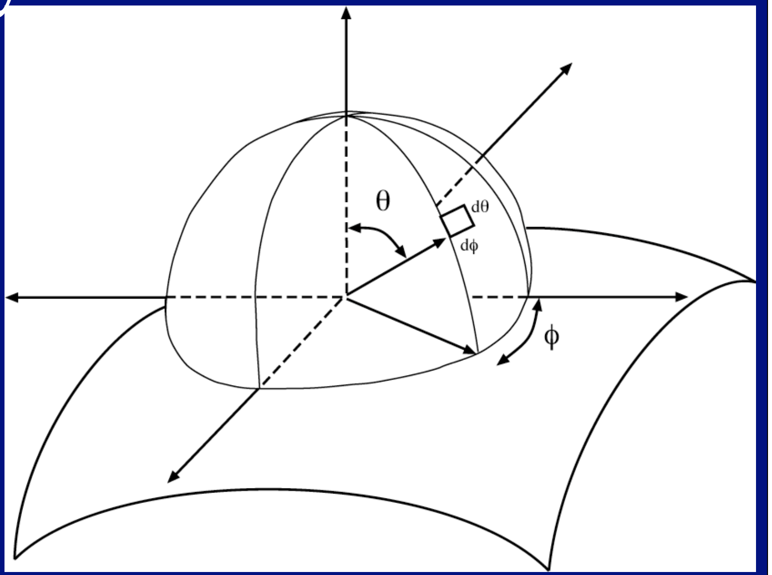
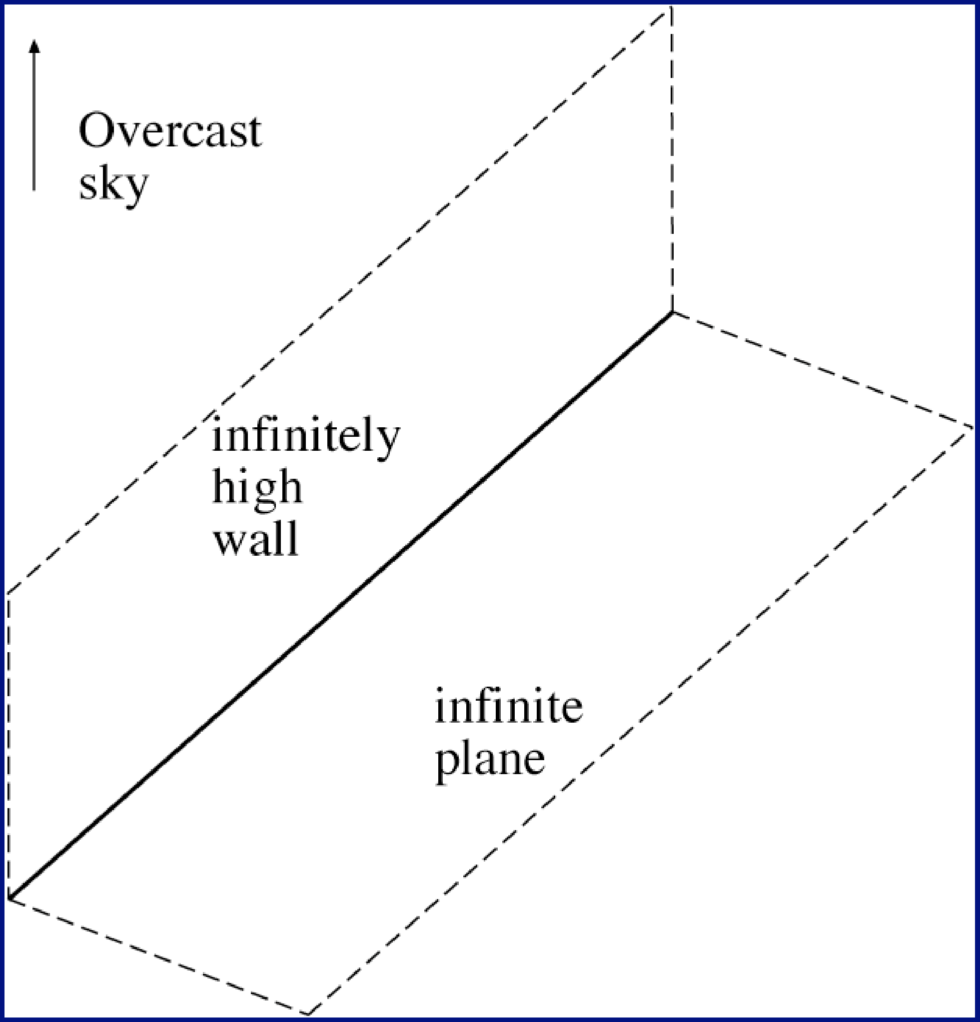
Radiometry

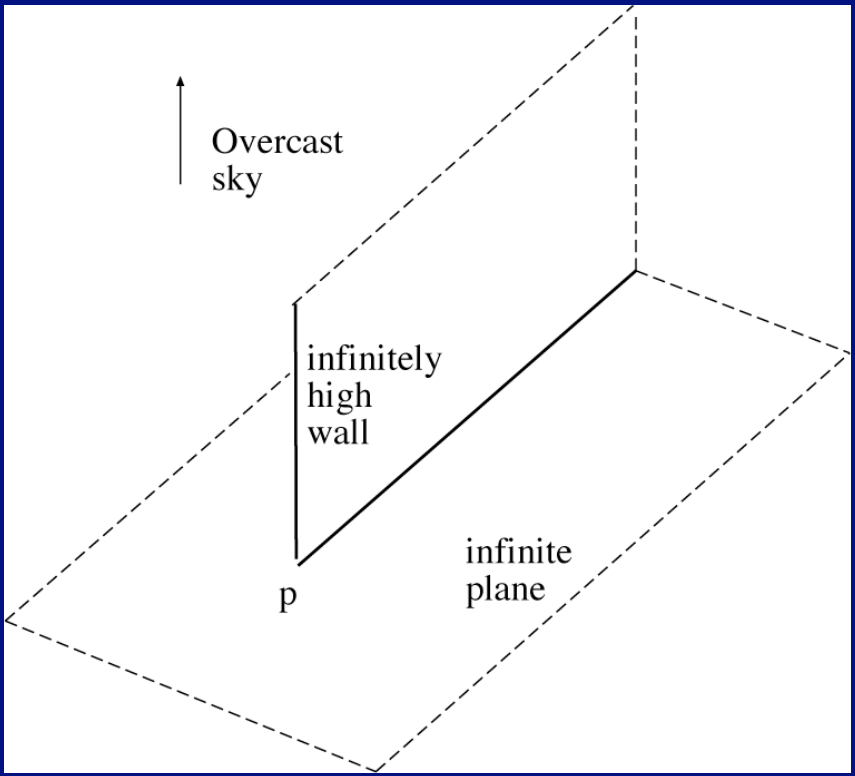


- Questions:
  - how “bright” will surfaces be?
  - what is “brightness”?
    - measuring light
    - interactions between light and surfaces
- Core idea - think about light arriving at a surface
- around any point is a hemisphere of directions
- Simplest problems can be dealt with by reasoning about this hemisphere

