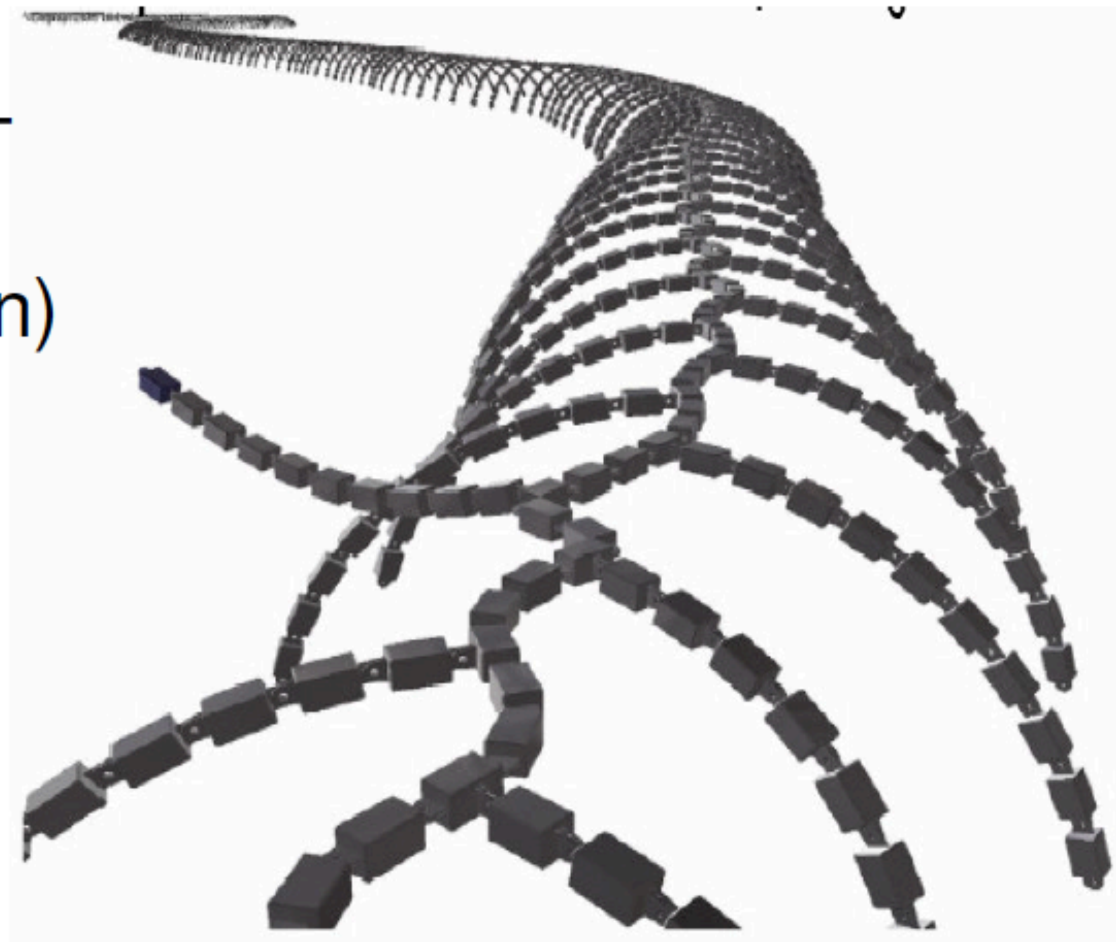
Motion Planning II

D.A. Forsyth
(with a lot of H. Choset, and some J. Li)

Large C-Space Dimension

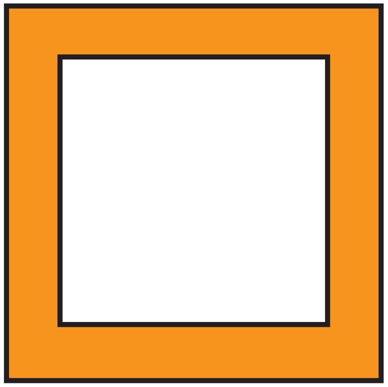
Millipede-like robot (S. Redon)



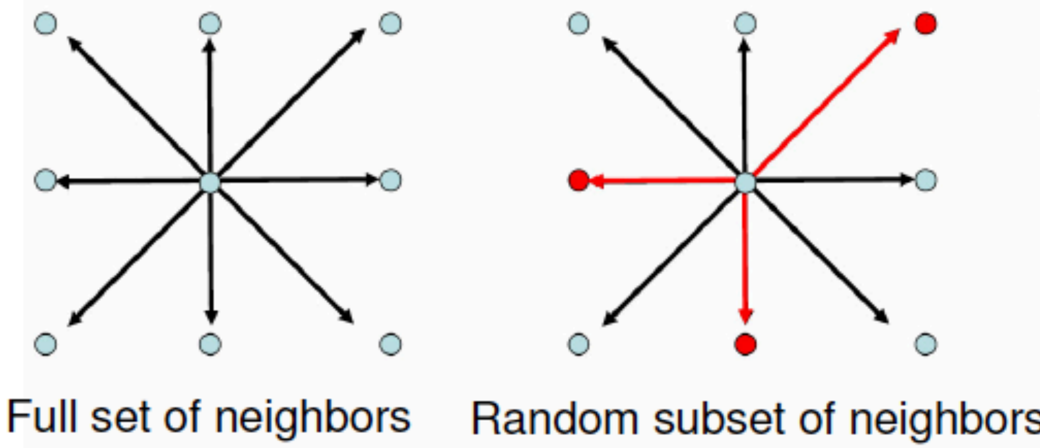
~13,000 DOFs !!!

