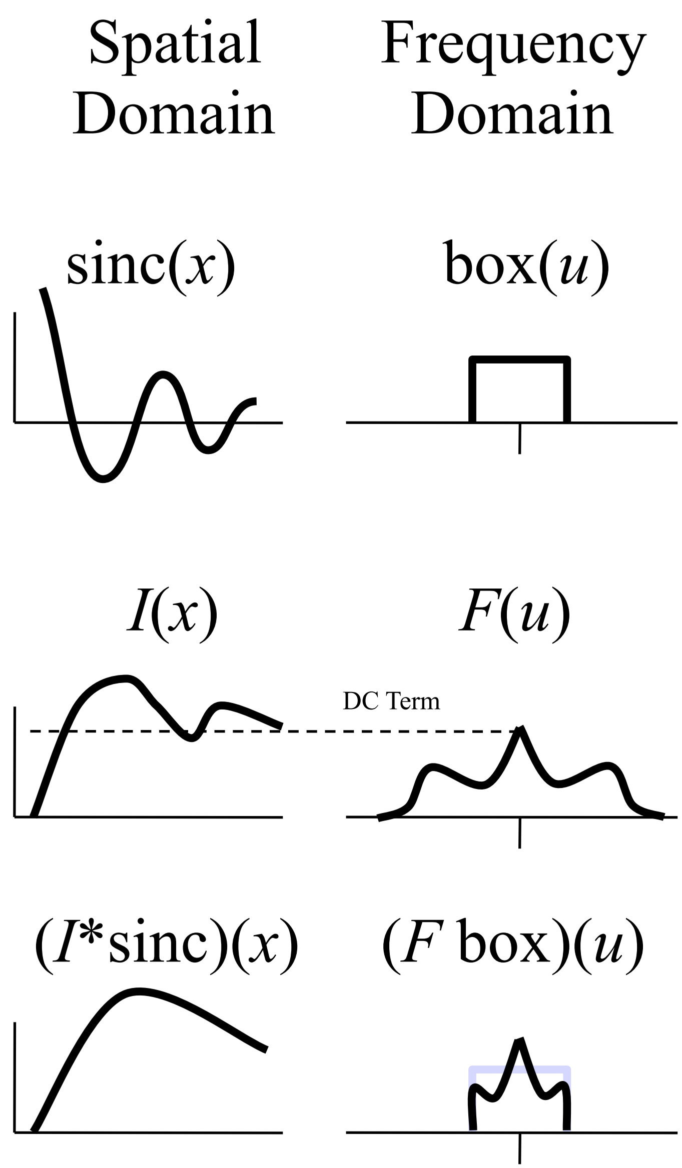


Prefiltering

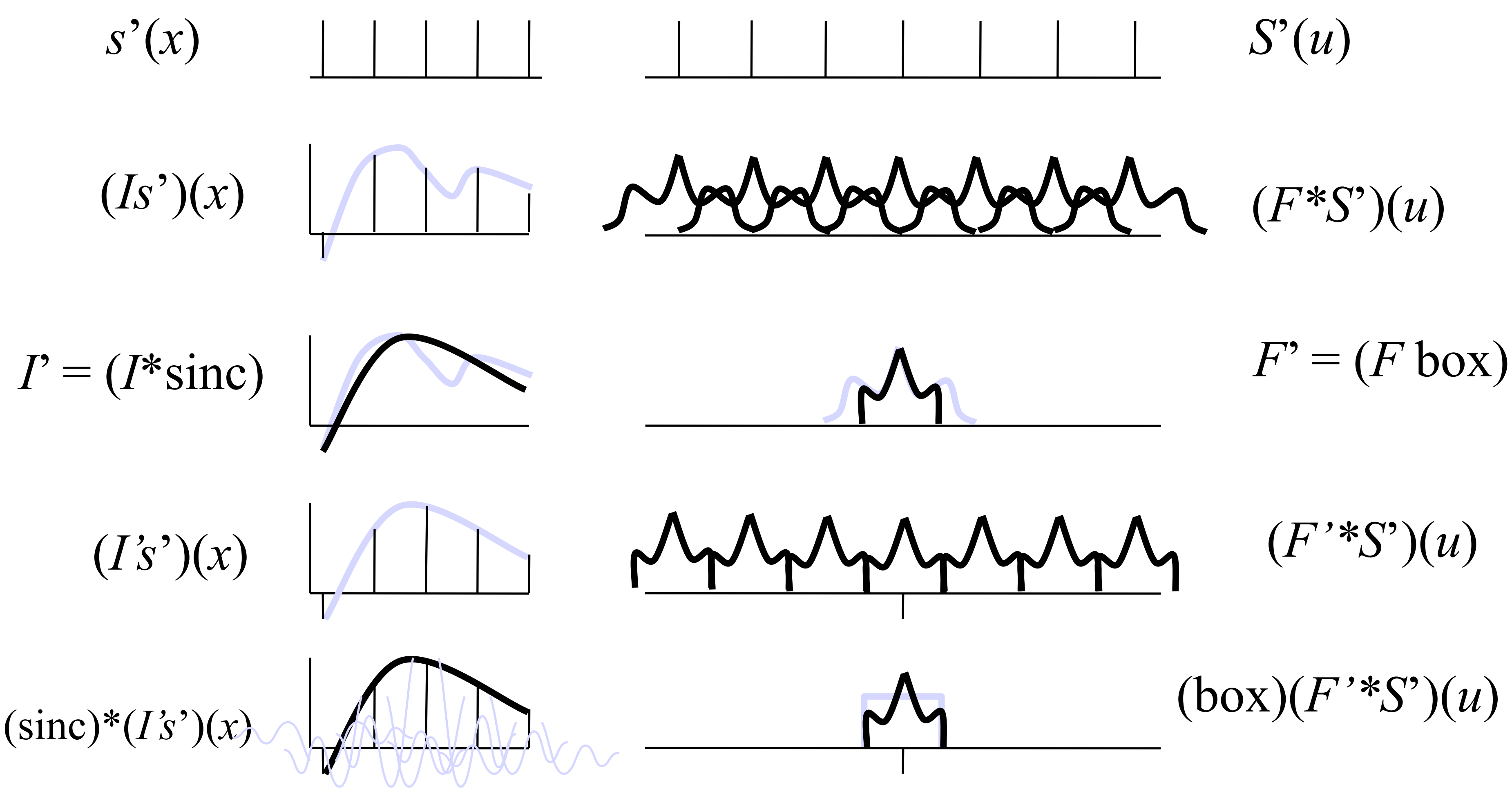
- Aliases occur at high frequencies
  - Sharp features, edges
  - Fences, stripes, checkerboards
- Prefiltering removes the high frequency components of an image before it is sampled
- Box filter (in frequency domain) is an ideal low pass filter
  - Preserves low frequencies
  - Zeros high frequencies
- Inverse Fourier transform of a box function is a sinc function