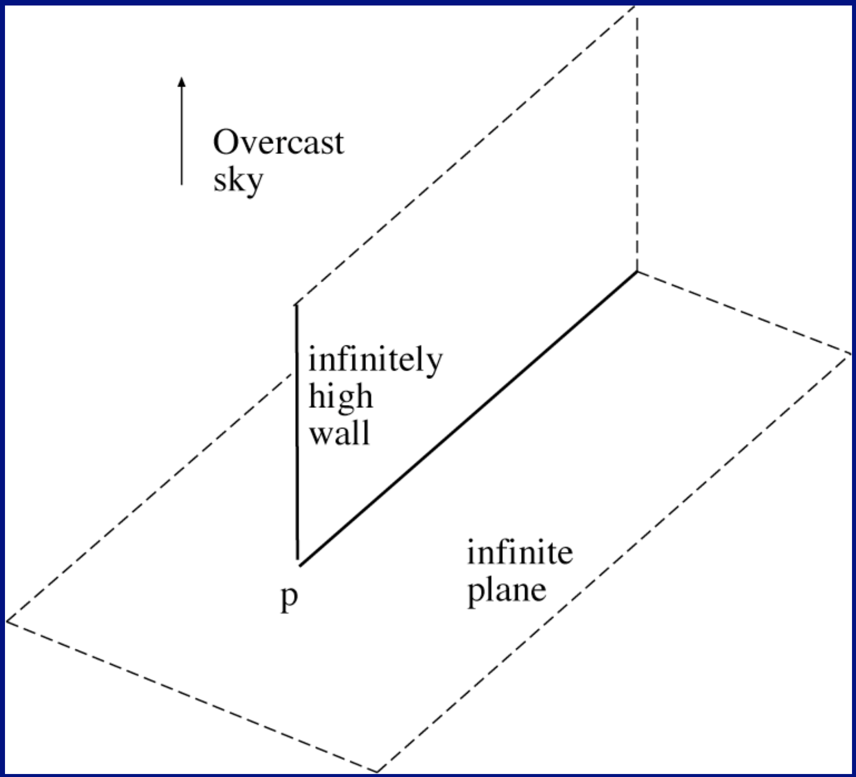
Lambert's wall

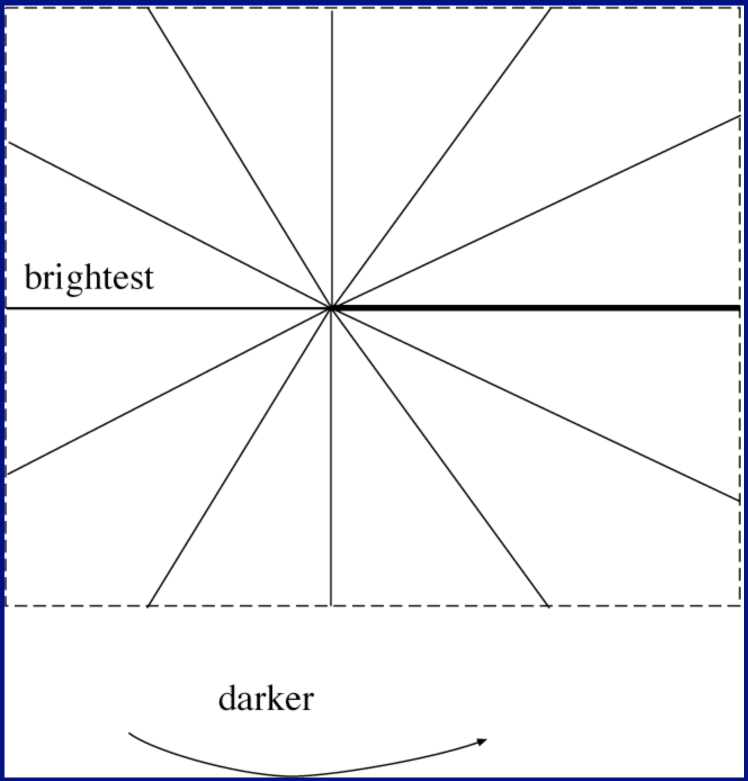


More complex wall



More complex wall

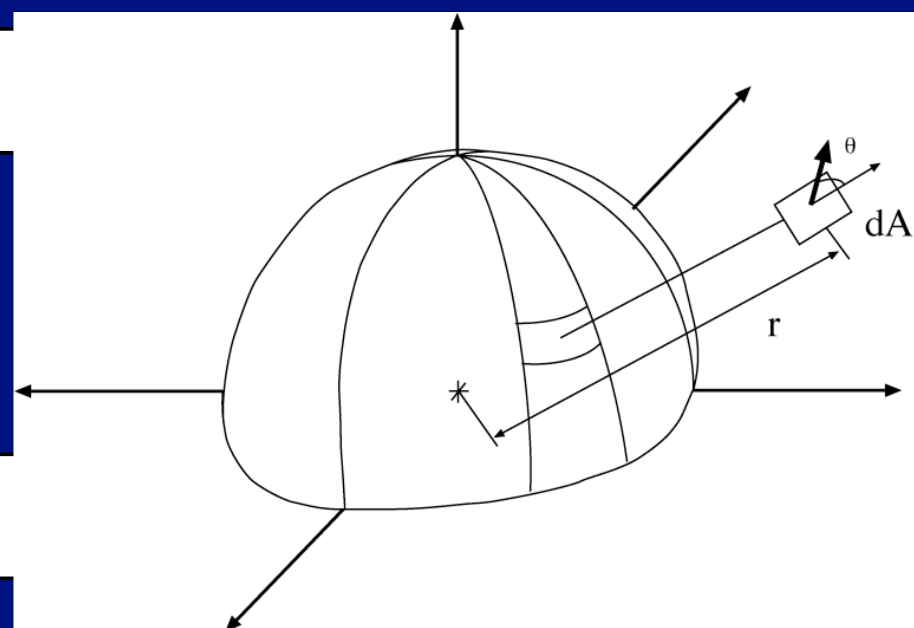




Solid Angle

- By analogy with angle (in radians)
- The solid angle subtended by a patch area dA is given by

$$d\omega = \frac{dA\cos\vartheta}{r^2}$$



- Another useful expression:

$$d\omega = \sin\vartheta (d\vartheta)(d\phi)$$

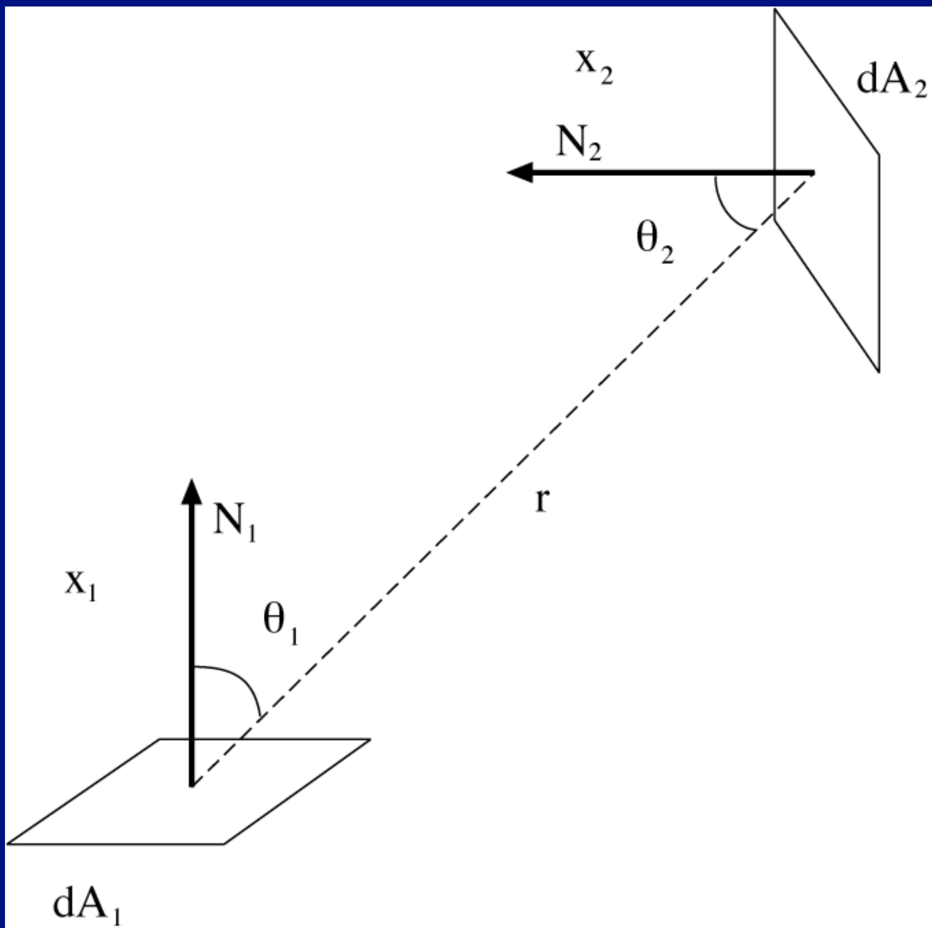
Radiance

- Measure the “amount of light” at a point, in a direction
- Property is:
  Radiant power per unit foreshortened area per unit solid angle
- Units: watts per square meter per steradian (wm-2sr-1)
- Usually written as:

$$L\left(\underline{x}, \vartheta, \varphi\right)$$

• Crucial property:
In a vacuum, radiance leaving p in the direction of q is the same as radiance arriving at q from p
– hence the units

Radiance is constant along straight lines



- Power 1->2, leaving 1:

$$L(\underline{x}_1,\vartheta,\varphi)(dA_1\cos\vartheta_1)\left(\frac{dA_2\cos\vartheta_2}{r^2}\right)$$

- Power 1->2, arriving at 2:

$$L(\underline{x}_2,\vartheta,\varphi)(dA_2\cos\vartheta_2)\left(\frac{dA_1\cos\vartheta_1}{r^2}\right)$$

Irradiance

- How much light is arriving at a surface?
- Sensible unit is Irradiance
- Incident power per unit area not foreshortened
- This is a function of incoming angle.
- A surface experiencing radiance L(x,θ,φ) coming in from dω experiences irradiance

$$L(\underline{x},\vartheta,\varphi)\cos\vartheta d\omega$$

$$\int_{\Omega} L(\underline{x},\vartheta,\varphi)\cos\vartheta\sin\vartheta d\vartheta d\varphi$$

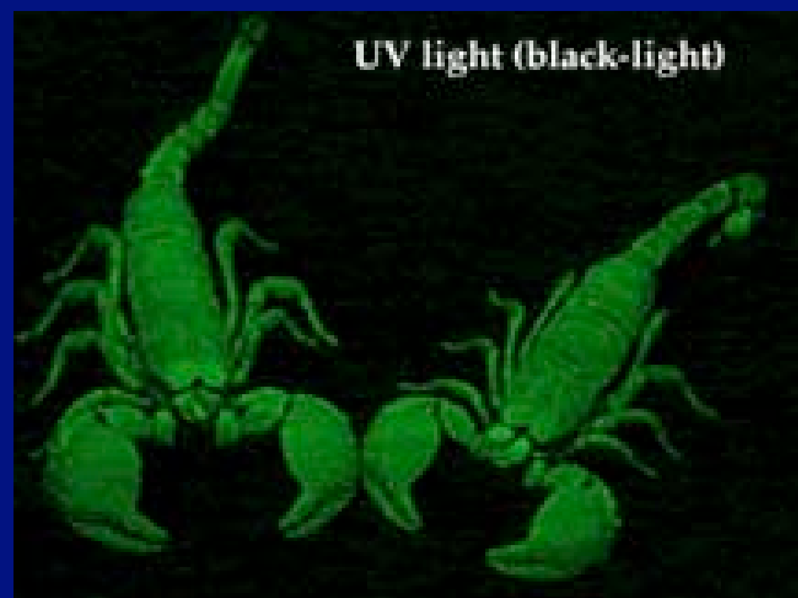
• Crucial property:
Total power arriving at the surface is given by adding irradiance over all incoming angles --- this is why it's a natural unit

Surfaces and the BRDF

- Many effects when light strikes a surface -- could be:
  - absorbed; transmitted. reflected; scattered
- Assume that
  - surfaces don't fluoresce
  - surfaces don't emit light (i.e. are cool)
  - all the light leaving a point is due to that arriving at that point
- Can model this situation with the Bidirectional Reflectance Distribution Function (BRDF)
- the ratio of the radiance in the outgoing direction to the incident irradiance

$$\rho_{bd}(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i,) = \frac{L_o(\underline{x},\vartheta_o,\varphi_o)}{L_i(\underline{x},\vartheta_i,\varphi_i)\cos\vartheta_i d\omega}$$


White light (indoor)

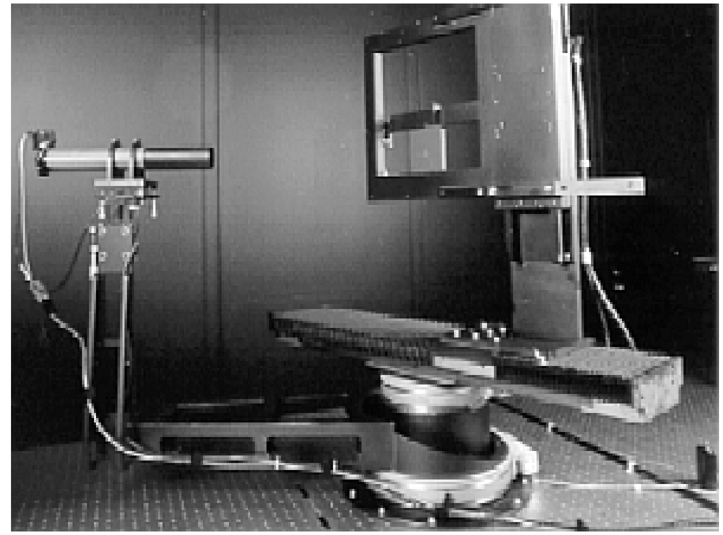

UV light (black-light)