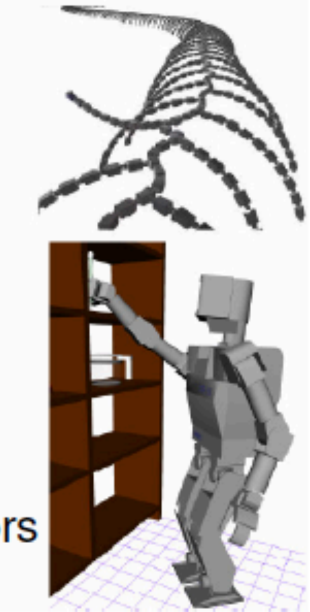
Dimension and its nuisances

- Counting:
  - A d-dimensional cube has 2^d vertices
- Volume:
  - your intuitions about volume are wrong in high dimension
  - consider cubical "orange" in high d
    - skin depth e/2
    - pulp (1-e)
    - volume of pulp:
      - (1-e)^d
    - volume of skin:
      - 1-(1-e)^d
    - IT'S ALL SKIN!
  - Almost all the volume of high d objects is very close to surface



Dealing with C-Space Dimension





- We should evaluate all the neighbors of the current state, but:
- Size of neighborhood grows exponentially with dimension
- Very expensive in high dimension

Solution:

- Evaluate only a random subset of K of the neighbors
- Move to the lowest potential neighbor

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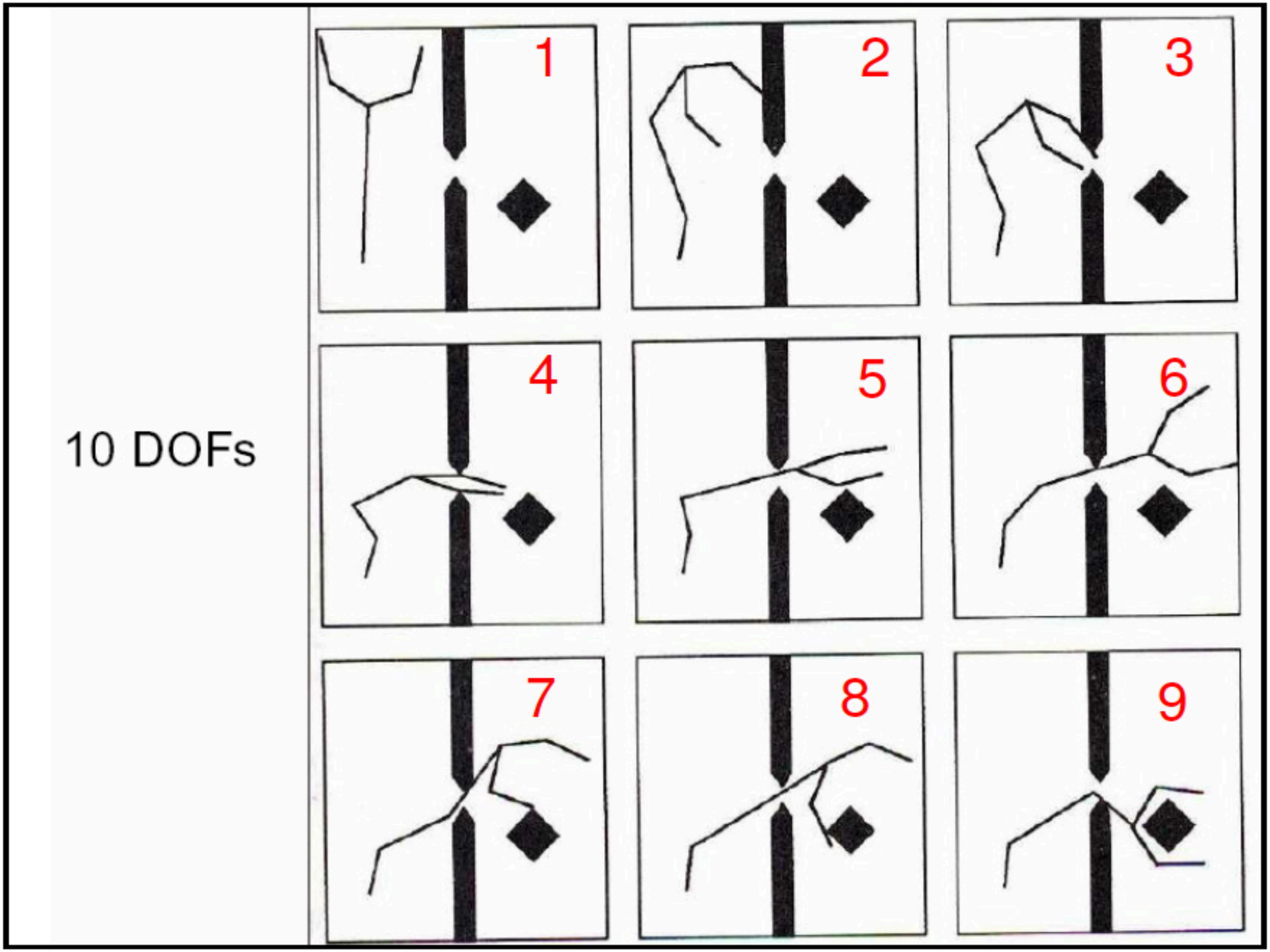

