Radiosity methods

Core ideas: Neumann series

- We have

$$B(x) = E(x) + \rho_d(x) \int_{\substack{\text{all other}\\ \text{surfaces}}} B(u) \frac{\cos\theta_i \cos\theta_s}{\pi r(x,u)^2} Vis(x,u)\, dA_u$$

- Can write:

$$B = E + \rho K B$$

- Which gives

$$B = E + (\rho K)E + (\rho K)(\rho K)E + (\rho K)^3 E + \ldots$$

Exitance

Source term

One bounce

Two bounces

Core ideas: gathering

- Recall definition: $$\rho K F = \rho(x) \int K(x,u) F(u) du$$
- How to evaluate this integral at a point?
  - obtain $$u_i \sim p(u)$$
  - Form: $$\frac{1}{N} \sum K(x, u_i) F(u_i)$$
- Similar to evaluating illumination from area source

Core ideas: gathering

- What is a good p(u)?
  - p(u) should be big when K(x, u) F(u) is big
  - this helps to control variance
  - known as importance sampling
  - Significant considerations:
    - fast variation in F(u)
    - fast variation in K
      - usually due to visibility
- How many samples?
  - fixed number
    - may be expensive, ineffective
  - by estimate of variance
    - this goes down as 1/N, which is very bad news

Core ideas: the final gather

- Notice:

$$B = E + (\rho K)B$$

- Assume that I have a very rough estimate of B
  - I could render this using

$$B = E + (\rho K)\hat{B}$$

  - This isn't such a good idea, instead use

$$B = E + (\rho K)E + (\rho K)(\hat{B} - E)$$

  - This is a very good idea indeed, because K smooths

Computing the integrals

- Two terms

$$\rho(x) \int K(x,u)E(u)du$$

  - source term
    - we expect to need multiple samples, some large values, large changes over space
    - large variance will be ugly - should compute this term carefully at each point to render
  - indirect term
    - this term should change slowly over space, and should be smaller in value
    - large variance less ugly - we can use fewer samples and pool samples

$$\rho(x) \int K(x,u)(\hat{B}(u) - E(u))du$$

Obtaining an estimate: Finite elements

- Divide domain into patches
- Radiosity will be constant on each patch
  - patch basis function, or element

$$\phi_i(x) = \begin{cases} 1 & \text{if } x \text{ is on patch } i \\ 0 & \text{otherwise} \end{cases}$$

- Now write
  - B_i for radiosity at patch i
  - E_i for exitance at patch i
- Equation becomes:

$$\left(\sum_i B_i \phi_i(x)\right) - \left(\sum_j E_j \phi_j(x)\right) - \left(\rho(x) \int K(x,u) \sum_j B_j \phi_j(u) du\right) = R(x) = 0$$

Obtaining an estimate: Finite elements

- But in what sense is it zero?
  - Galerkin method

$$\int R(x)\phi_k(x)dx = 0 \forall k$$

- Apply to:

$$\left(\sum_i B_i\phi_i(x)\right) - \left(\sum_j E_j\phi_j(x)\right) - \left(\rho(x)\int K(x,u)\sum_j B_j\phi_j(u)du\right) = R(x) = 0$$

- And get

$$B_k A_k = E_k A_k + \sum_j \left(\int_{\text{patch } k} \rho(x) \int_{\text{patch } j} K(x,u)dudx\right) B_j$$

Finite Element Radiosity Equation

- Start with:

$$B_k A_k = E_k A_k + \sum_j \left(\int_{\text{patch } k} \rho(x) \int_{\text{patch } j} K(x,u) du dx \right)$$

- Divide through by A_k, assume constant albedo patches, get

$$B_k = E_k + \sum_k \rho_k F_j k B_j$$

- Where geometric effects are concentrated in the form factor

$$F_j k = \frac{1}{A_k} \int_{\text{patch } k} \int_{\text{patch } j} K(x,u) du dx$$

Form factors

- if patches are all flat, then:

$$F_{ii} = 0$$

- if i can't see j at all, then:

$$F_{ij} = 0$$

- reciprocity:

$$A_k F_{jk} = A_j F_{kj}$$

- interpretation:
  - Fjk is percentage of energy leaving k that arrives at j
  - this gives:

$$\sum_j F_{jk} = 1$$

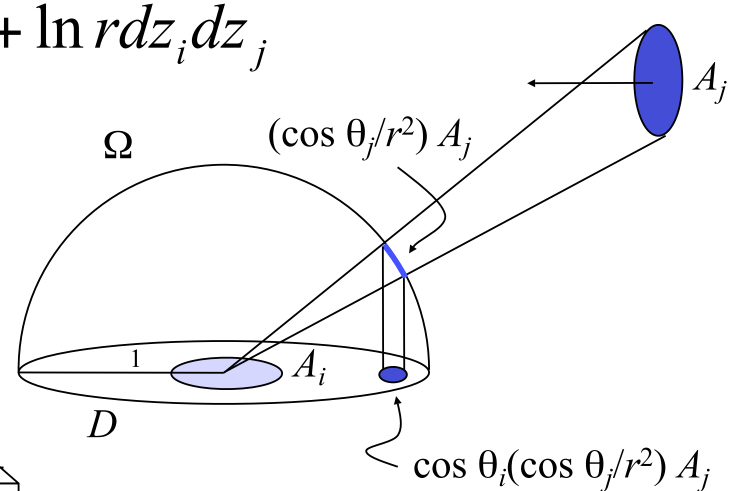
Computing Form Factors

- Stokes Theorem [Lambert 1760, Goral *et al.* S84]

$$F_{ij} = \frac{1}{2\pi A_i} \oint_{\partial A_i} \oint_{\partial A_j} \ln r\, dx_i dx_j + \ln r\, dy_i dy_j + \ln r\, dz_i dz_j$$