BRDF

- Units: inverse steradians (sr-1)
- Symmetric in incoming and outgoing directions
- Radiance leaving in a particular direction:
  - add contributions from every incoming direction

$$\int_{\Omega} \rho_{bd}\left(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i,\right)L_i\left(\underline{x},\vartheta_i,\varphi_i\right)\cos\vartheta_i d\omega_i$$

Modeling BRDF's



- Mathematical derivation
  - Use laws of physics, geometry
  - Statistical model of idealized material
- Simulation
  - Model material directly
  - Render light reflected onto hemisphere
- Measurement
  - Reflect real light off of real material
  - Gonioreflectometer

Suppressing Angles - Radiosity

- In many situations, we do not really need angle coordinates
  - e.g. cotton cloth, where the reflected light is not dependent on angle
- Appropriate radiometric unit is radiosity
  - total power leaving a point on the surface, per unit area on the surface (Wm-2)
- Radiosity from radiance?
  - sum radiance leaving surface over all exit directions

$$\boxed{B(\underline{x}) = \int_{\Omega} L_o(\underline{x}, \vartheta, \varphi) \cos\vartheta \, d\omega}$$

Radiosity

- Important relationship:
  - radiosity of a surface whose radiance is independent of angle (e.g. that cotton cloth)

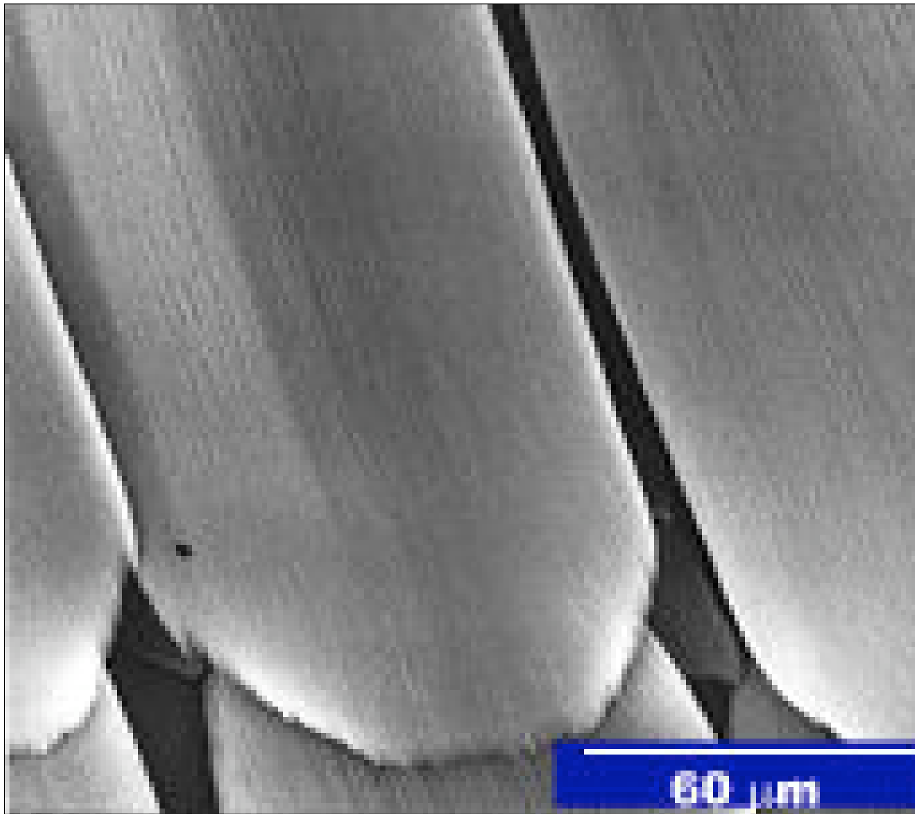
$$
\begin{aligned}
B(\underline{x}) &= \int_{\Omega} L_o(\underline{x},\vartheta,\varphi)\cos\vartheta\, d\omega \\
&= L_o(\underline{x}) \int_{\Omega} \cos\vartheta\, d\omega \\
&= L_o(\underline{x}) \int_0^{\pi/2} \int_0^{2\pi} \cos\vartheta \sin\vartheta\, d\varphi\, d\vartheta \\
&= \pi L_o(\underline{x})
\end{aligned}
$$

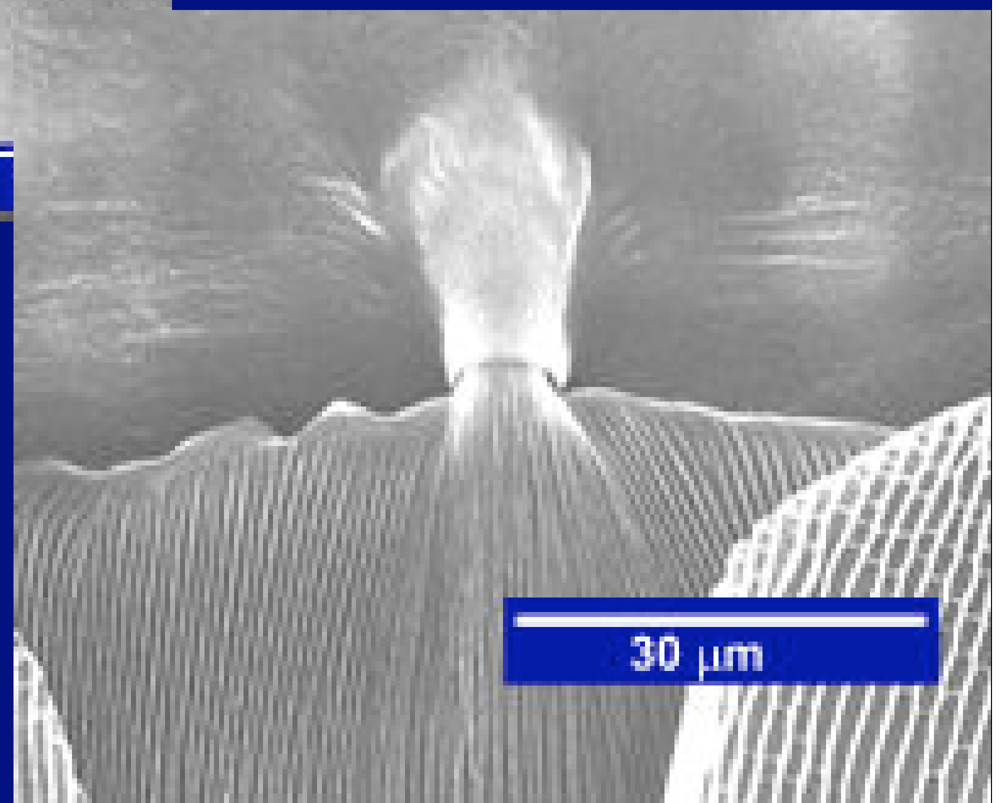
Directional hemispheric reflectance

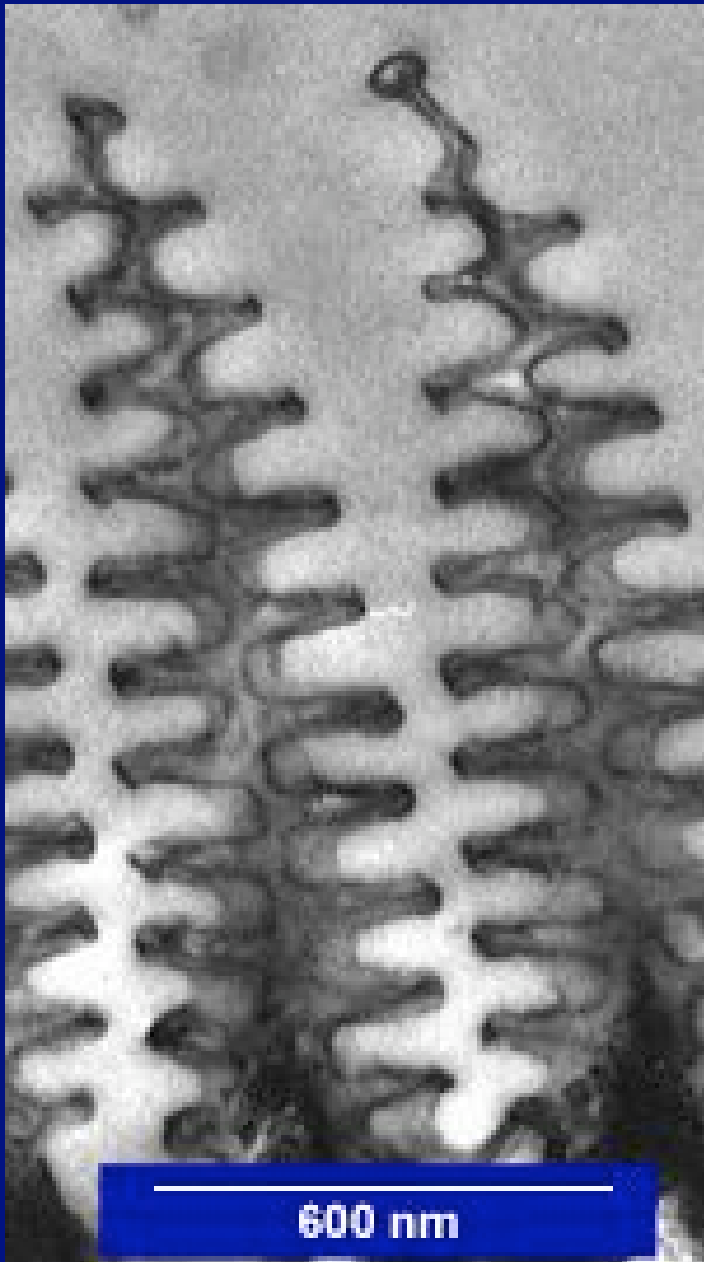
- BRDF is a very general notion
  - some surfaces need it (underside of a CD; tiger eye; etc)
  - very hard to measure and very unstable
  - for many surfaces, light leaving the surface is largely independent of exit angle (surface roughness is one source of this property)
- Directional hemispheric reflectance:
  - the fraction of the incident irradiance in a given direction that is reflected by the surface (whatever the direction of reflection)
  - unitless, range 0-1

$$\rho_{dh}(\vartheta_i, \varphi_i) = \frac{\int_{\Omega} L_o(\underline{x}, \vartheta_o, \varphi_o) \cos\vartheta_o d\omega_o}{L_i(\underline{x}, \vartheta_i, \varphi_i) \cos\vartheta_i d\omega_i}$$

$$= \int_{\Omega} \rho_{bd}(\underline{x}, \vartheta_o, \varphi_o, \vartheta_i, \varphi_i,) \cos\vartheta_o d\omega_o$$