$$\text{sinc}(x) = \sin(x)/x$$

- Convolution with a sinc function removes high frequencies



Prefiltering Can Prevent Aliasing



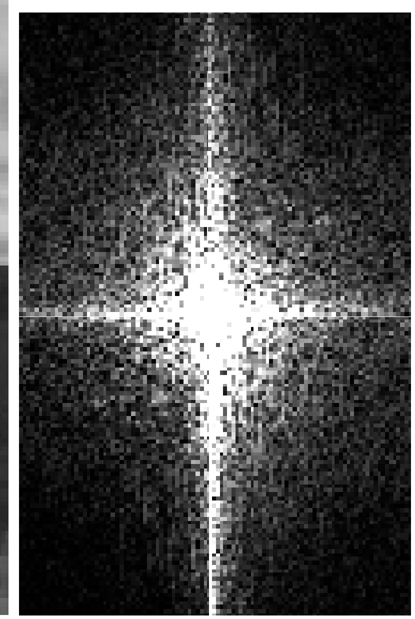
2-D Fourier Transform

- Converts spatial image into frequency spectrum image
- Distance from origin corresponds to frequency
- Angle about origin corresponds to direction frequency occurs

$$F(u,v) = \frac{1}{2\pi}\iint I(x,y)\exp(-j(ux+vy))dxdy$$

$$I(x,y) = \frac{1}{2\pi}\iint F(u,v)\exp(-j(ux+vy))dxdy$$

diag. \ freq.	vert. freq.	diag. / freq.
hor. freq.	DC	hor. freq.
diag. / freq.	vert. freq.	diag. \ freq.



Analytic Diamond

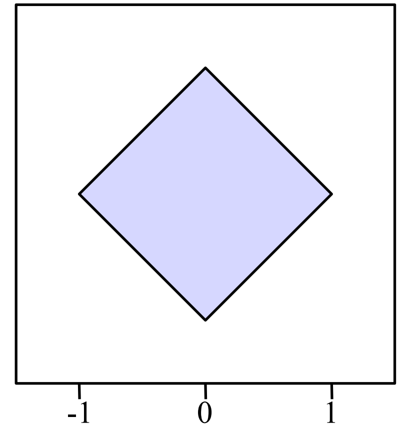
- Example: a Gaussian diamond function

$$I(x,y) = e^{-1} - e^{-|x| - |y|}$$

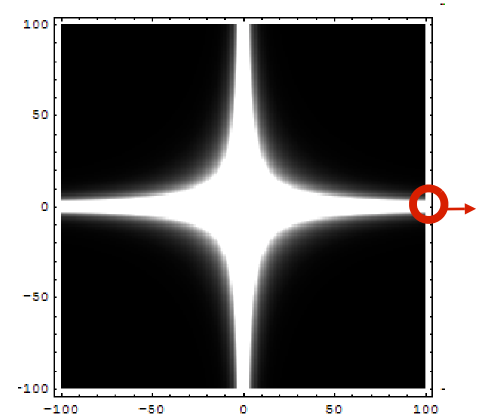
- Fourier transform of I yields

$$F(u,v) = -\frac{0.637}{(1+u^2)(1+v^2)}$$

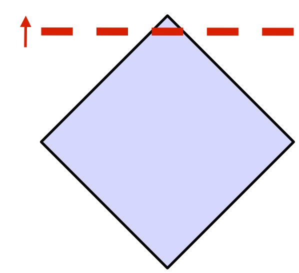
- Spectrum has energy at infinitely high horizontal and vertical frequencies
- Limited bandwidth for diagonal frequencies
- Dashed line can be placed arbitrarily close to tip of diamond, yielding a signal with arbitrarily high frequencies