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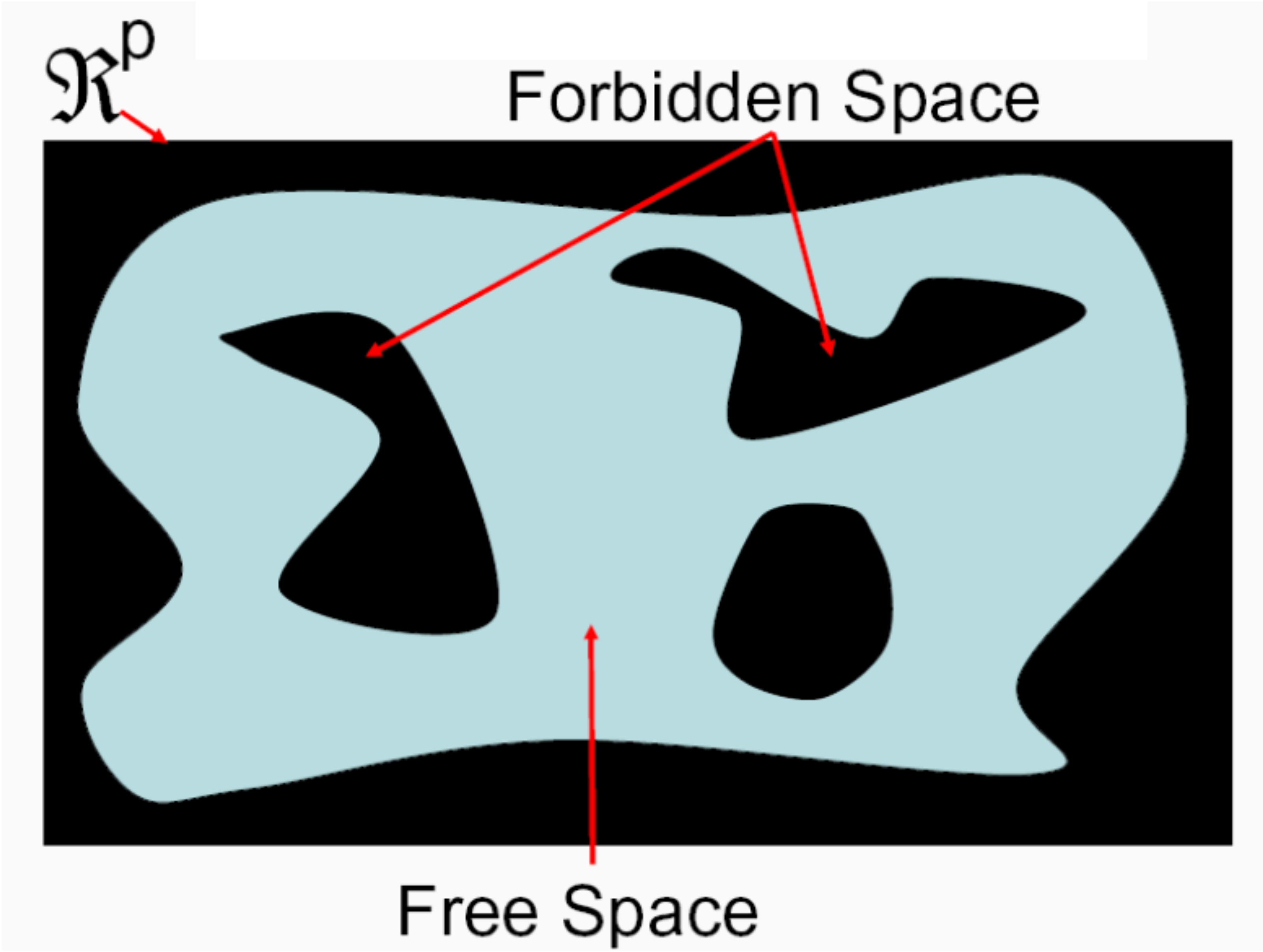
Sampling Techniques