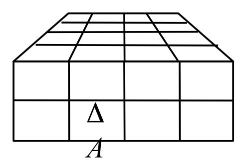
- Nusselt analog

$$F_{ij} = \mathrm{proj}_D(\mathrm{proj}_\Omega(A_j))/\mathrm{Area}(D)$$

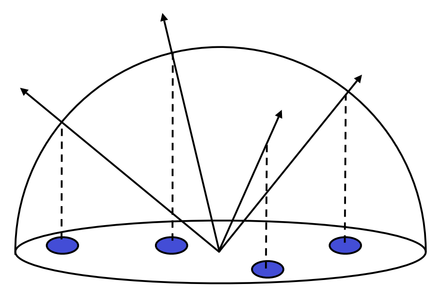


- Hemicube

$$\Delta F_{dAiAj} = \frac{\cos\phi_i \cos\phi_j}{\pi r^2} \Delta A$$



- Monte-Carlo Ray Casting
  - Uniformly sample disk
  - F_{ij} = # of rays hitting A_j / # of rays

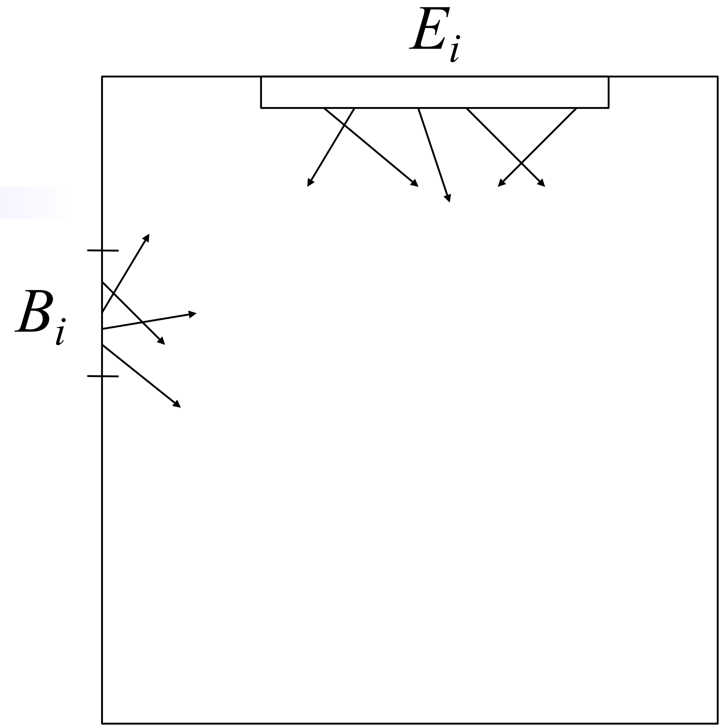


Matrix Radiosity

$$B_i A_i = E_i A_i + \rho_i \sum_{j=1}^{n} F_{ji} B_j A_j$$

$$A_i F_{ij} = A_j F_{ji}$$

$$\boxed{B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} B_j}$$



$$B_i - \rho_i \sum_{j=1}^{n} F_{ij} B_j = E_i$$

$$R = \begin{bmatrix} \rho_1 & & & \\ & \rho_2 & & \\ & & \ddots & \\ & & & \rho_n \end{bmatrix}$$