60 µm


30 µm


600 nm

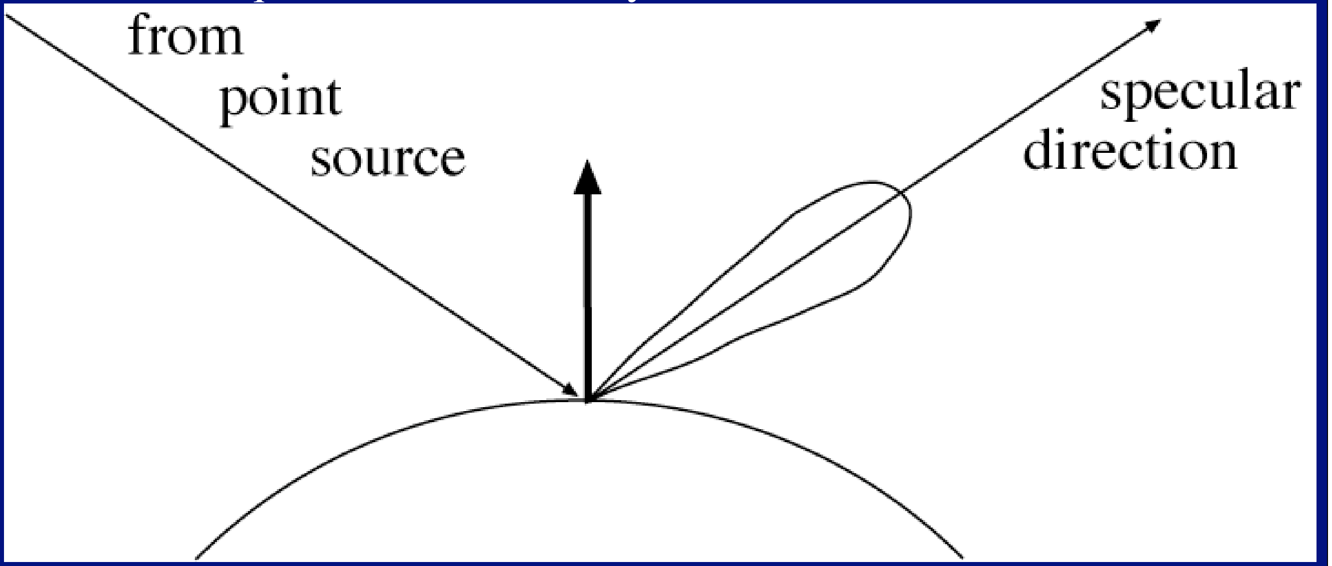
Lambertian surfaces and albedo

- For some surfaces, the DHR is independent of direction
  - cotton cloth, carpets, matte paper, matte paints, etc.
  - radiance leaving the surface is independent of angle
  - Lambertian surfaces (same Lambert) or ideal diffuse surfaces
  - Use radiosity as a unit to describe light leaving the surface
  - DHR is often called diffuse reflectance, or albedo
- for a Lambertian surface, BRDF is independent of angle, too.
- Useful fact:

$$\boxed{\rho_{brdf} = \frac{\rho_d}{\pi}}$$

Specular surfaces

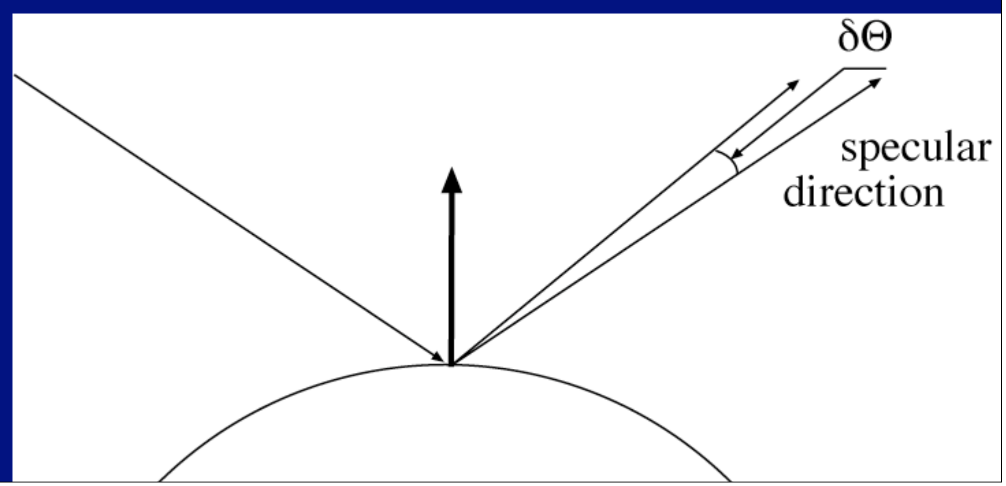
- Another important class of surfaces is specular, or mirror-like.
  - radiation arriving along a direction leaves along the specular direction
  - reflect about normal
  - some fraction is absorbed, some reflected
  - on real surfaces, energy usually goes into a lobe of directions
  - can write a BRDF, but requires the use of funny functions



Phong's model

- There are very few cases where the exact shape of the specular lobe matters.
- Typically:
  - very, very small --- mirror
  - small -- blurry mirror
  - bigger -- see only light sources as "specularities"
  - very big -- faint specularities
- Phong's model
  - reflected energy falls off with

$$\boxed{\cos^n(\delta\vartheta)}$$



Lambertian + specular

- Widespread model
  - all surfaces are Lambertian plus specular component
- Advantages
  - easy to manipulate
  - very often quite close true
- Disadvantages
  - some surfaces are not
    - e.g. underside of CD’s, feathers of many birds, blue spots on many marine crustaceans and fish, most rough surfaces, oil films (skin!), wet surfaces
  - Generally, very little advantage in modelling behaviour of light at a surface in more detail -- it is quite difficult to understand behaviour of L+S surfaces

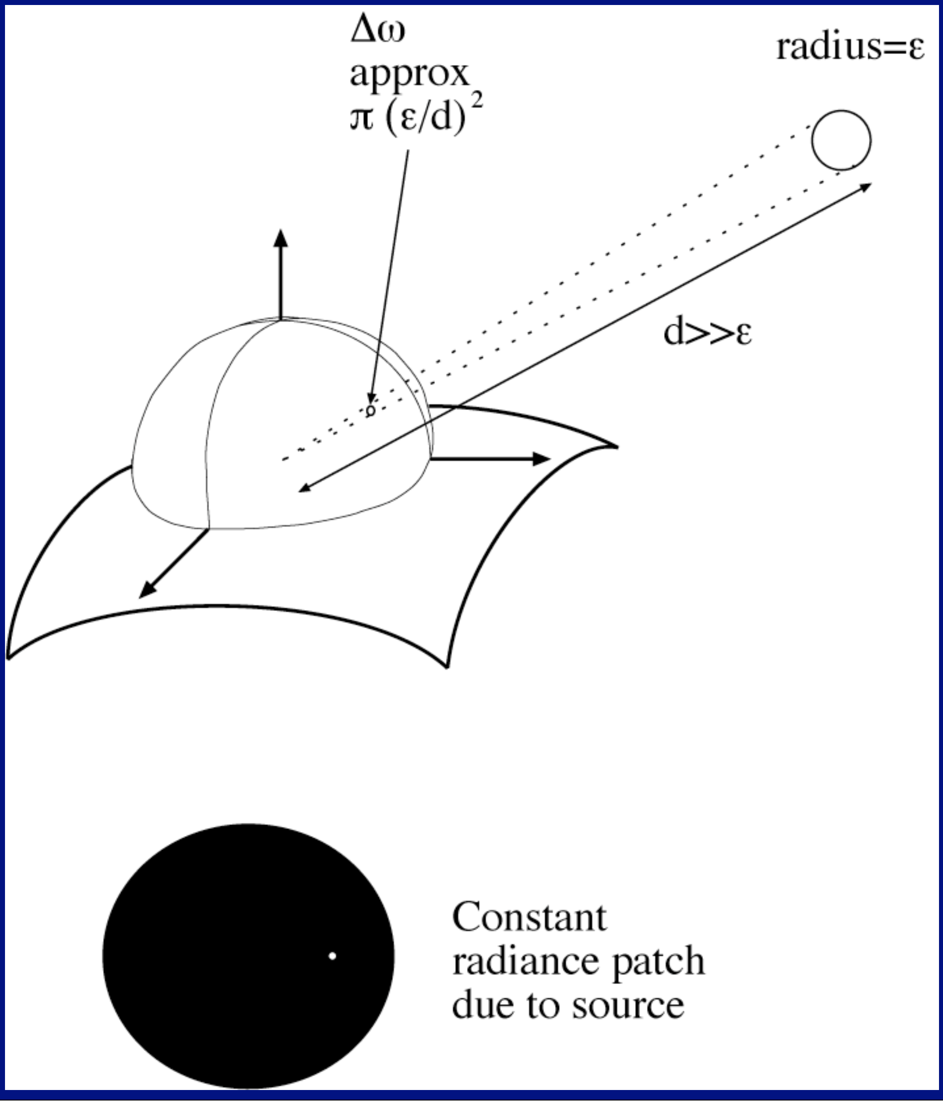
Sources, shadows and shading

- But how bright (or what colour) are objects?
- One more definition: Exitance of a source is
  - the internally generated power radiated per unit area on the radiating surface
- similar to radiosity: a source can have both
  - radiosity, because it reflects
  - exitance, because it emits

Sources, shadows and shading

$$B(x) = E(x) + \int_{\Omega} \left\{ \begin{matrix} \text{radiosity due to} \\ \text{incoming radiance} \end{matrix} \right\} d\omega$$

Radiosity due to point sources



• small, distant sphere radius ε and exitance E, which is far away subtends solid angle of about

$$\pi\left(\frac{\varepsilon}{d}\right)^2$$

Radiosity due to a point source

- Radiosity is

$$
\begin{aligned}
B(x) &= \pi L_o(x) \\
&= \rho_d(x)\int_{\Omega} L_i(x,\omega)\cos\theta_i d\omega \\
&= \rho_d(x)\int_{D} L_i(x,\omega)\cos\theta_i d\omega \\
&\approx \rho_d(x)(\text{solid angle})(\text{Exitance term})\cos\theta_i \\
&= \frac{\rho_d(x)\cos\theta_i}{r(x)^2}(\text{Exitance term and some constants})
\end{aligned}
$$