$I(x,y) < 0$

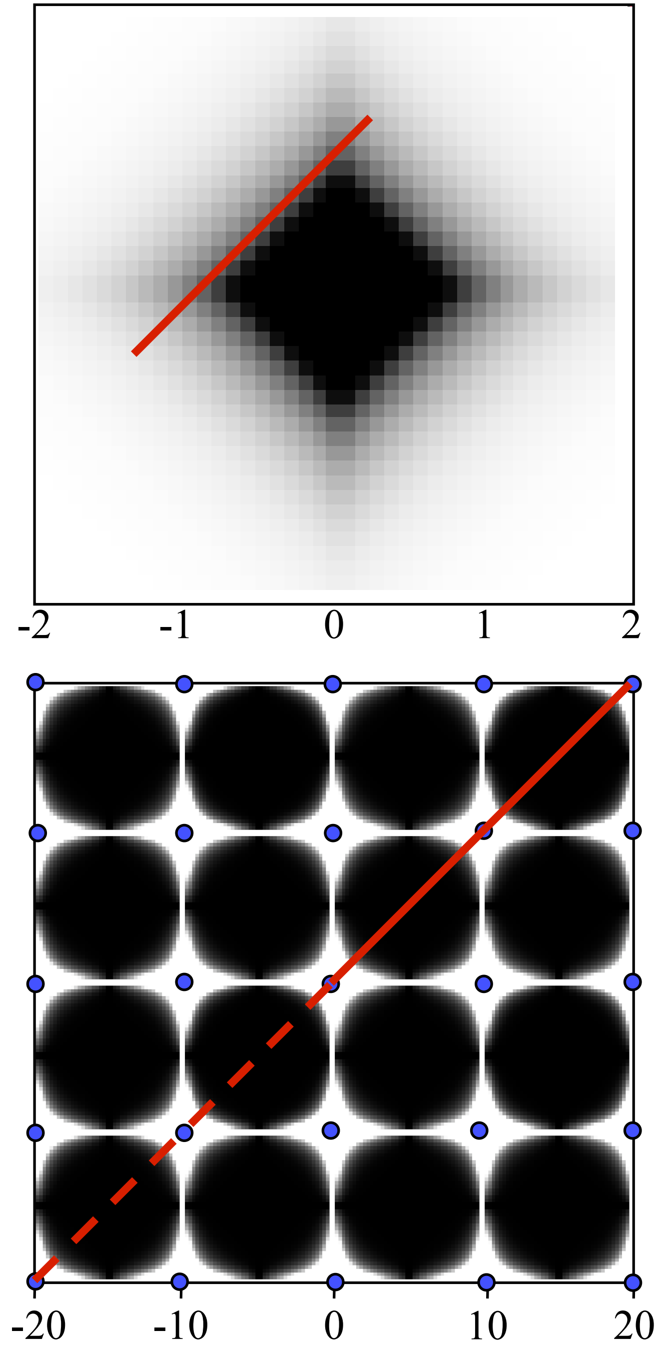


$F(u,v)$



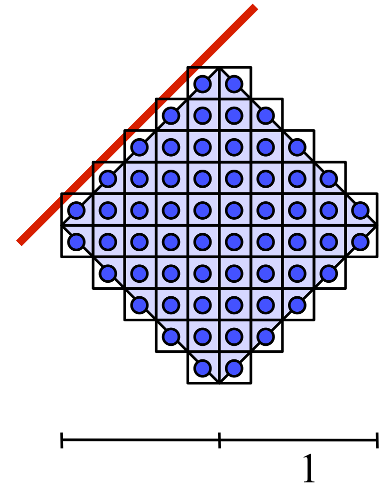
Sampled Diamond

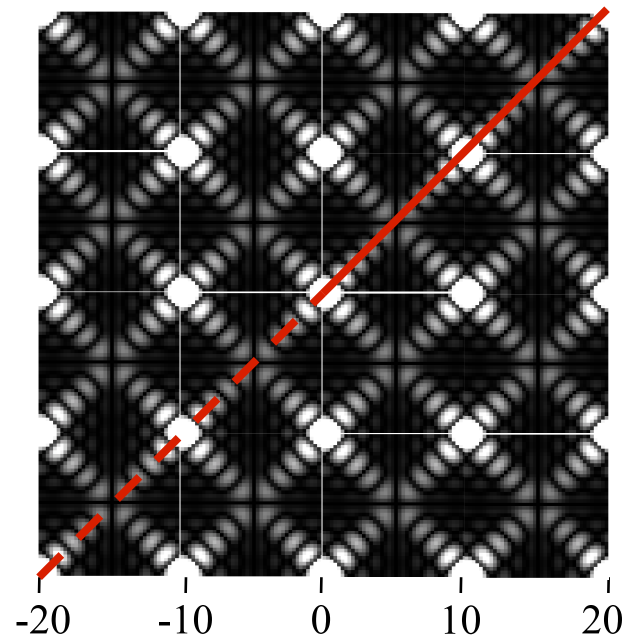
- Sample every 0.1 units: Sampling frequency is 10 Hz (samples per unit length)
- Frequencies overlap with replicas of diamond's spectrum centered at 10Hz
- Aliasing causes blocking quantization artifacts
- Frequencies of staircase edge include ~10 Hz and ~20 Hz copies of diamond's analytical spectrum centered about the DC (0Hz) term



Diamond Edges

- Plot one if $I(x,y) \leq 0$, otherwise zero
- Aliasing causes “jaggy” staircase edges
- Frequencies of staircase edge include ~10 Hz and ~20 Hz copies of diamond’s analytical spectrum centered about the DC (0Hz) term