$$(I - RF)B = E$$

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

Solving the radiosity system: Gathering

- Neumann series (again!) $(I - \rho K)B = E$

$$B = E + \rho K E + (\rho K)^2 E + ...$$

$$B^{(0)} = E$$

$$B^{(n+1)} = E + \rho K B^{(n)}$$

Gathering with iterative methods

- Linear system Ax=b

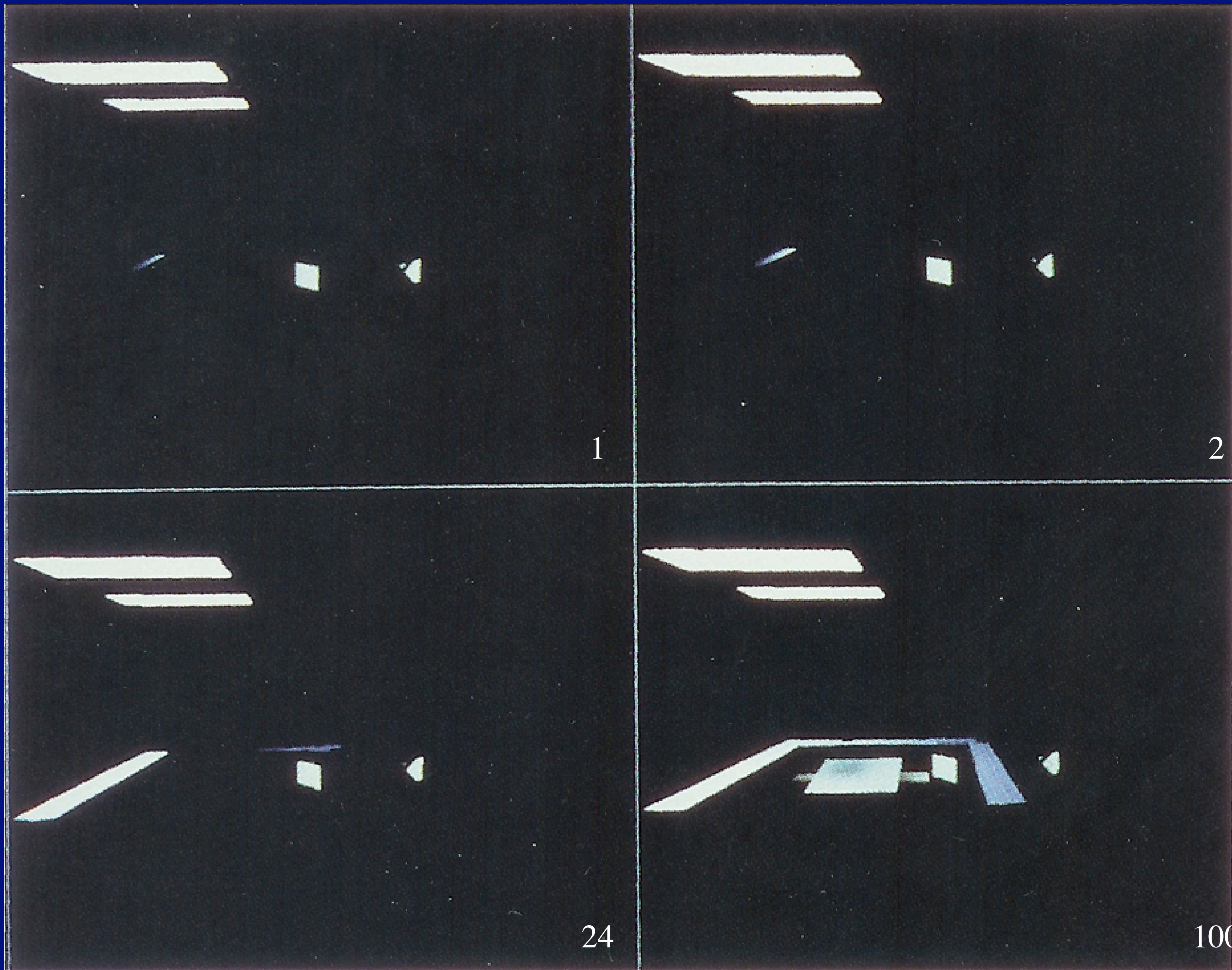
$$\sum_j a_{ij} x_j = b_i$$

- Jacobi iteration
  - reestimate each x

$$x_j^{(n+1)} = \frac{1}{a_{jj}} \left(b_i - \sum_{l \neq j} a_{il} x_l^{(n)} \right)$$

- Gauss-Seidel
  - reuse new estimates

$$x_j^{(n+1)} = \frac{1}{a_{jj}} \left(b_i - \sum_{l<j} a_{il} x_l^{(n+1)} - \sum_{l>j} a_{il} x_l^{(n)} \right)$$



From Cohen, SIGGRAPH 88

Southwell iteration: Progressive radiosity

- Gauss-Seidel, Jacobi, Neumann require us to evaluate whole kernel at each iteration
  - this is vilely expensive 10^6x 10^6 matrix?
  - it's also irrational
    - in G-S, Jacobi, for one pass through the variables,
      - we gather at each patch, from each patch
        - but some patches are not significant sources
    - we should like to gather only from bright patches
      - or rather, patches should "shoot"
- This is Southwell iteration

Southwell iteration: update x

- Define a residual: $R = (b - Ax)$
  - whose elements are $$r_i^{(n)} = b_i - \sum_j a_{ij} x_j^{(n)}$$
- now choose the largest r_i
  - and adjust the corresponding x component to make it zero

$$r_i^{(n+1)} = 0$$

$$x_l^{(n+1)} = \left\{ \begin{array}{cc} x_l^{(n)} & \text{if } l \neq i \\ \frac{1}{a_{ii}} \left(r_i^{(n)} + a_{ii} x_i^{(n)} \right) & \text{if } l = i \end{array} \right\}$$

Southwell iteration: update r

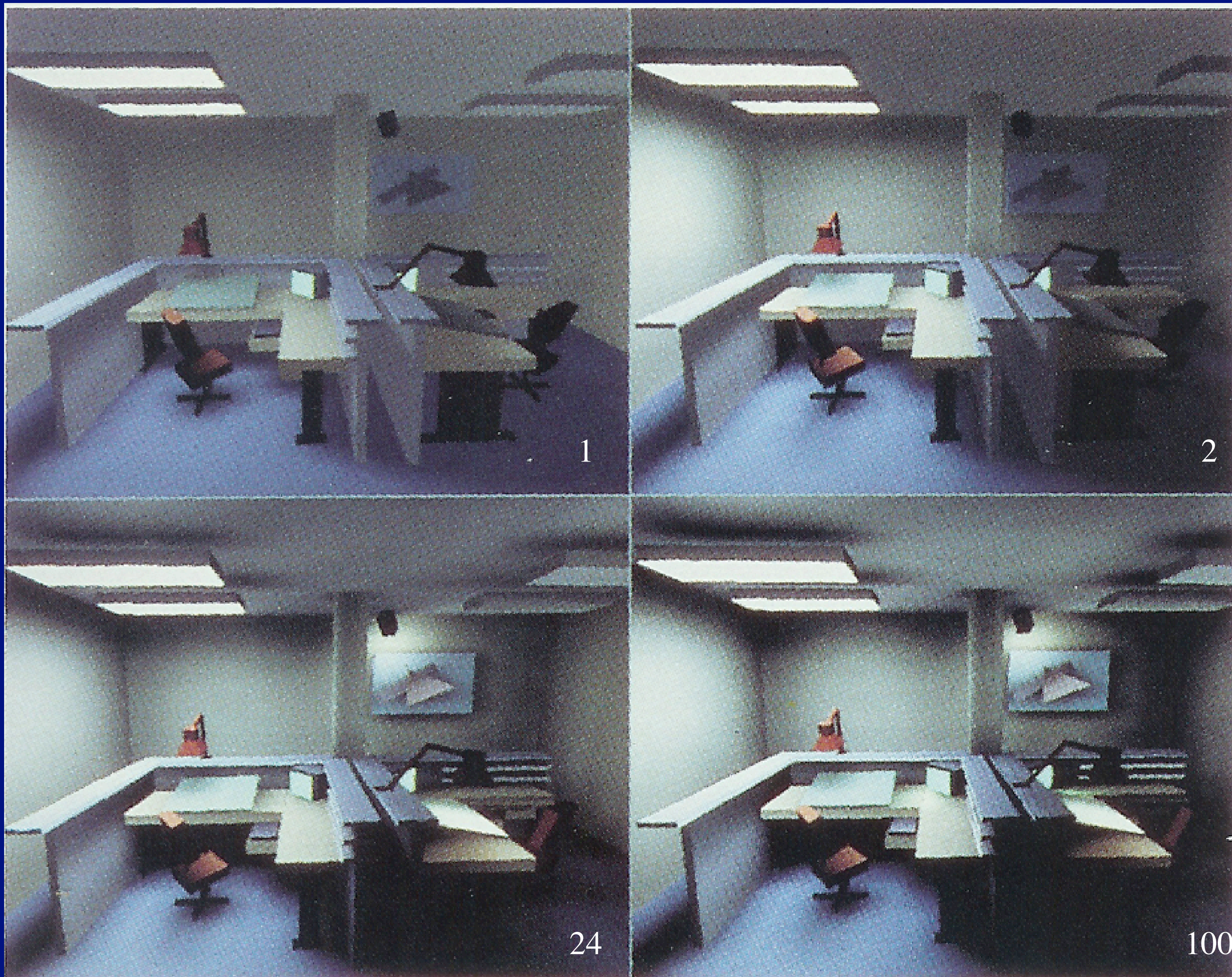
- Update the residual by adding old x col, subtracting new