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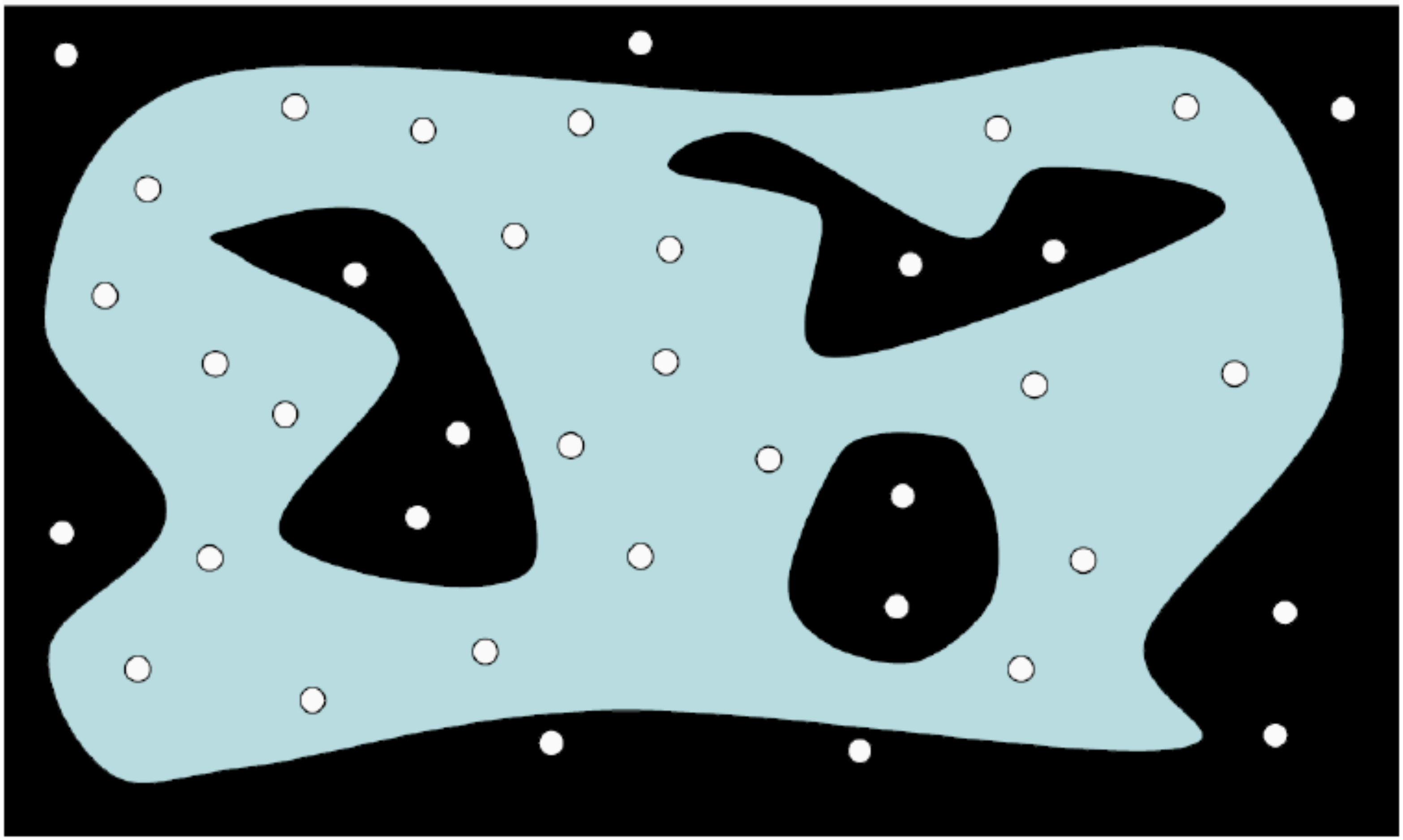
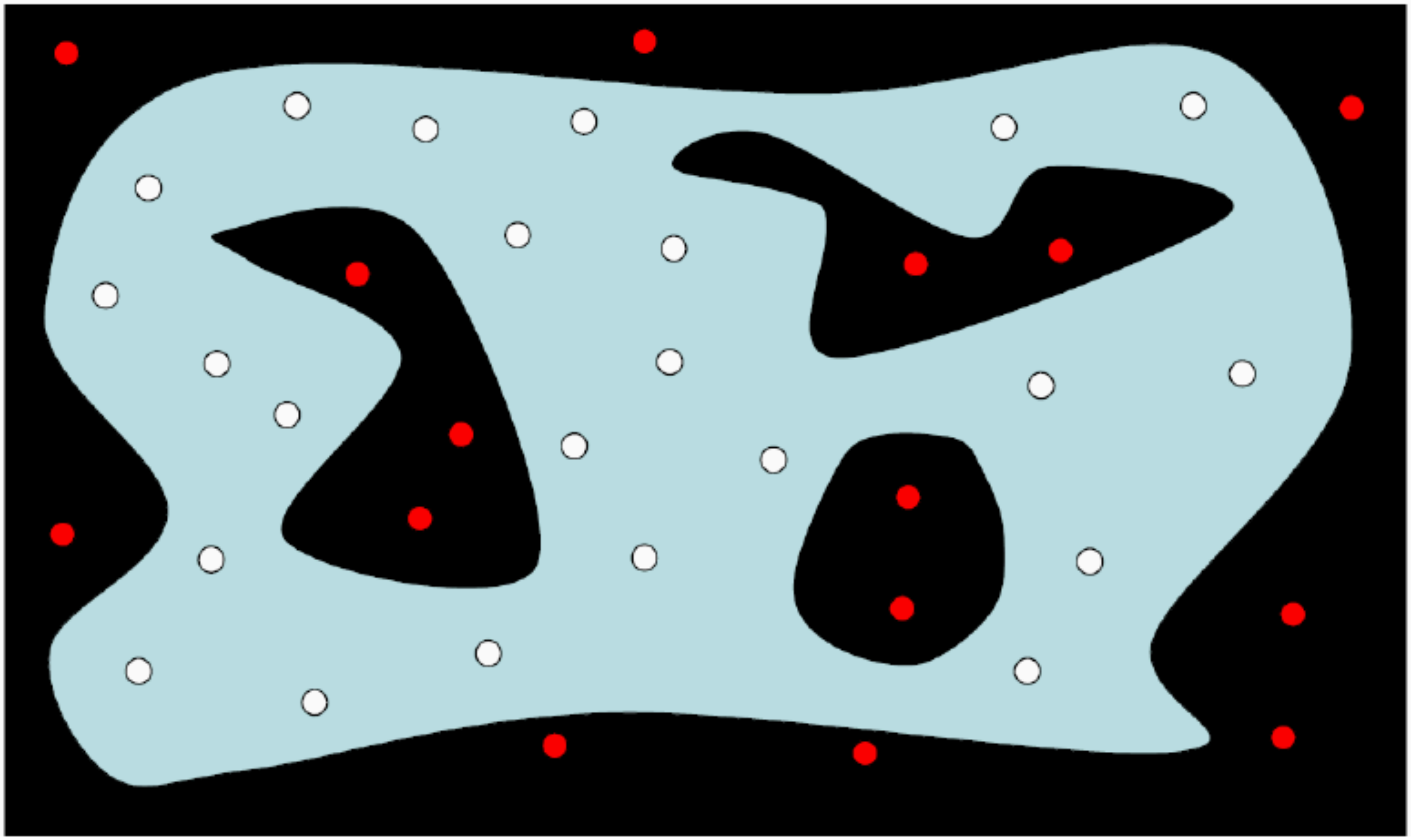
Sampling Techniques