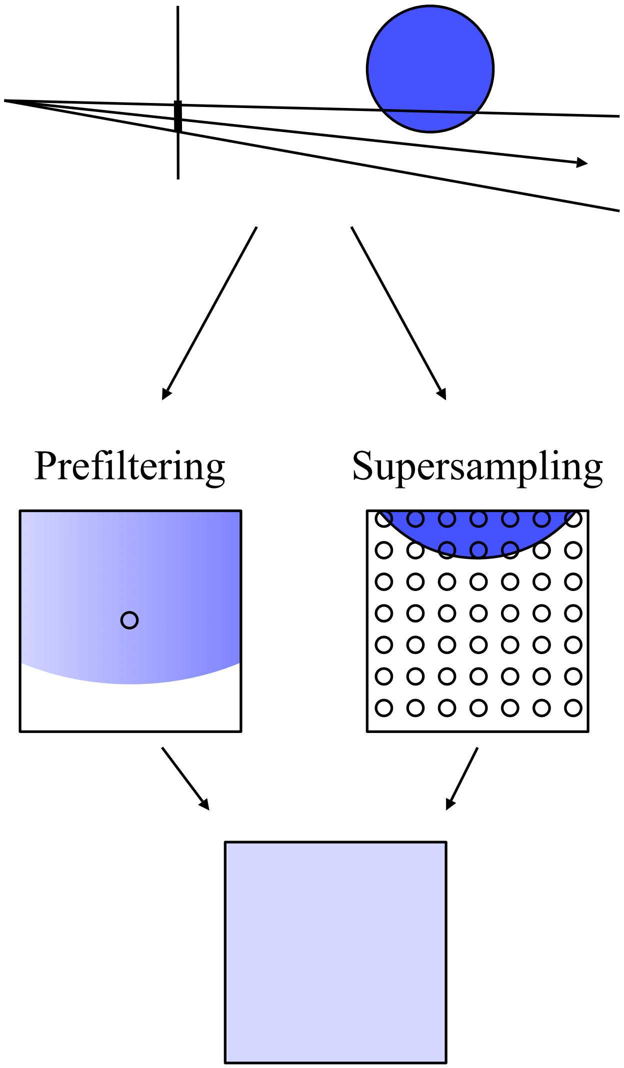




Antialiasing Strategies

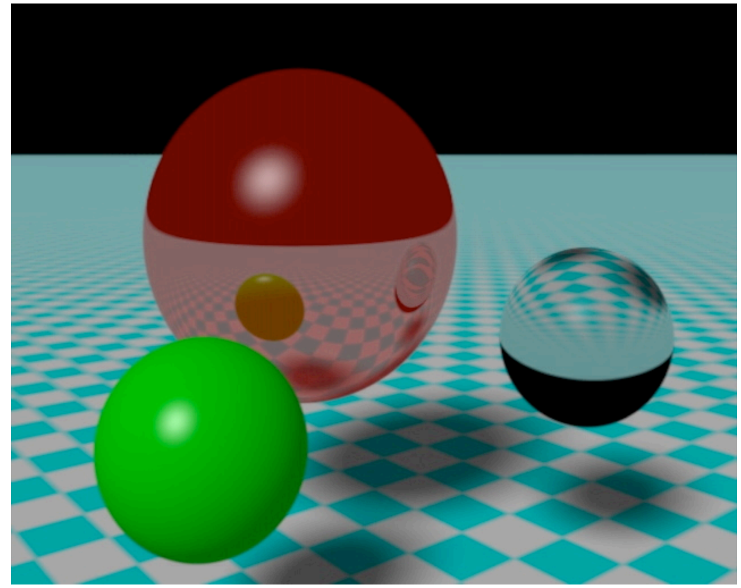
Pixel needs to represent average color over its entire area

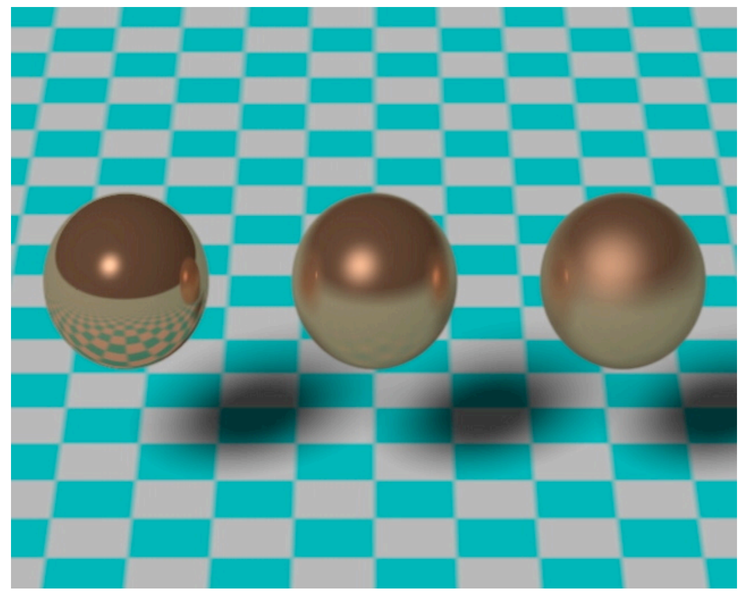
1. Prefiltering
   - averages the image function so a single sample represents the average color
   - Limits bandwidth of image signal to avoid overlap
2. Supersampling
   - Supersampling averages together many samples over pixel area
   - Moves the spectral replicas farther apart in frequency domain to avoid overlap



Cone Tracing

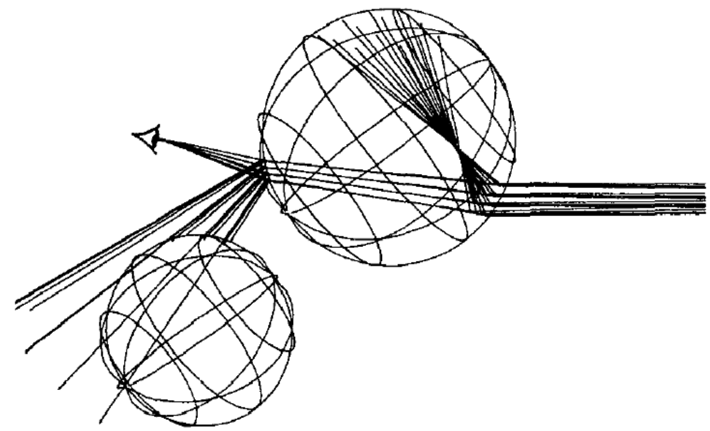
- Amanatides SIGGRAPH 84
- Replace rays with cones
- Cone samples pixel area
- Intersect cone with objects
  - Analytic solution of cone-object intersection similar to ray-object intersection
  - Expensive