$$r_l^{(n+1)} = r_l^{(n)} + a_{li}(x_i^{(n)} - x_i^{(n+1)})$$

- but this takes an easy form

$$r_l^{(n+1)} = r_l^{(n)} - \frac{a_{li}}{a_{ii}} r_i^{(n)}$$

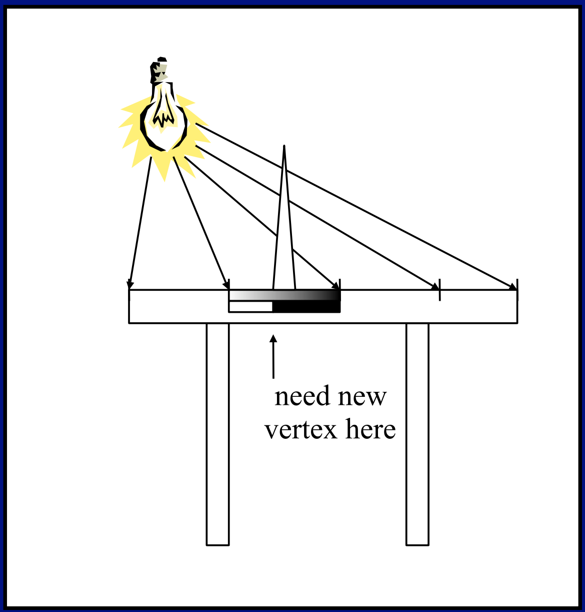
- Notice we can update variables in order of large residual, using only one col of kernel to do so
  - this converges (non-trivial) rather fast (non-trivial)
  - to get a solution, we need evaluate only a small proportion of the kernel (non-trivial)