Standard nearby point source model

- N is the surface normal
- rho is diffuse albedo
- S is source vector - a vector from x to the source, whose length is the intensity term
  - works because a dot-product is basically a cosine

$$\rho_d(x)\left(\frac{N(x)\bullet S(x)}{r(x)^2}\right)$$

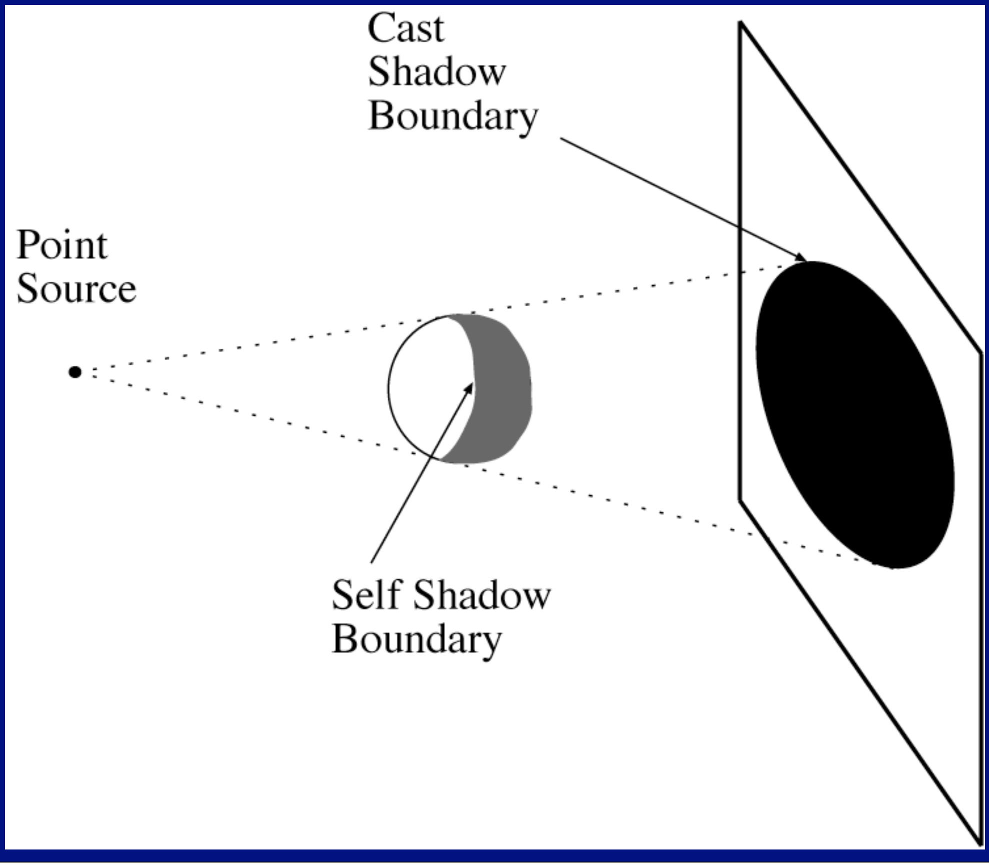
Standard distant point source model

- Issue: nearby point source gets bigger if one gets closer
  - the sun doesn't for any reasonable binding of closer
- Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn't vary much, and the distance doesn't vary much either, and we can roll the constants together to get:

$$\boxed{\rho_d(x)\left(N(x) \bullet S_d(x)\right)}$$

Shadows cast by a point source

- A point that can’t see the source is in shadow
- For point sources, the geometry is simple



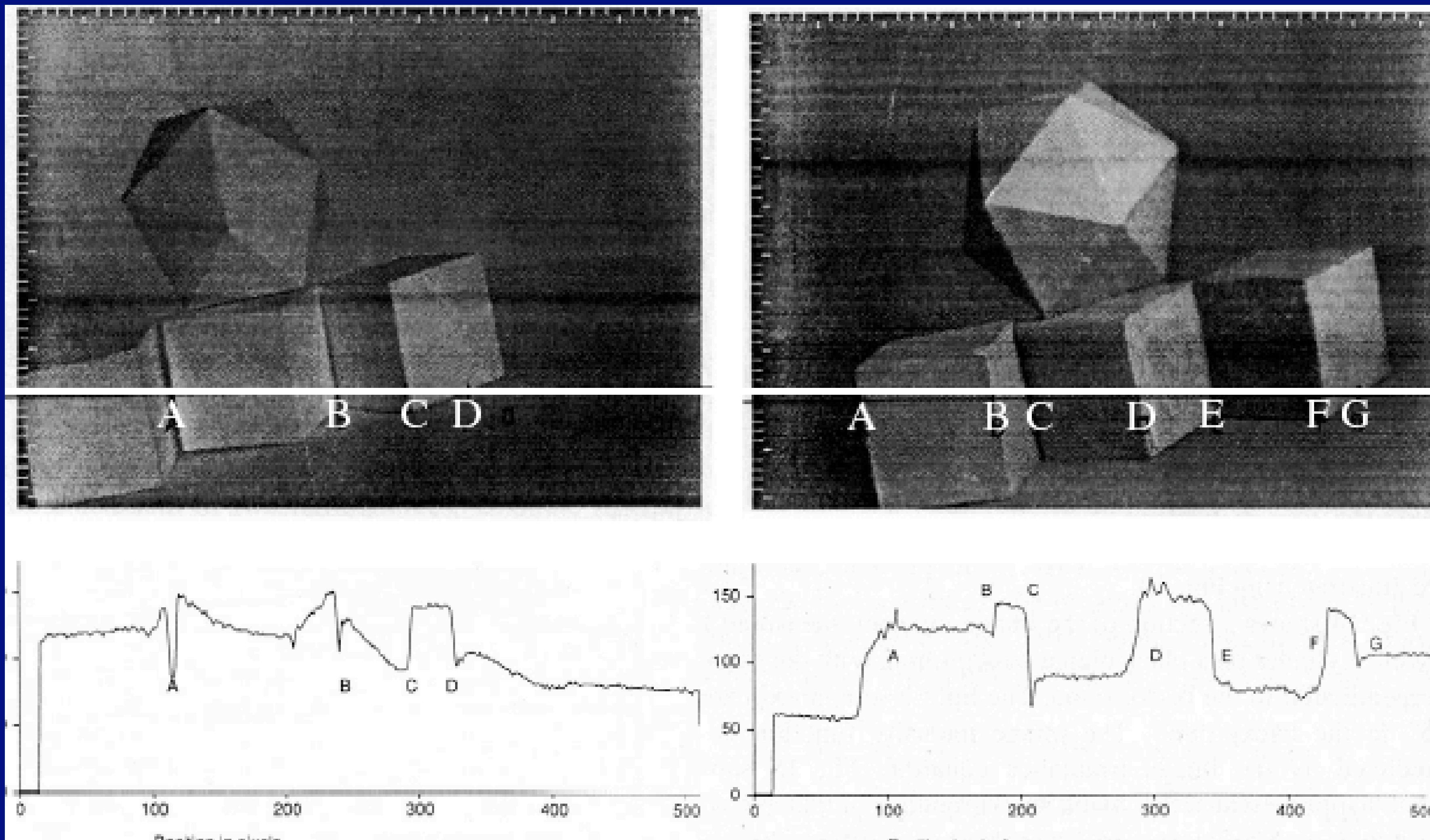
Shading models

- Local shading model
  - Surface has radiosity due only to sources visible at each point
  - Advantages:
    - often easy to manipulate, expressions easy
    - supports quite simple theories of how shape information can be extracted from shading
- Global shading model
  - Surface has radiosity due to radiance reflected from other surfaces as well as from sources
  - Advantages
    - Very accurate, compelling
  - Disadvantage
    - Extremely difficult to infer anything from shading values

Curious Experimental Fact

- Prepare two rooms, one with white walls and white objects, one with black walls and black objects
- Illuminate the black room with bright light, the white room with dim light
- People can tell which is which (due to Gilchrist)

- Why? (a local shading model predicts they can't).


A B C D
A B C D E F G
A B C D
0 100 200 300 400 500
Position in pixels
150 100 50 0
A B C D E F G
0 100 200 300 400 500
Position in pixels

Interreflections

- Issue:
  - local shading model is a poor description of physical processes that give rise to images
    - because surfaces reflect light onto one another
  - This is a major nuisance; the distribution of light (in principle) depends on the configuration of every radiator; big distant ones are as important as small nearby ones (solid angle)
  - The effects are easy to model
  - It appears to be hard to extract information from these models