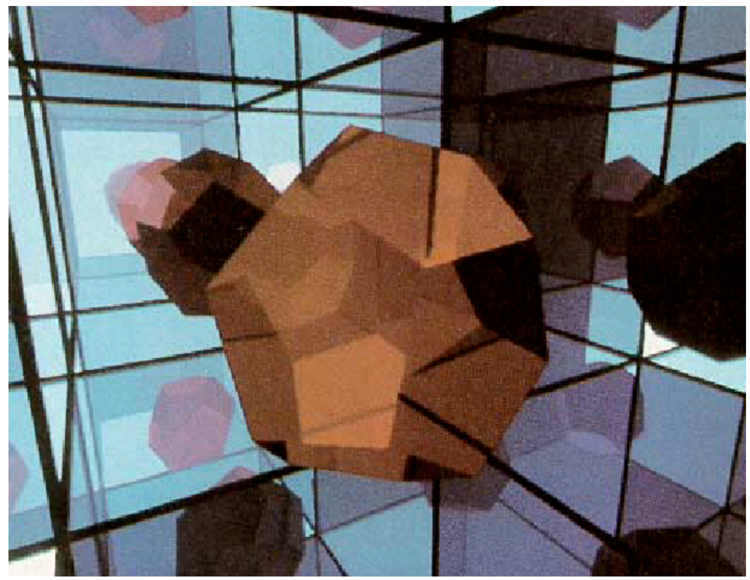


Images courtesy John Amanatides

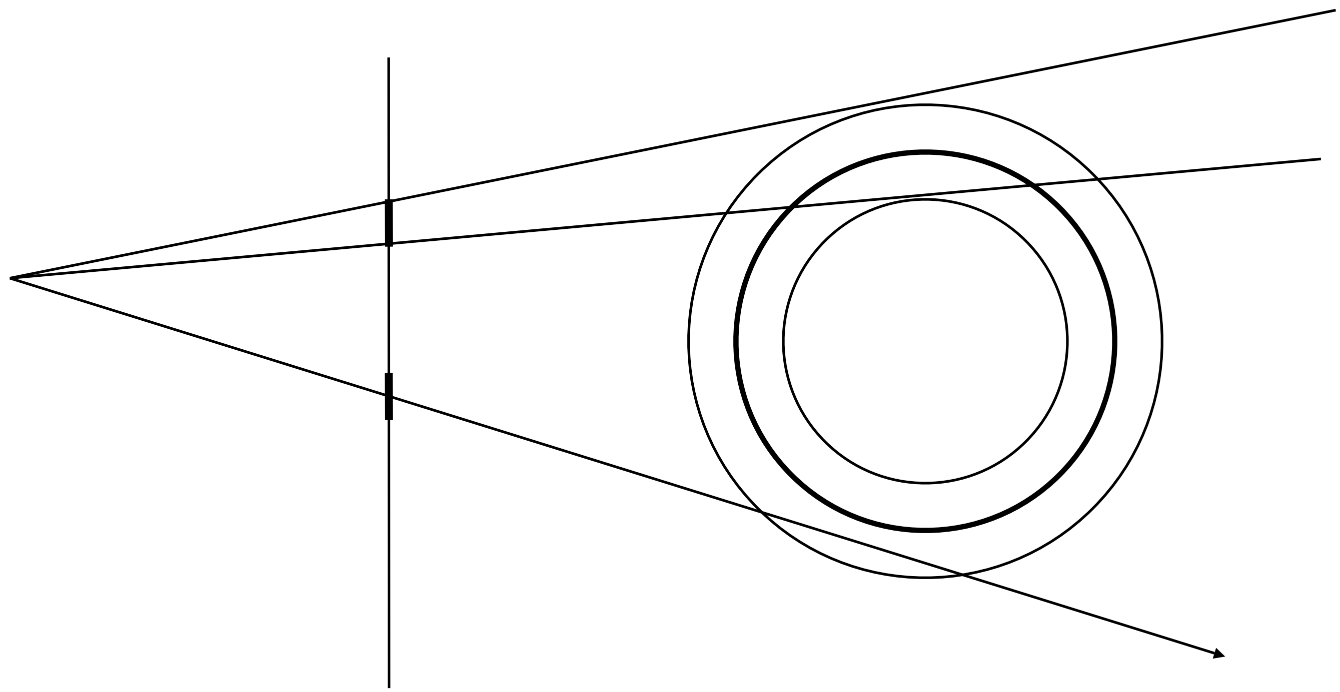
Beam Tracing



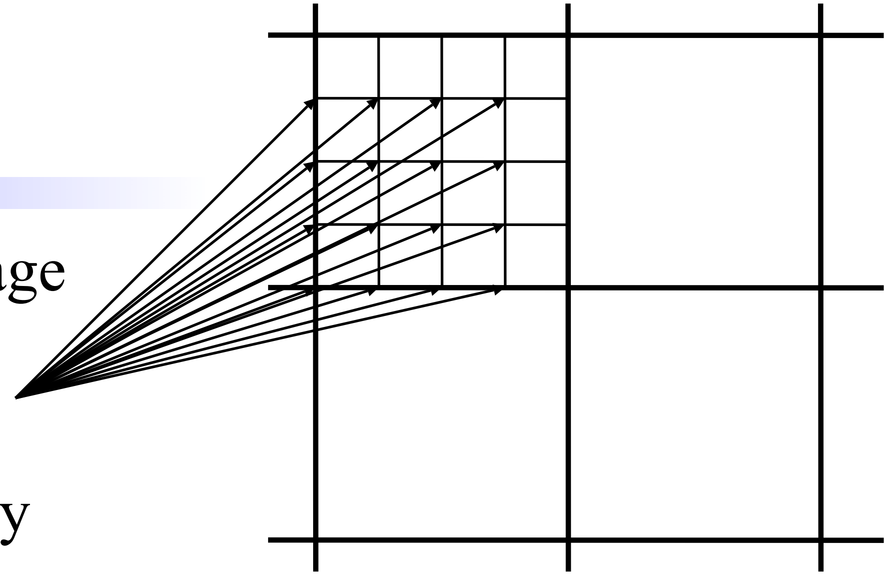
- Heckbert & Hanrahan SIGGRAPH 84
- Replace rays with generalized pyramids
- Intersection with polygonal scenes
  - Plane-plane intersections easy, fast
  - Existing scan conversion antialiasing
- Can perform some recursive beam tracing
  - Scene transformed to new viewpoint
  - Result clipped to reflective polygon