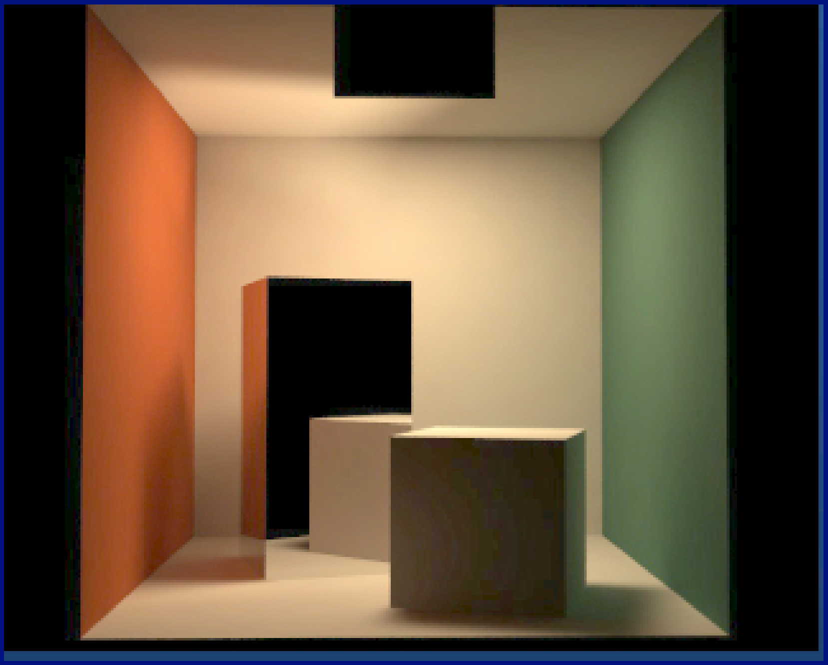


From Cohen, SIGGRAPH 88

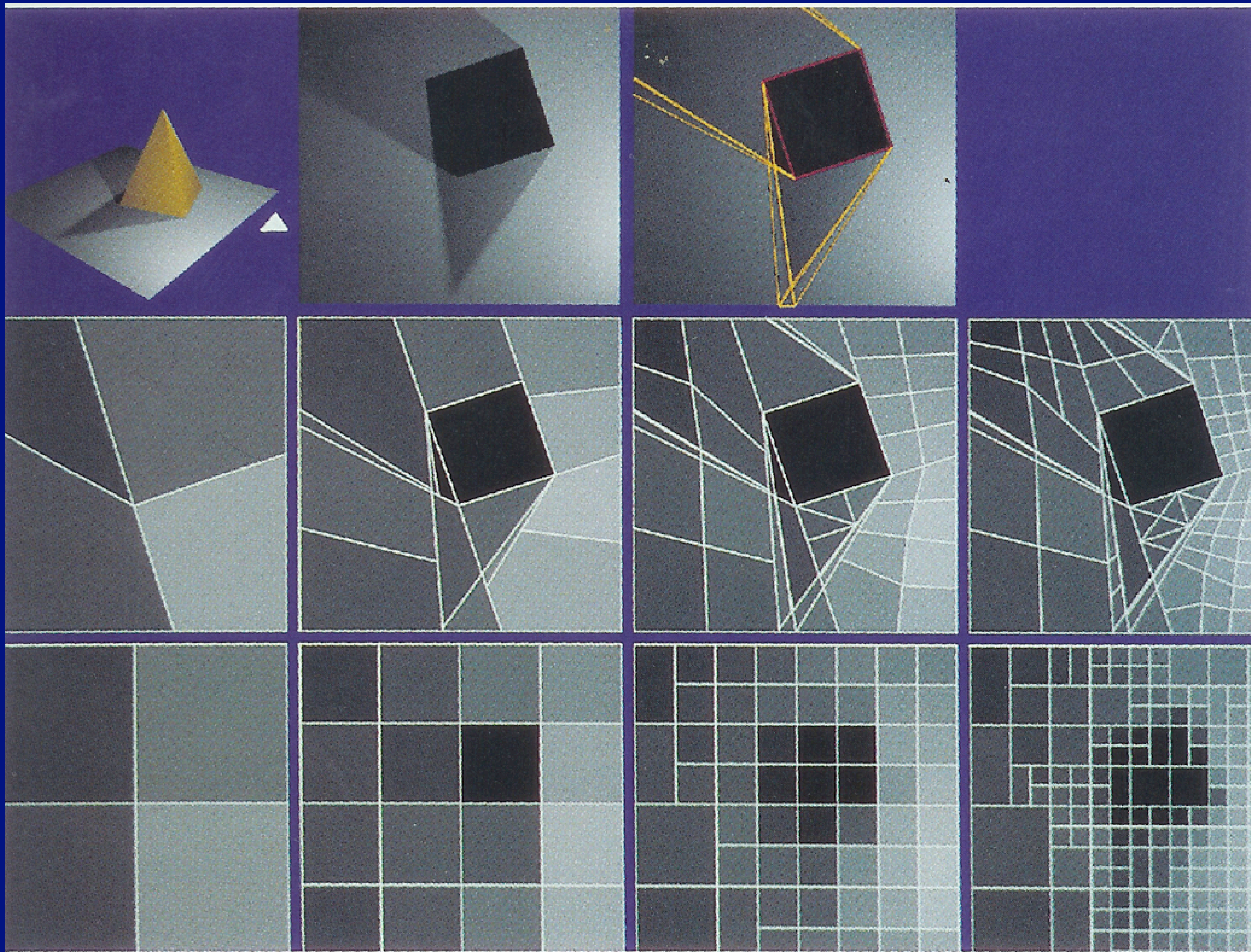
Important nuisances

- Light leaks
- Shadow problems
- Mesh complexity

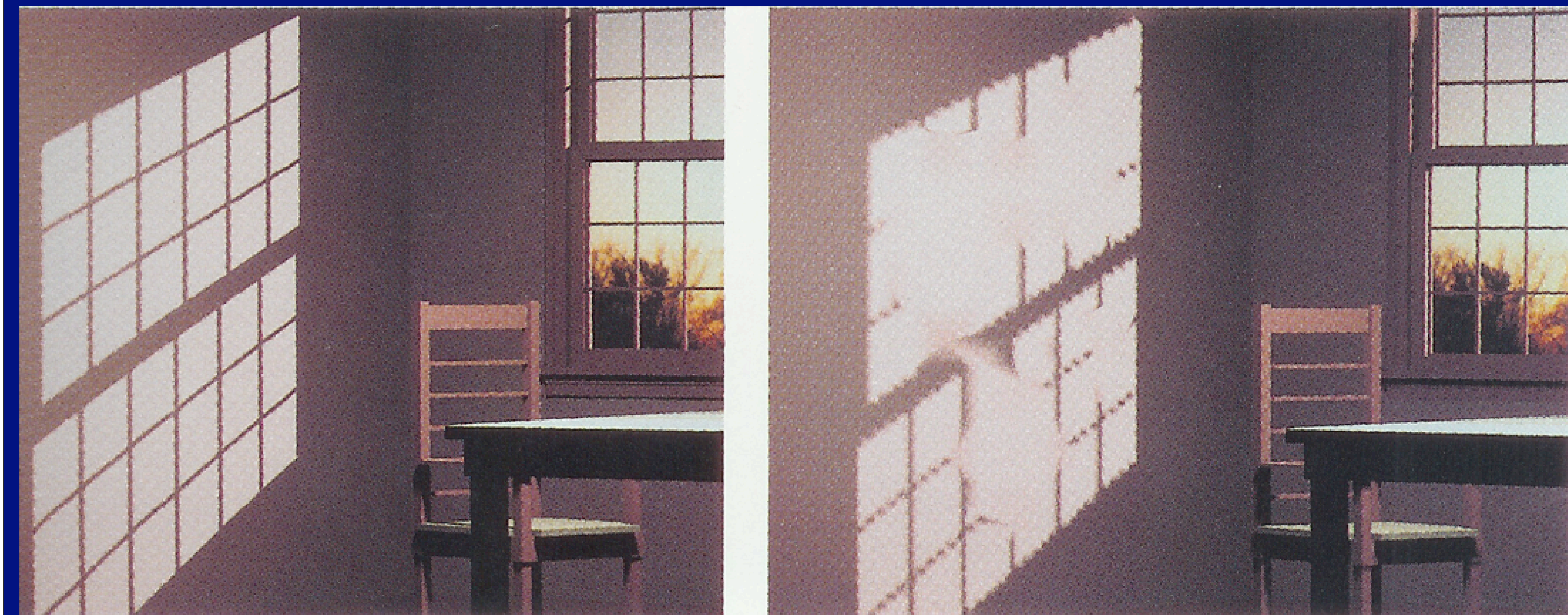




Cornell Program of Computer Graphics



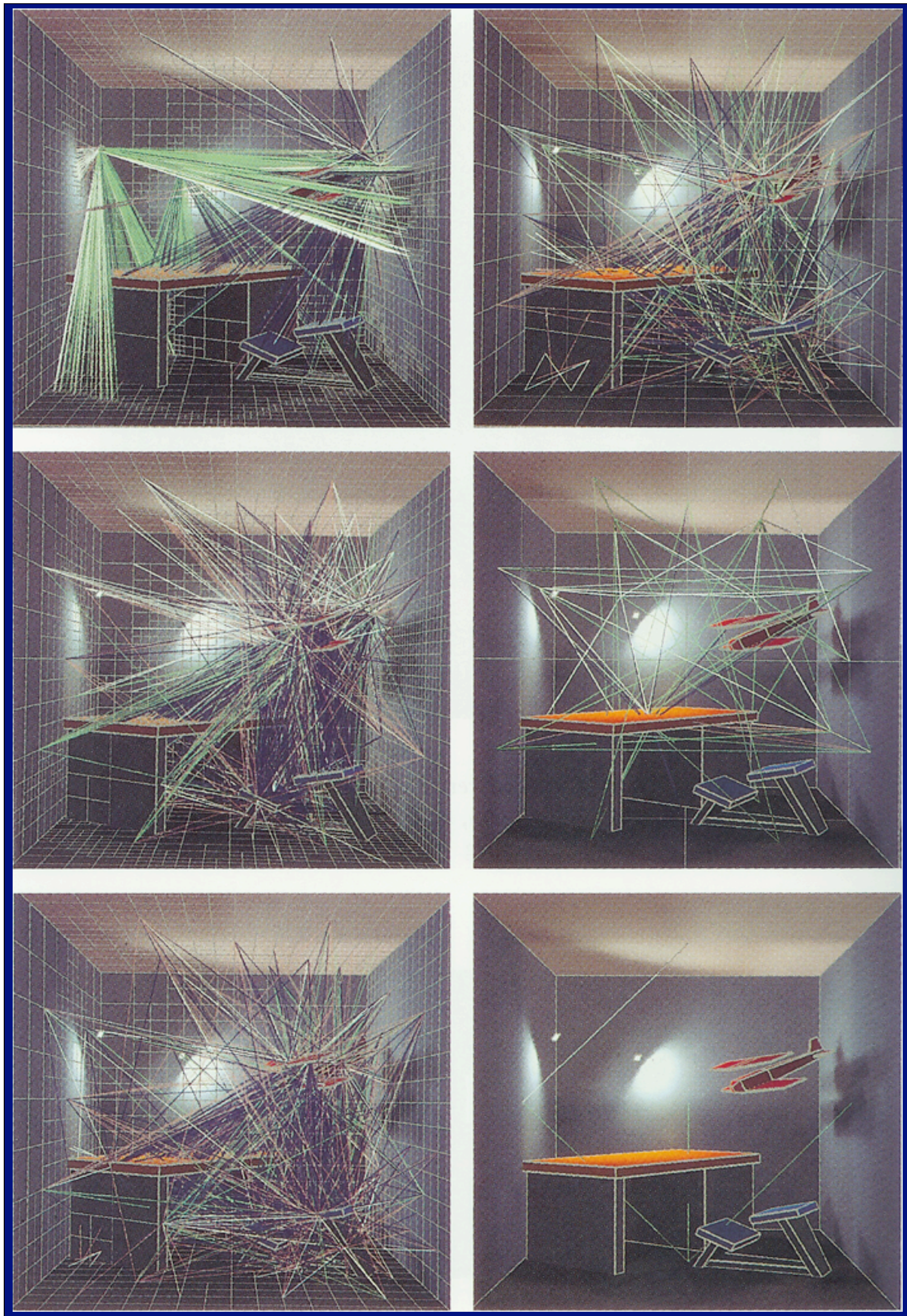
Lischinski et al S93



Lischinski et al S93

Hierachical radiosity

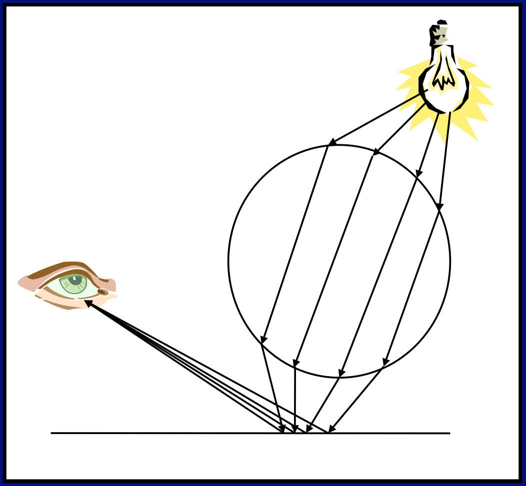
- Radiosity similar to n-body problems
  - gathering can be grouped
- Build mesh hierarchy using link oracle
  - F-linking
  - BF-linking
- Solve by
  - iterate
    - gather along links
    - push to leaves
    - pull from leaves
  - this (with work) is a Neumann series (again!)