Shading models

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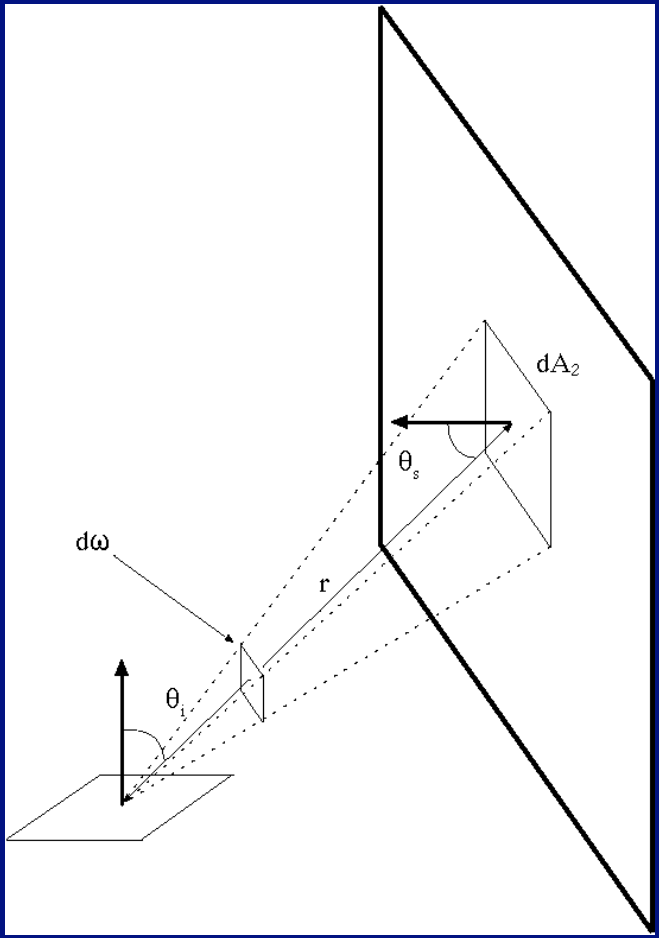
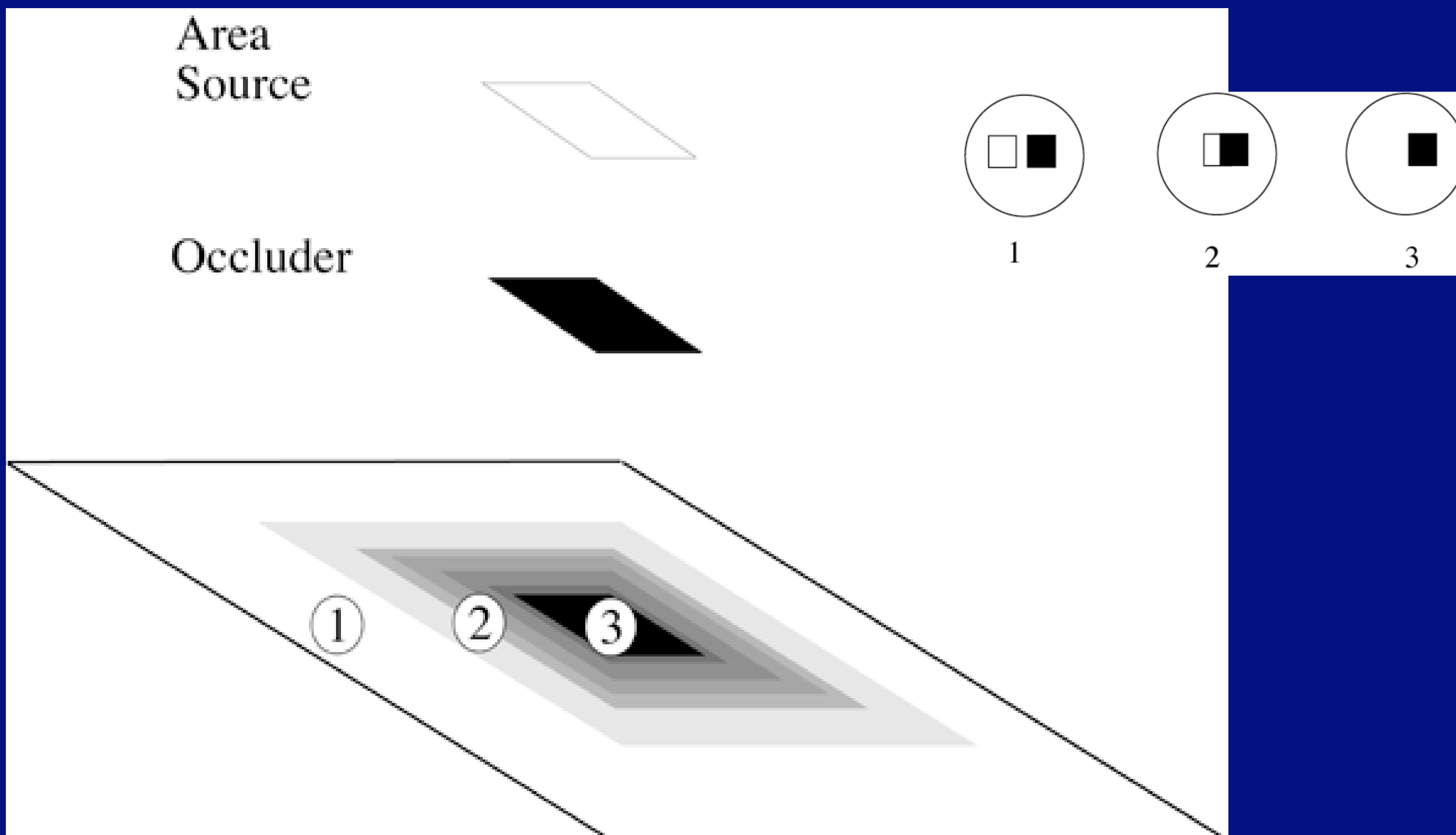
Area sources



- Examples: diffuser boxes, white walls.
- The radiosity at a point due to an area source is obtained by adding up the contribution over the section of view hemisphere subtended by the source
  - change variables and add up over the source

Area Source Shadows



Radiosity due to an area source

- rho is albedo
- E is exitance
- r(x, u) is distance between points
- u is a coordinate on the source

$$
\begin{aligned}
B(x) &= \rho_d(x)\int_{\Omega} L_i(x, u \to x)\cos\theta_i d\omega \\
&= \rho_d(x)\int_{\Omega} L_e(x, u \to x)\cos\theta_i d\omega \\
&= \rho_d(x)\int_{\Omega}\left(\frac{E(u)}{\pi}\right)\cos\theta_i d\omega \\
&= \rho_d(x)\int_{source}\left(\frac{E(u)}{\pi}\right)\cos\theta_i\left(\cos\theta_s\frac{dA_u}{r(x,u)^2}\right) \\
&= \rho_d(x)\int_{source} E(u)\frac{\cos\theta_i\cos\theta_s}{\pi r(x,u)^2}dA_u
\end{aligned}
$$

Interreflections

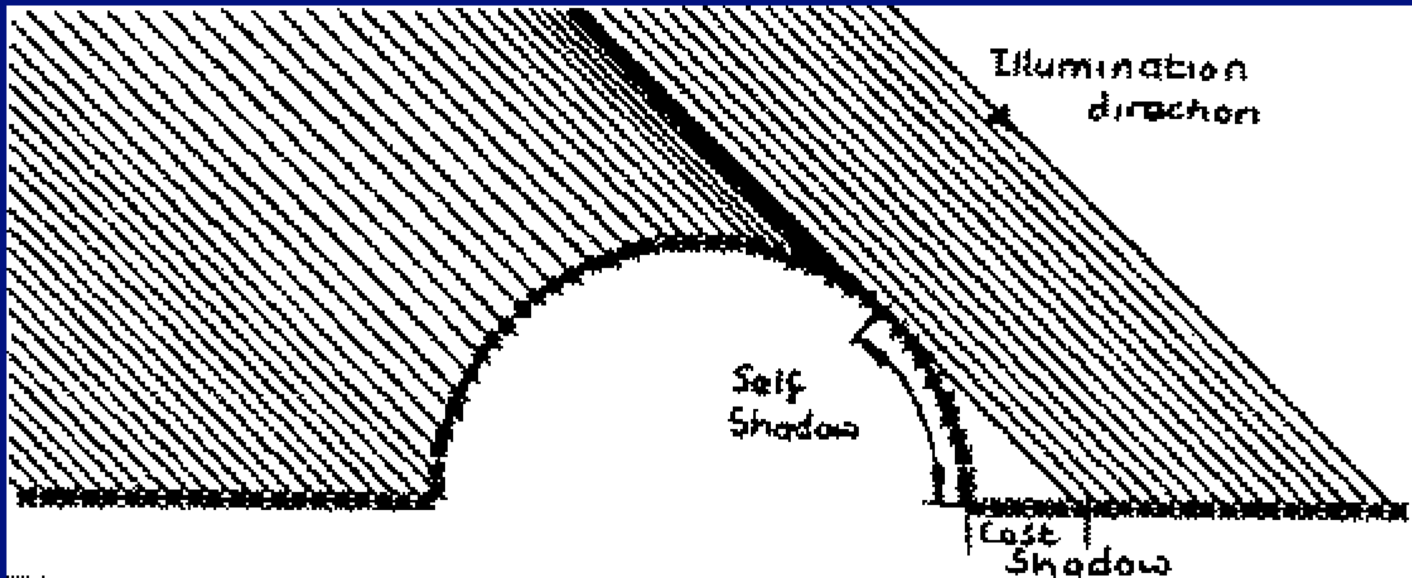
- Radiosity at surface=Exitance plus radiosity due to incoming radiosity from all other surfaces

- This gives an integral equation (below)
- Vis(x, u) is 1 if they can see each other, 0 if they can't
- Well understood by the graphics community, with many tricks known

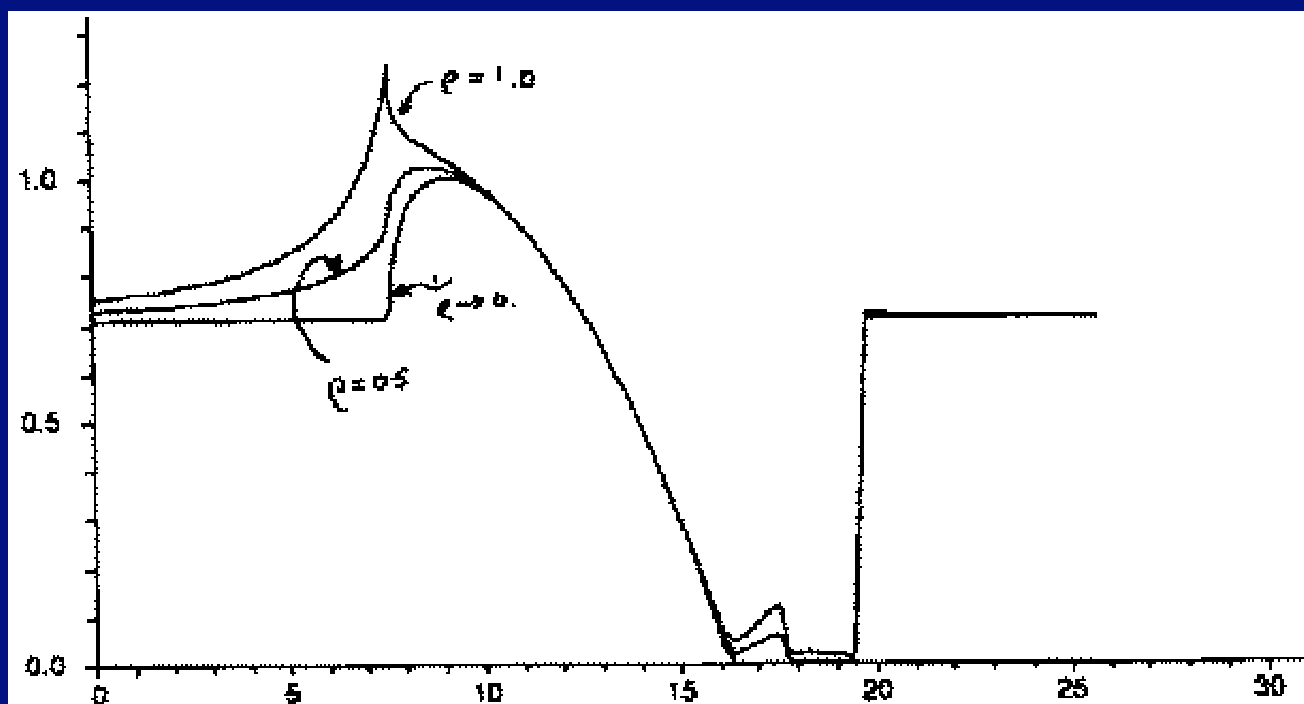
$$B(x) = E(x) + \rho_d(x) \int_{\substack{\text{all other} \\ \text{surfaces}}} B(u) \frac{\cos\theta_i \cos\theta_s}{\pi r(x,u)^2} Vis(x,u) dA_u$$

What do we do about this?