Covers

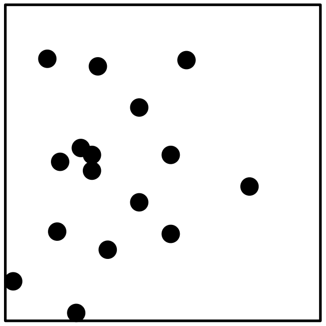


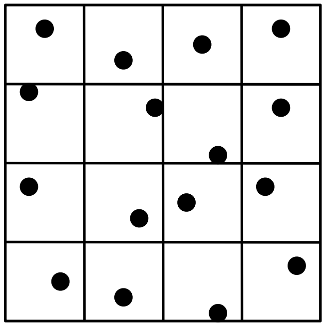
Supersampling

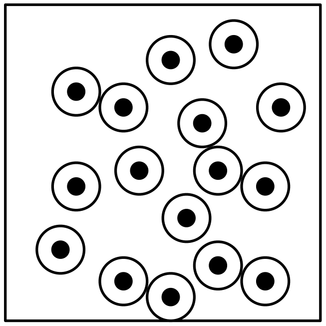


- Trace at higher resolution, average results
- Adaptive supersampling
  - trace at higher resolution only where necessary
- Problems
  - Does not eliminate aliases (e.g. moire patterns)
  - Makes aliases higher-frequency
  - Due to uniformity of samples

Stochastic Sampling

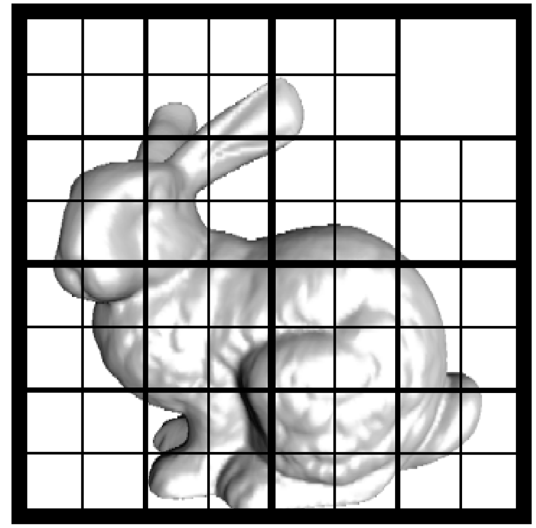






- Eye is extremely sensitive to patterns
- Remove pattern from sampling
- Randomize sampling pattern
- Result: patterns -> noise
- Some noises better than others
- *Jitter*: Pick *n* random points in sample space
  - Easiest, but samples cluster
- *Uniform Jitter*: Subdivide sample space into *n* regions, and randomly sample in each region
  - Easier, but can still cluster
- *Poisson Disk*: Pick *n* random points, but not too close to each other
  - Samples can't cluster, but may run out of room

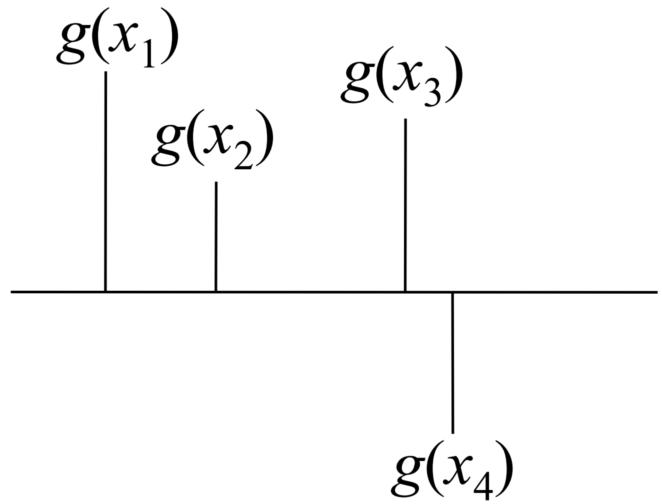
Adaptive Stochastic Sampling



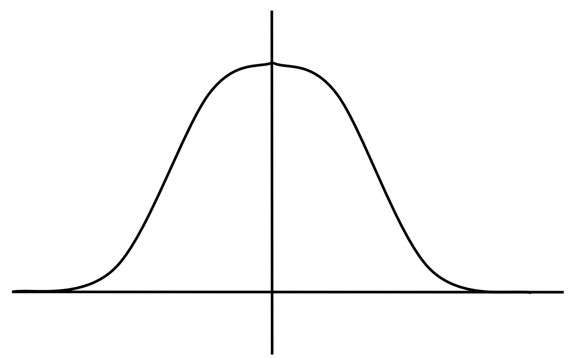
- Proximity inversely proportional to variance
- How to generate patterns at various levels?
  - Cook: Jitter a quadtree
  - Dippe/Wold: Jitter a k-d tree
  - Dippe/Wold: Poisson disk on the fly - too slow
  - Mitchell: Precompute levels - fast but granular