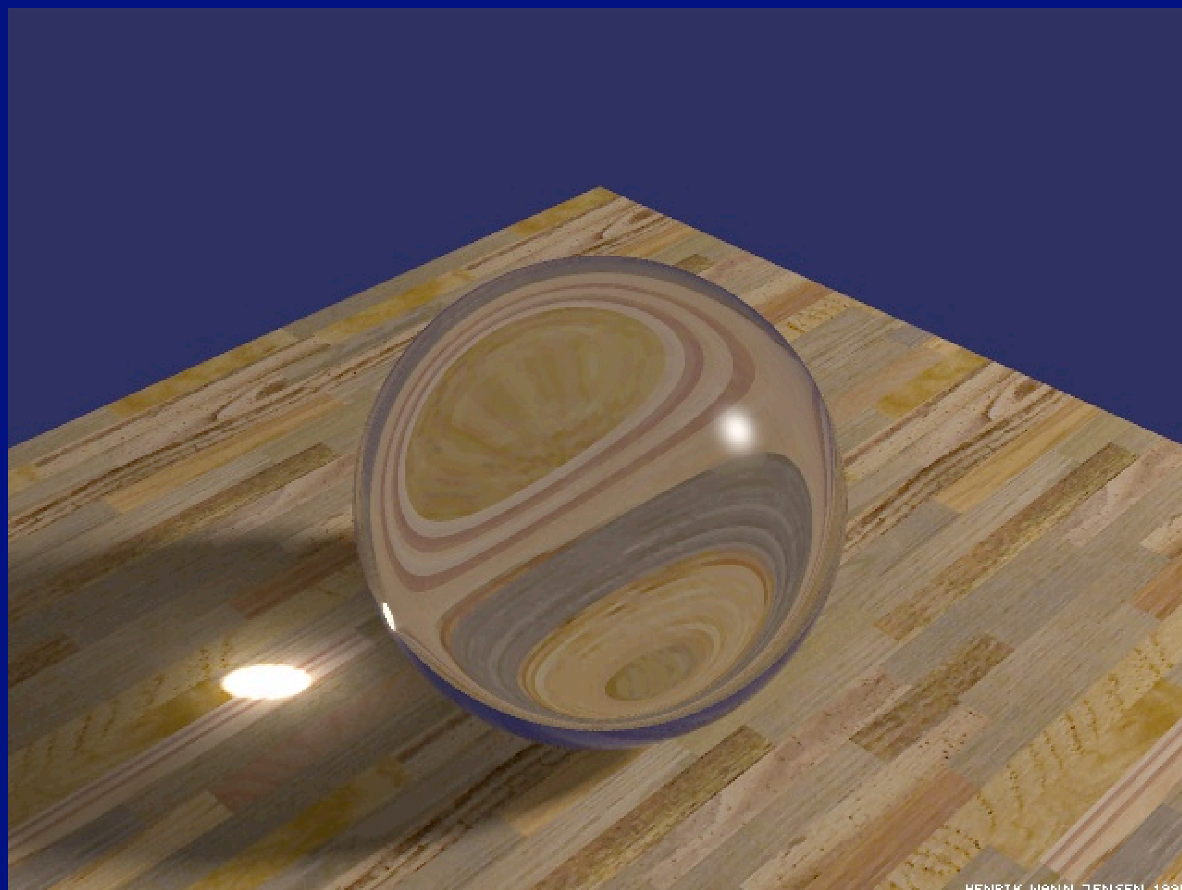


BIF links, from Hanrahan et al, 91

Other ways to get a rough solution

- Randomized integration (again!)
  - radiosity at a sample point is
    - a sum of contributions over paths that reach the light
    - these paths are fairly easily sampled
  - sample points are very highly correlated in space
    - radiosity values don't change much over space
- This viewpoint will allow us to deal with important effects
  - Refraction caustics
  - Reflection caustics

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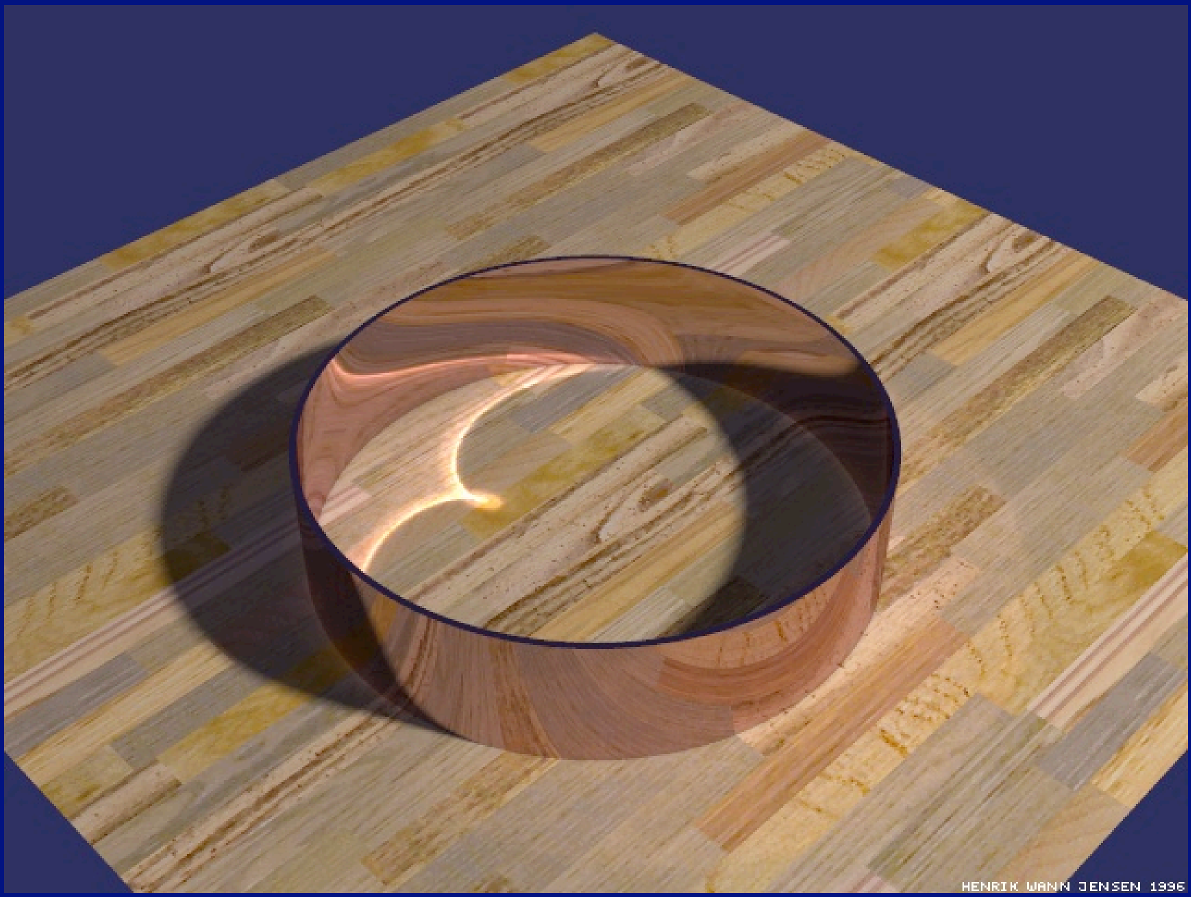
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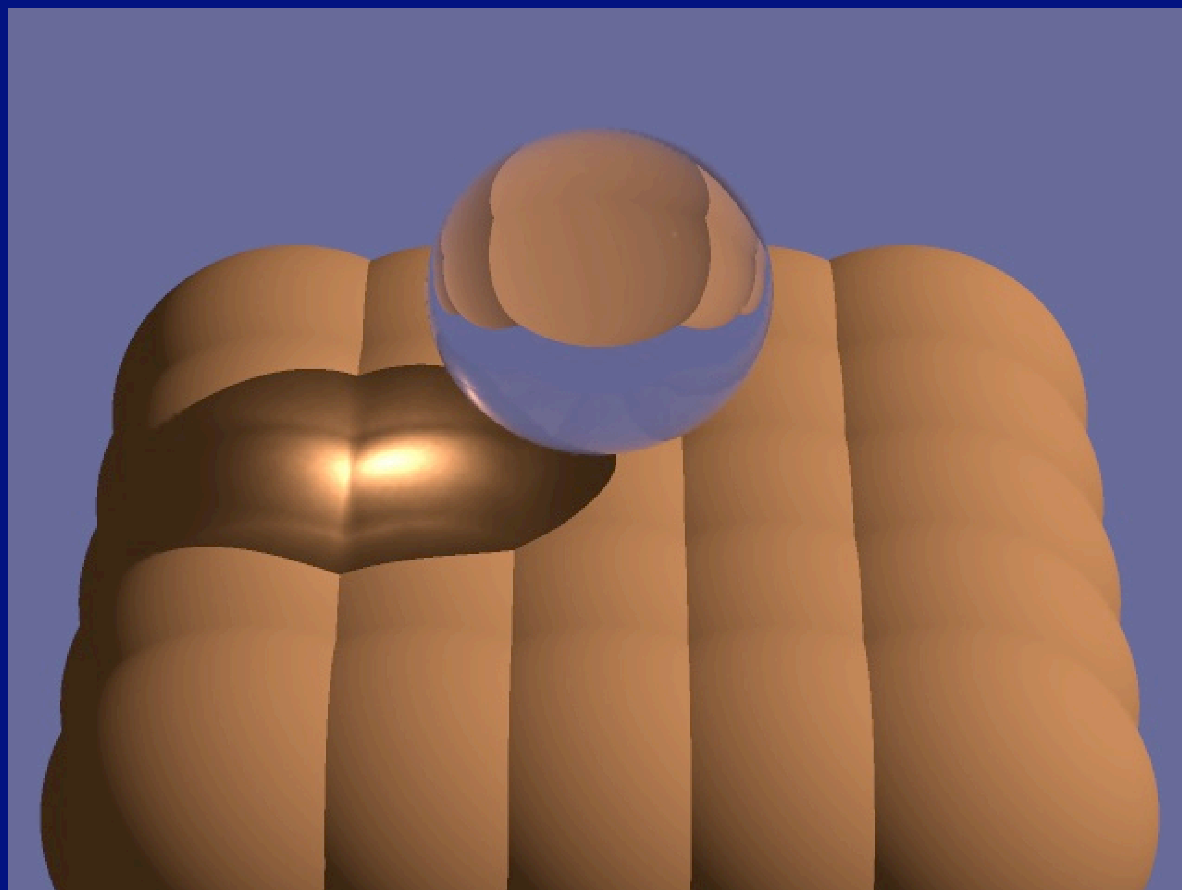