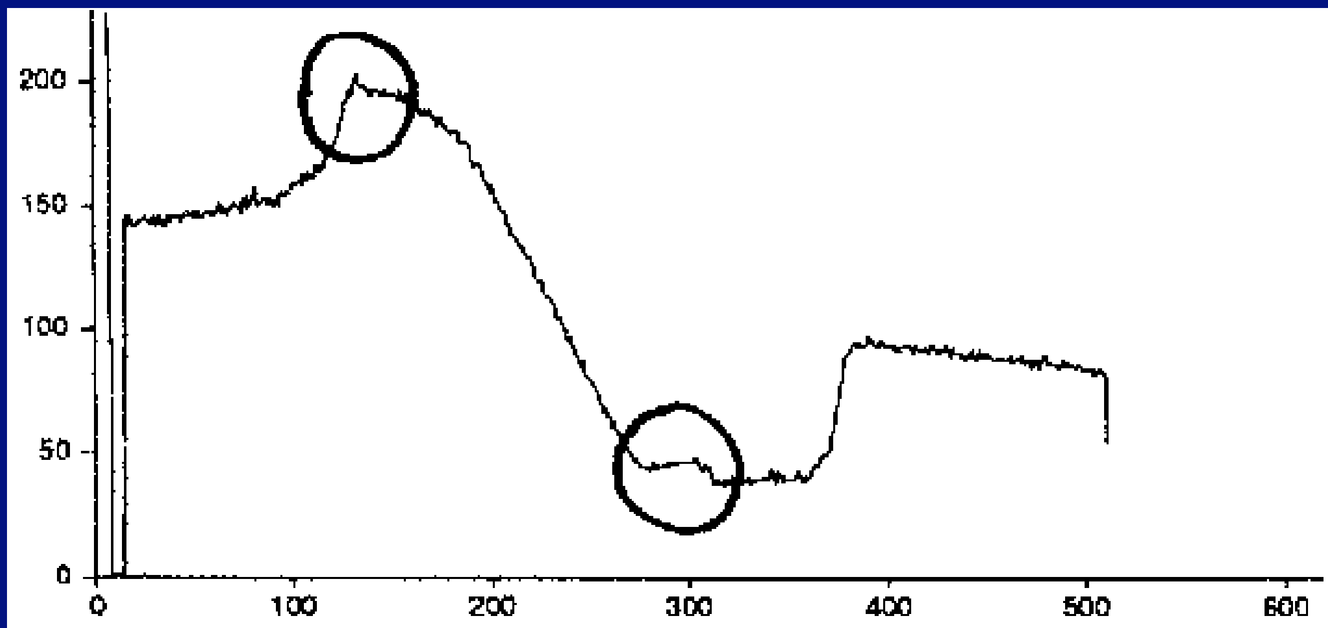
- Attempt to build approximations
  - Ambient illumination
- Study qualitative effects
  - reflexes
  - decreased dynamic range
  - smoothing
- Try to use other information to control errors



Illumination direction
Self Shadow
Cast Shadow



ρ = 1.0
ρ → 0.
ρ = 0.5
1.0
0.5
0.0
0
5
10
15
20
25
30


200
150
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50
0
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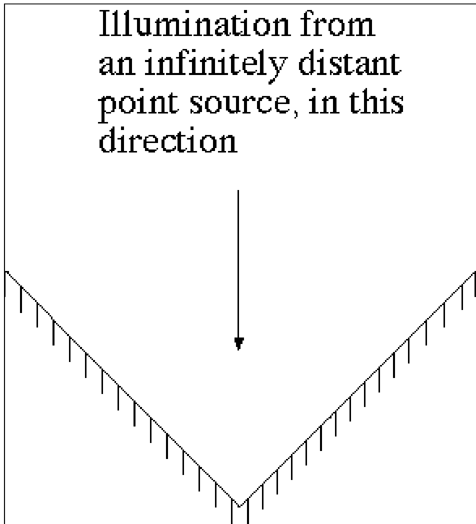
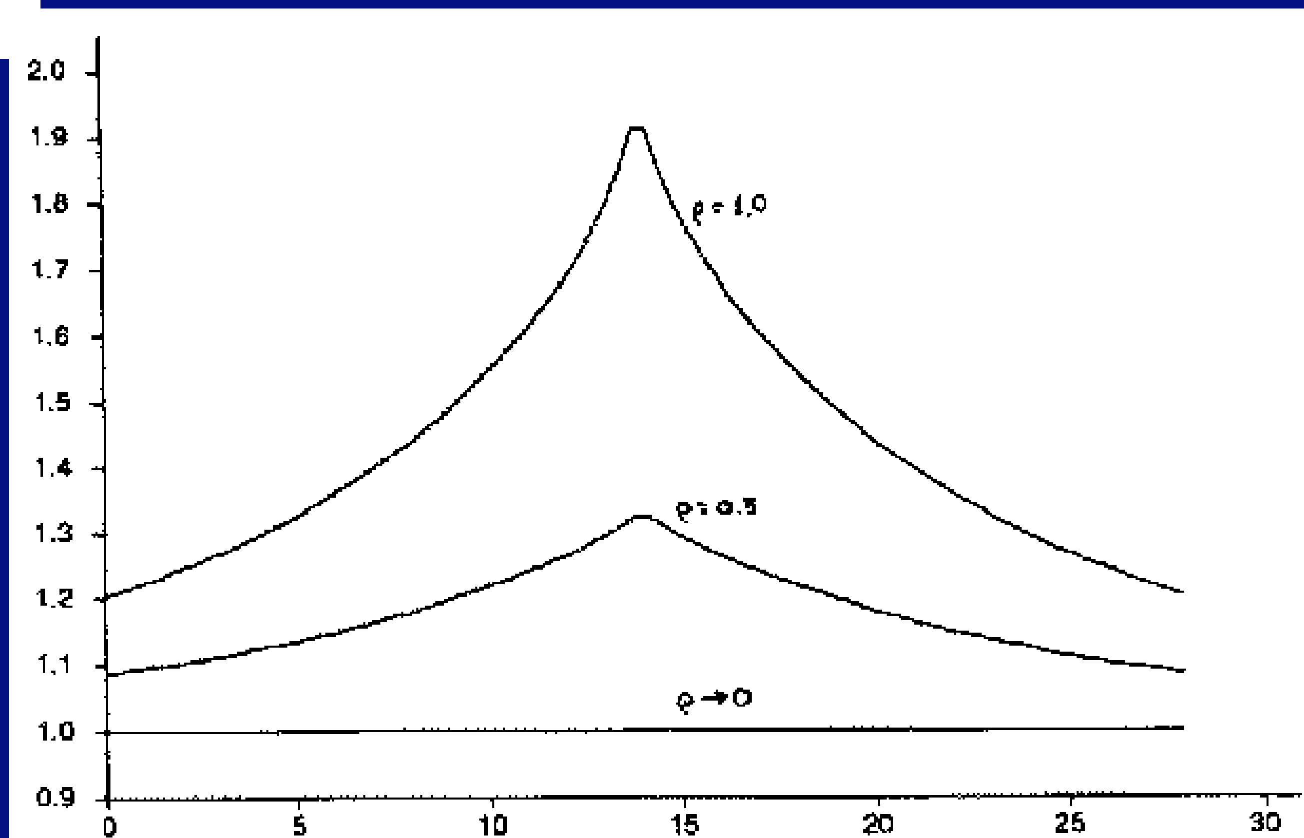
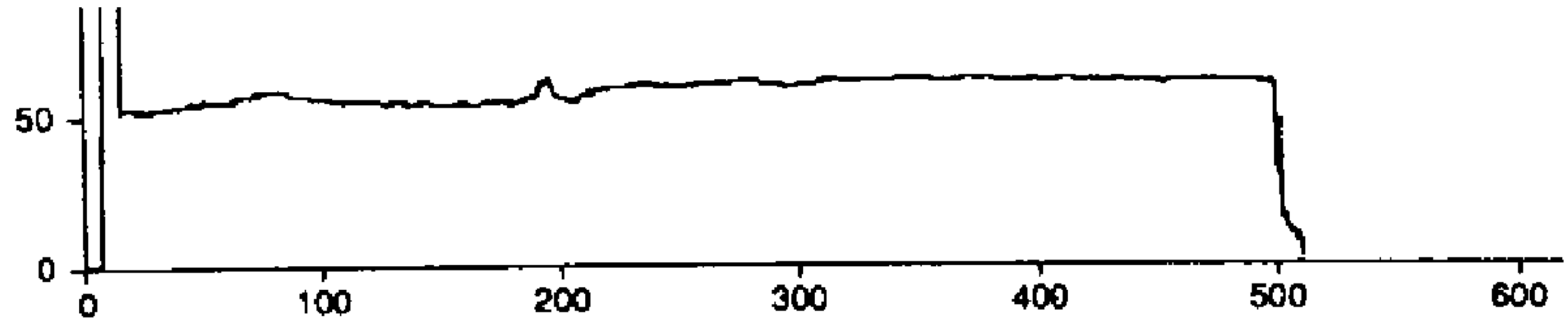

Illumination from an infinitely distant point source, in this direction


ρ = 1.0
ρ = 0.5
ρ → 0
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0
5
10
15
20
25
30