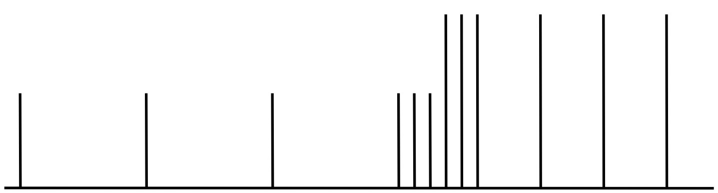
Reconstruction

$$g(x) = \frac{\sum_{n=-\infty}^{\infty} k(x - x_n)\, g(x_n)}{\sum_{n=-\infty}^{\infty} k(x - x_n)}$$



k



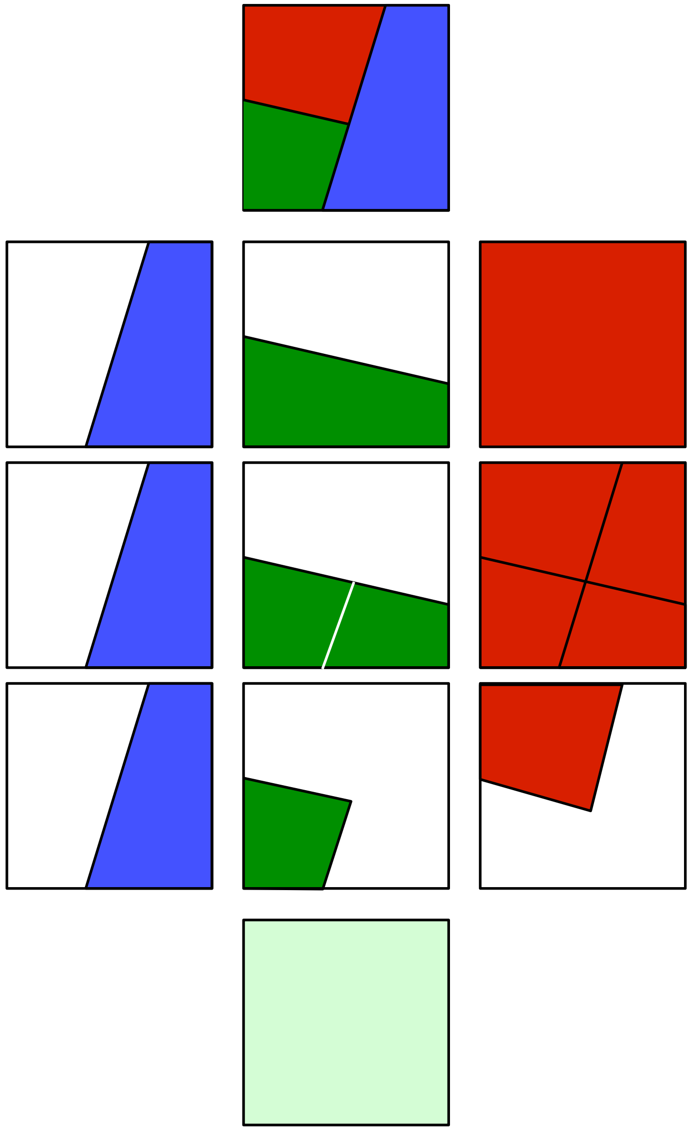


OpenGL Aliases

- Aliasing due to rasterization
- Opposite of ray casting
- New polygons-to-pixels strategies
- Prefiltering
  - Edge aliasing
    - Analytic Area Sampling
    - A-Buffer
  - Texture aliasing
    - MIP Mapping
    - Summed Area Tables
- Postfiltering
  - Accumulation Buffer

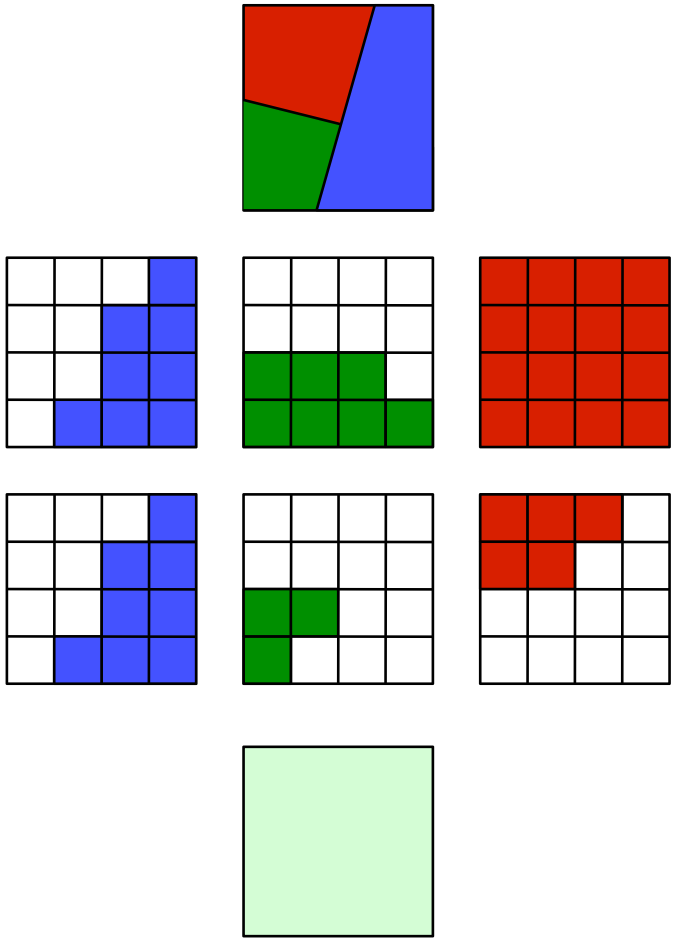
Analytic Area Sampling

- Ed Catmull, 1978
- Eliminates edge aliases
- Clip polygon to pixel boundary
- Sort fragments by depth
- Clip fragments against each other
- Scale color by visible area
- Sum scaled colors



A-Buffer

- Loren Carpenter, 1984
- Subdivides pixel into 4x4 bitmasks
- Clipping = logical operations on bitmasks
- Bitmasks used as index to lookup table

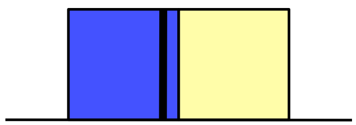


Texture Aliasing

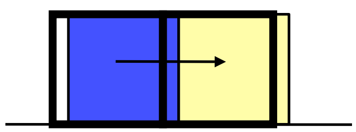
- Image mapped onto polygon
- Occur when screen resolution differs from texture resolution
- Magnification aliasing
  - Screen resolution finer than texture resolution
  - Multiple pixels per texel
- Minification aliasing
  - Screen resolution coarser than texture resolution
  - Multiple texels per pixel