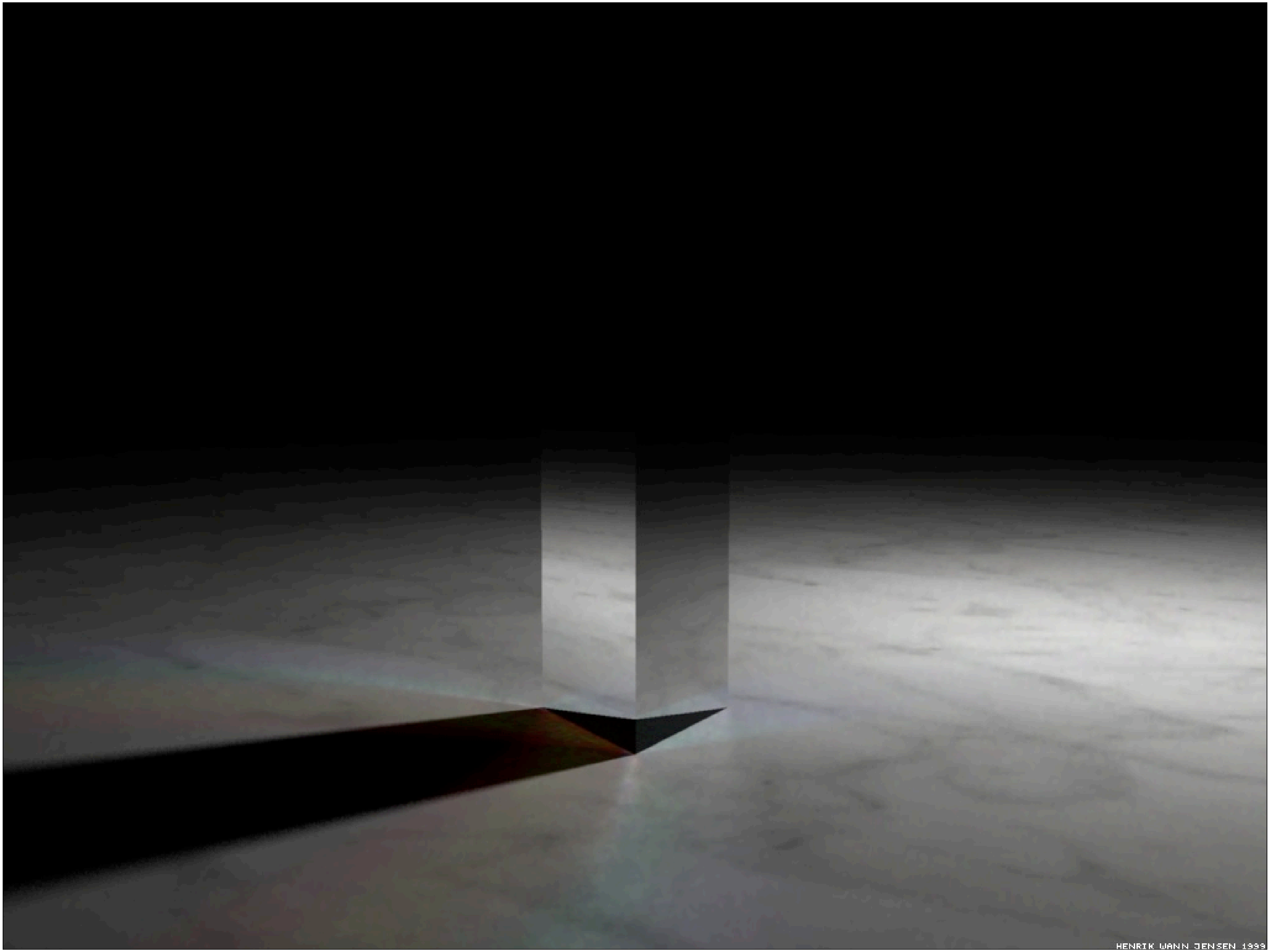
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