Sample random locations



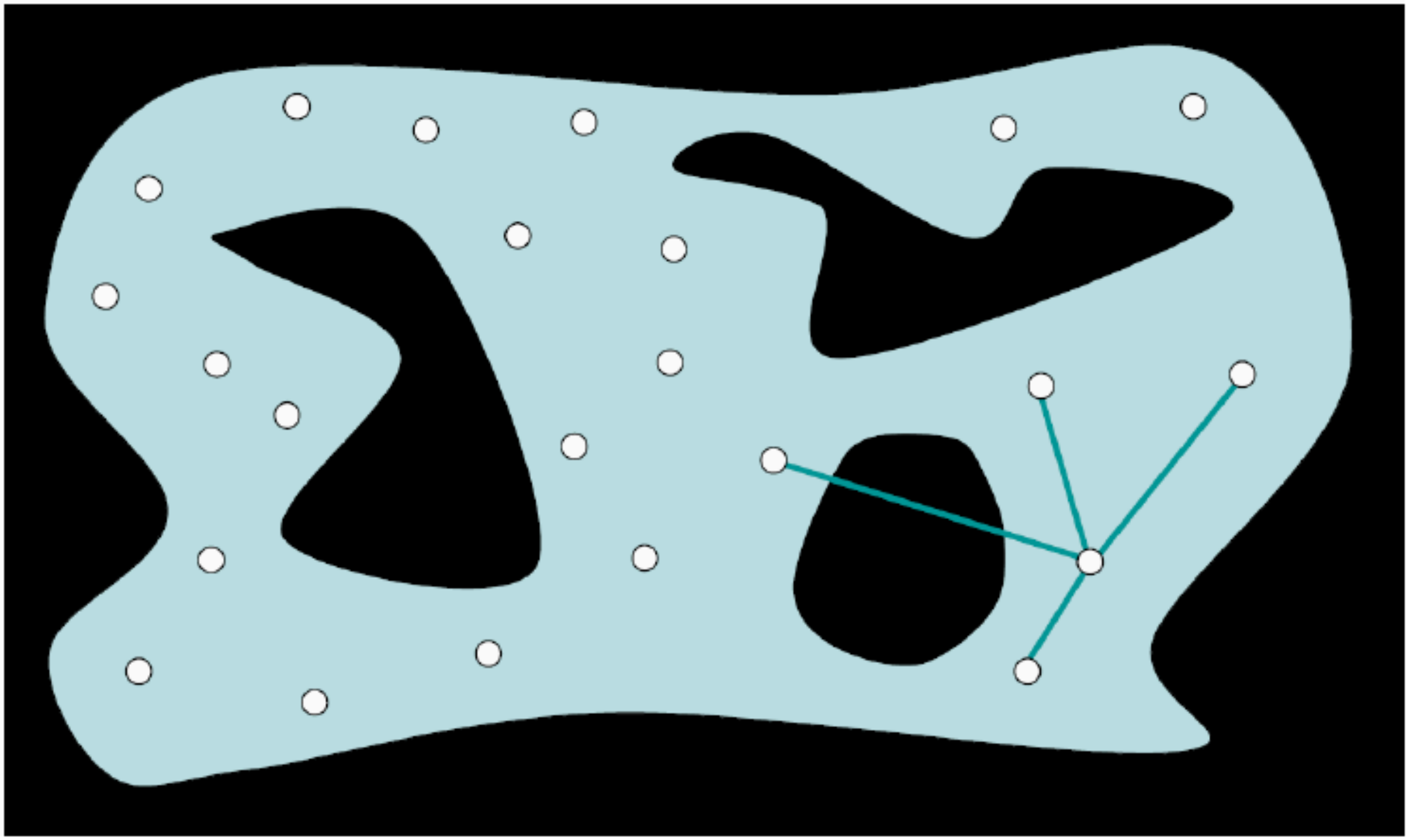
Sampling Techniques

Remove the samples in the forbidden regions



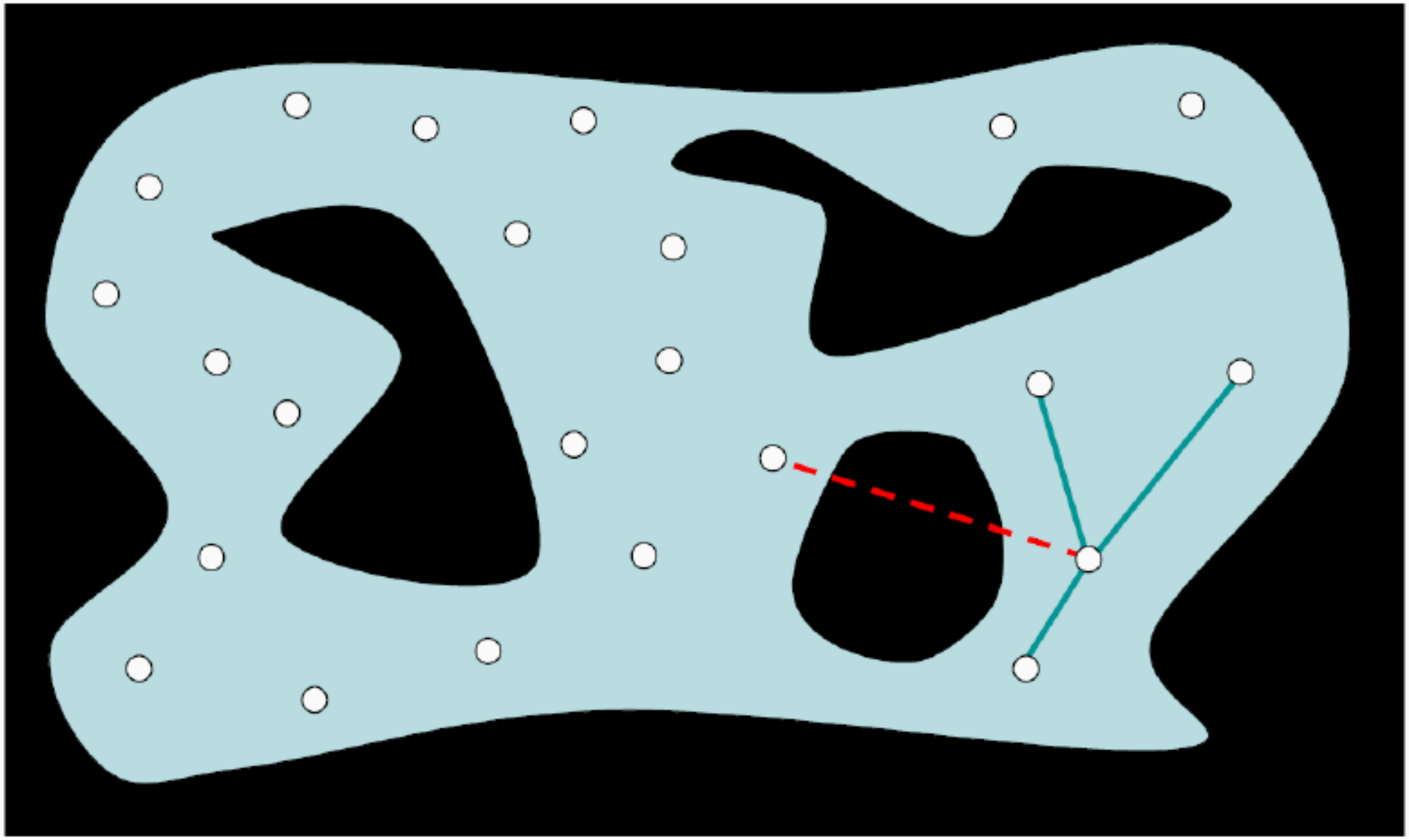
Sampling Techniques