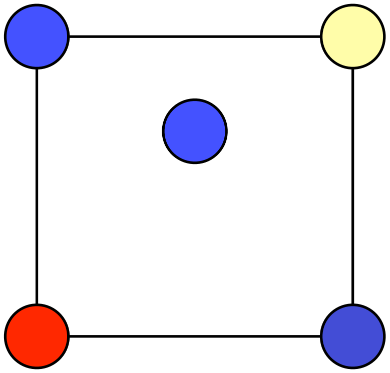
Magnification Filtering

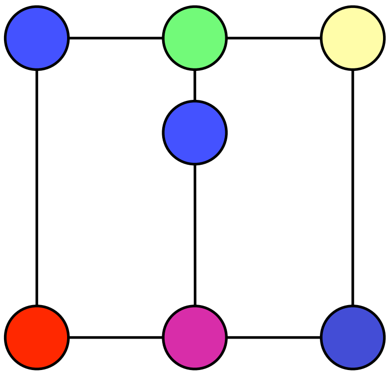
- Nearest neighbor
  - Equivalent to spike filter



- Linear interpolation
  - Equivalent to box filter





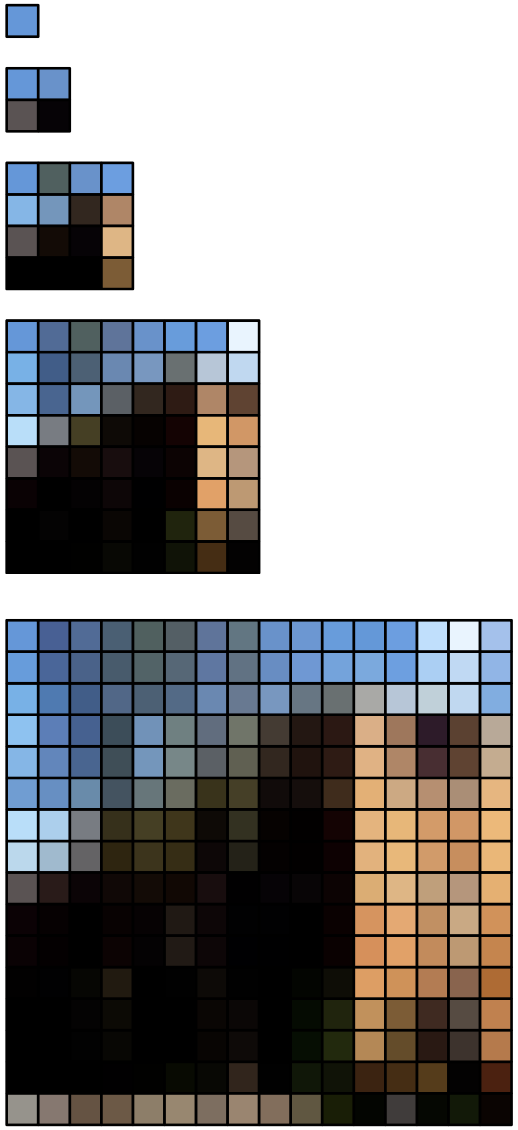


Minification Filtering

- Multiple texels per pixel
- Potential for aliasing since texture signal bandwidth greater than framebuffer
- Box filtering requires averaging of texels
- Precomputation
  - MIP Mapping
  - Summed Area Tables

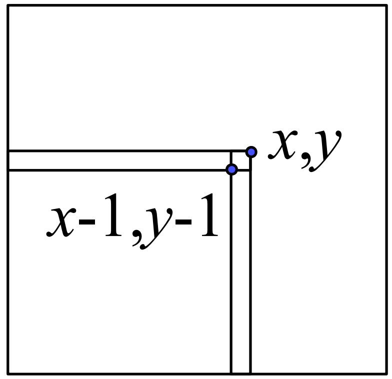
MIP Mapping

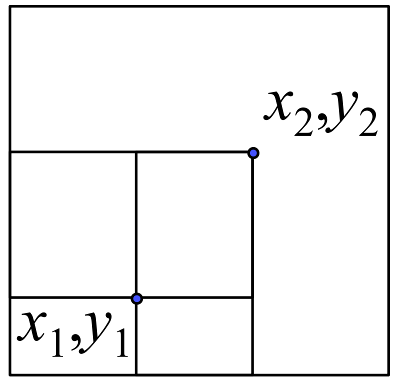
- Lance Williams, 1983
- Create a resolution pyramid of textures
  - Repeatedly subsample texture at half resolution
  - Until single pixel
  - Need extra storage space
- Accessing
  - Use texture resolution closest to screen resolution
  - Or interpolate between two closest resolutions



Summed Area Table

- Frank Crow, 1984
- Replaces texture map with summed-area texture map
  - $S(x,y)$ = sum of texels <= x,y
  - Need double range (e.g. 16 bit)
- Creation
  - Incremental sweep using previous computations
  - $S(x,y) = T(x,y) + S(x-1,y) + S(x,y-1) - S(x-1,y-1)$
- Accessing
  - $\Sigma\, T([x_1,x_2],[y_1,y_2]) = S(x_2,y_2) - S(x_1,y_2) - S(x_2,y_1) + S(x_1,y_1)$
  - Ave $T([x_1,x_2],[y_1,y_2])/((x_2 - x_1)(y_2 - y_1))$





Accumulation Buffer

- Increases OpenGL's resolution
- Render the scene 16 times
- Shear projection matrices
- Samples in different location in pixel
- Average result
- Jittered, but same jitter sampling pattern in each pixel