50
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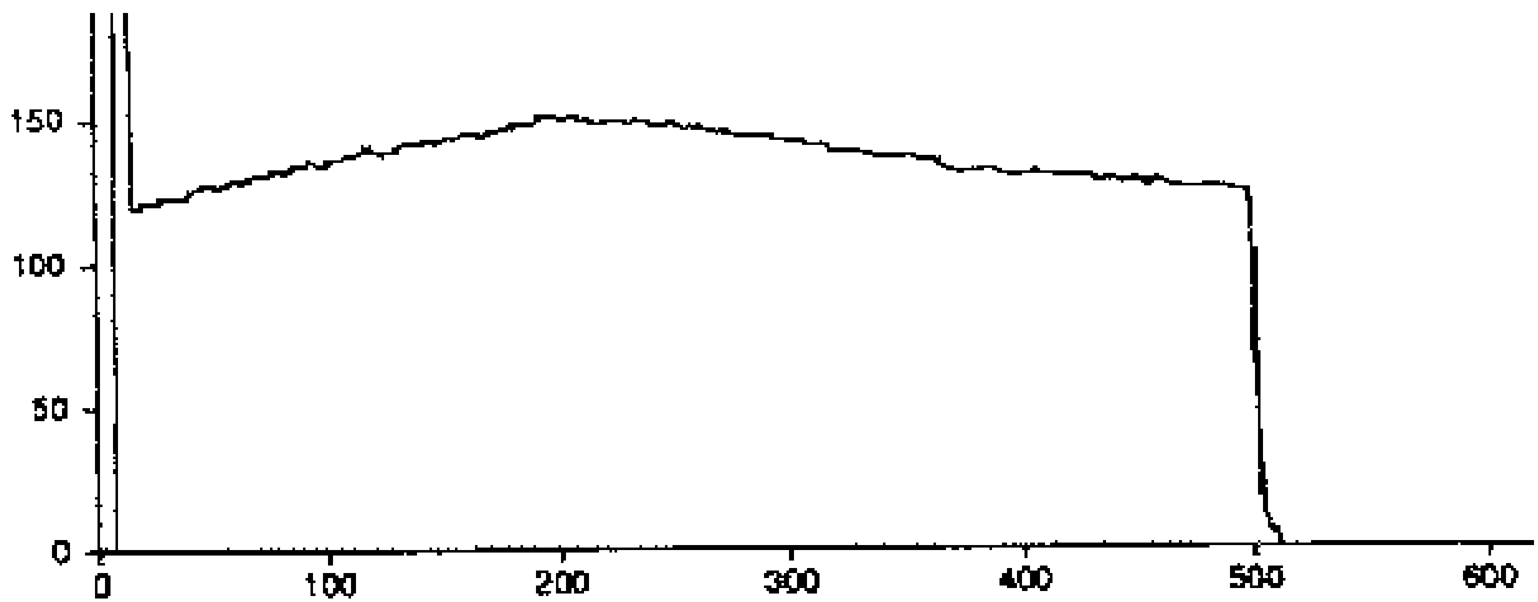
150
100
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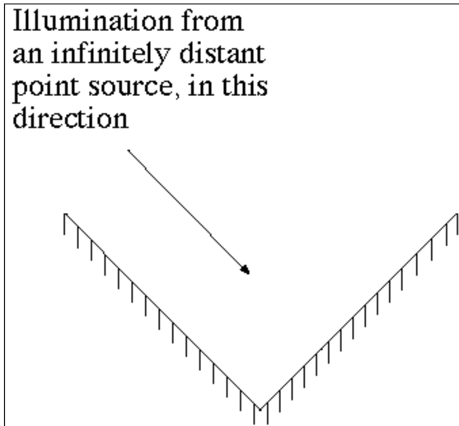
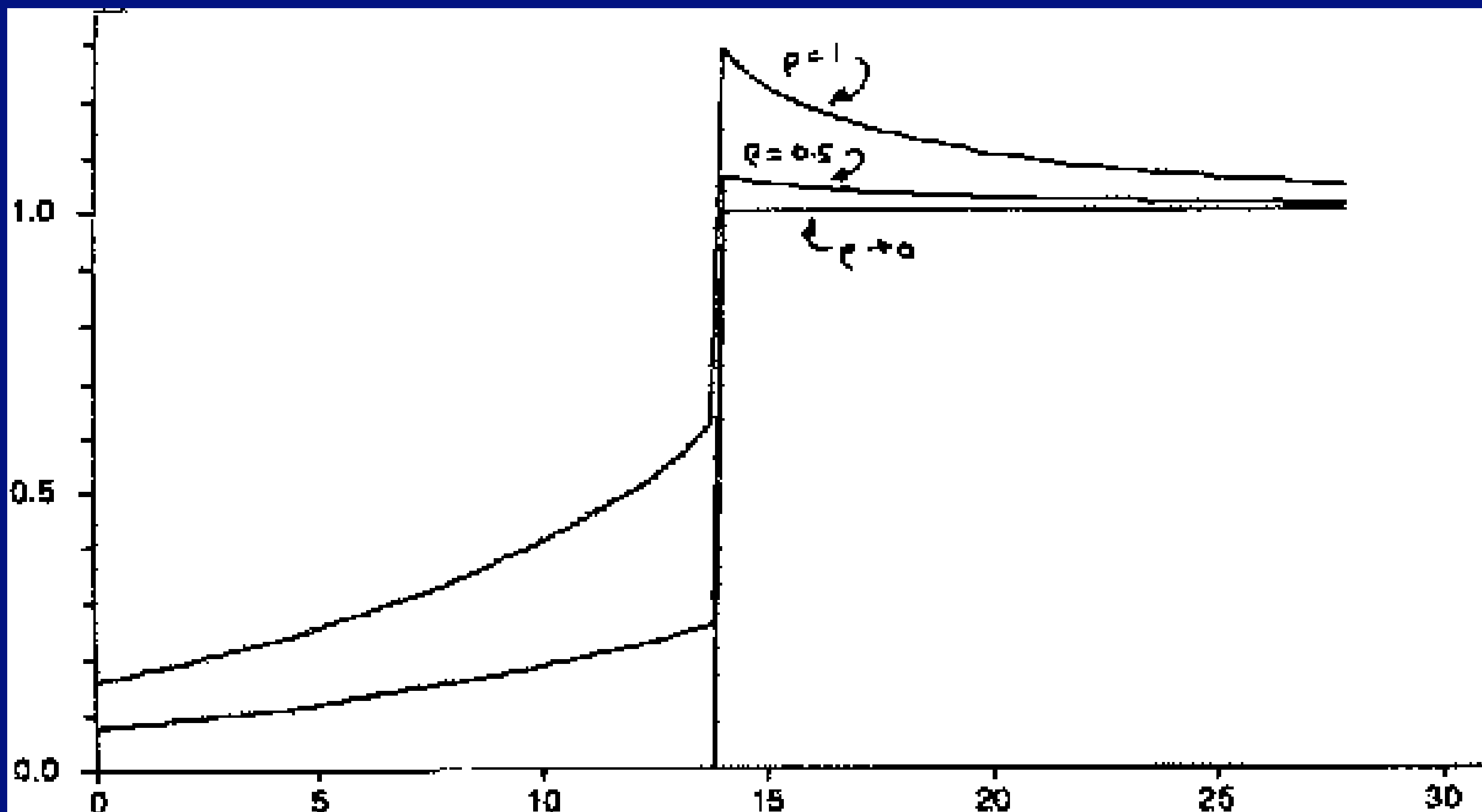

Illumination from
an infinitely distant
point source, in this
direction


1.0
0.5
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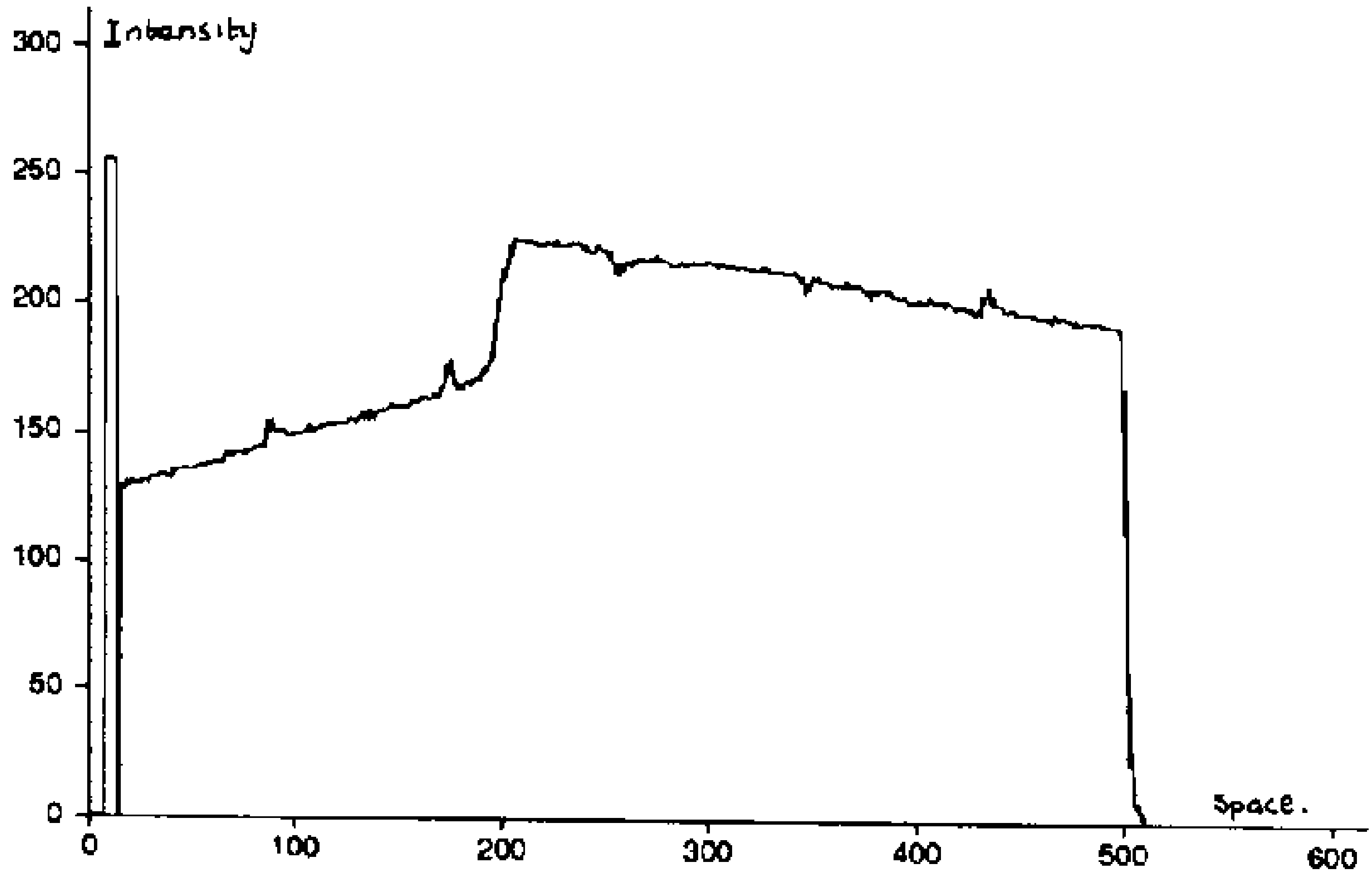

Intensity
300
250
200
150
100
50
0
0
100
200
300
400
500
600
Space.