Link each sample to its *K* nearest neighbors



Sampling Techniques

Remove the links that cross forbidden regions



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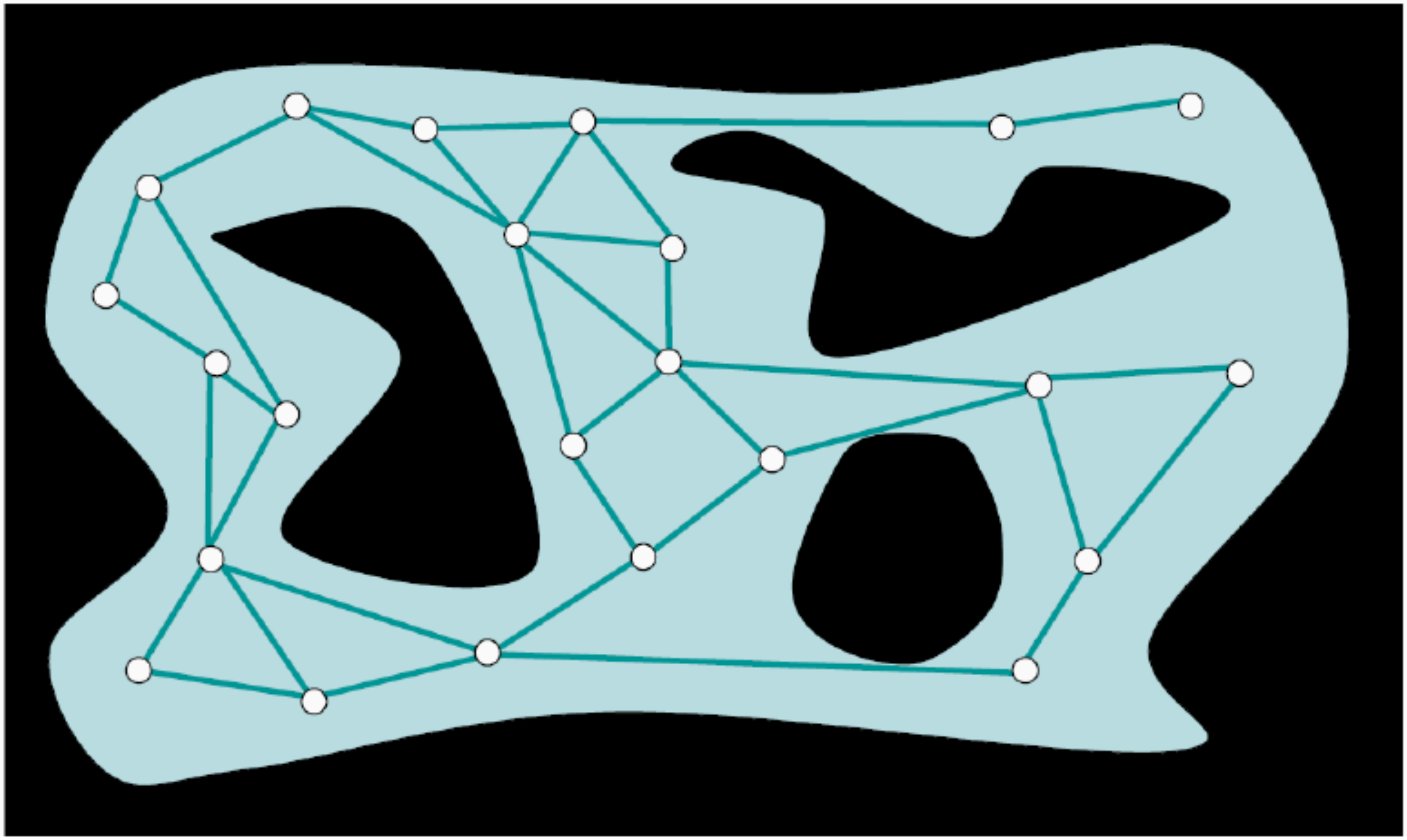
Sampling Techniques

Remove the links that cross forbidden regions



The resulting graph is a *probabilistic roadmap (PRM)*

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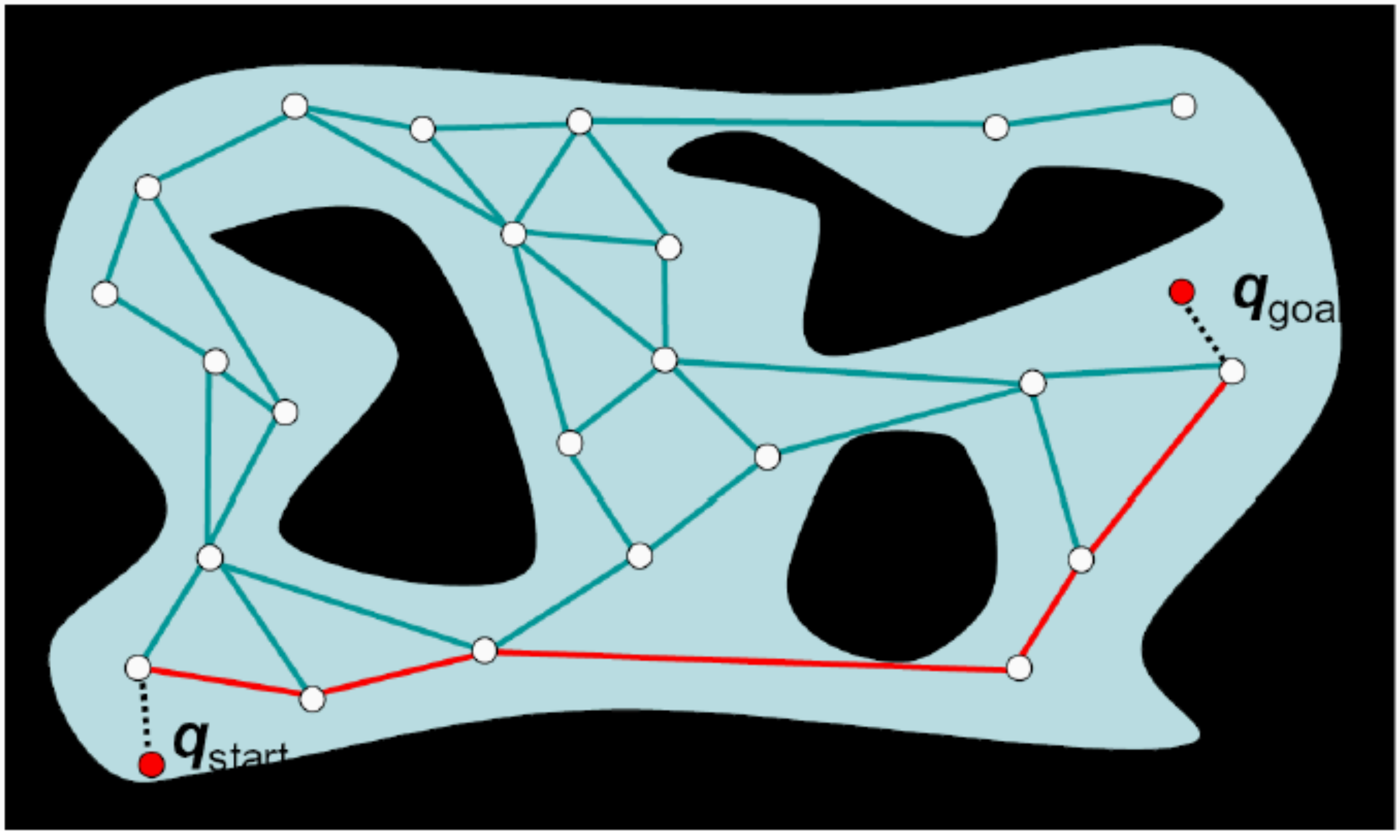
Sampling Techniques

Link the start and goal to the PRM and search using A*



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Sampling Techniques

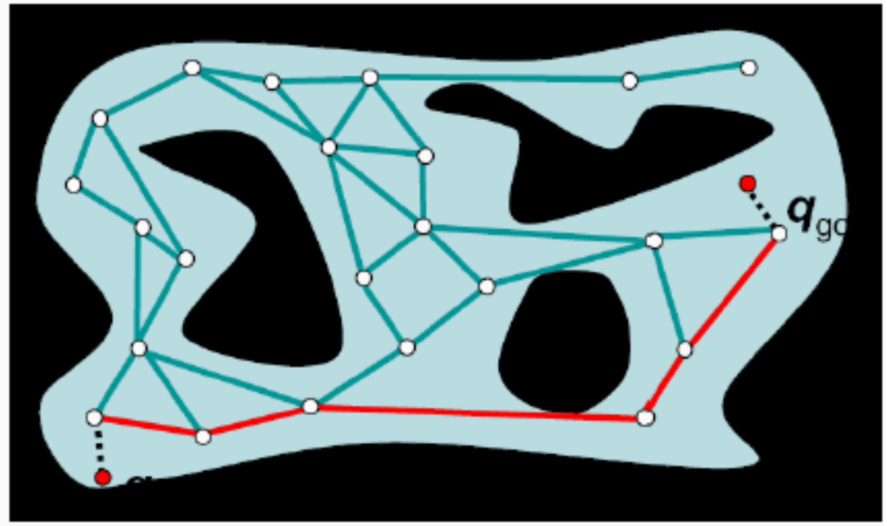
Continuous Space

↓

Discretization

↓

A* Search



- “Good” sampling strategies are important:
  - Uniform sampling
  - Sample more near points with few neighbors
  - Sample more close to the obstacles
  - Use pre-computed sequence of samples

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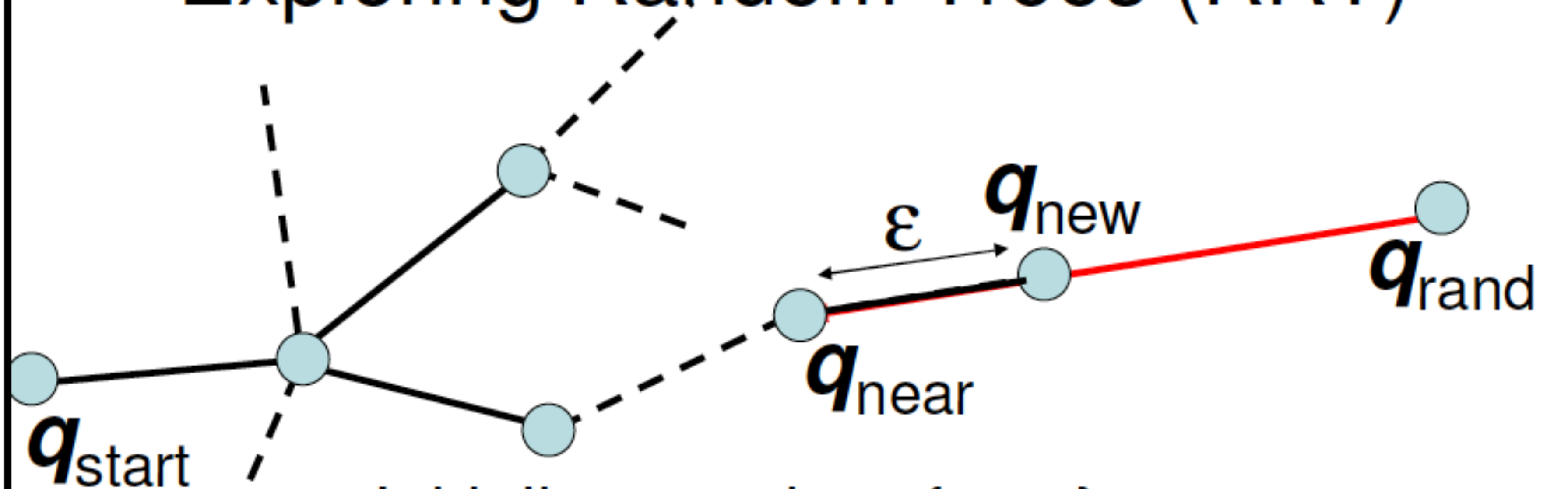
Sampling Techniques

- Remarkably, we can find a solution by using *relatively few randomly* sampled points.
- In most problems, a relatively small number of samples is sufficient to cover most of the feasible space with probability 1
- For a large class of problems:
  - Prob(finding a path) $\rightarrow$ 1 exponentially with the number of samples
- *But*, cannot detect that a path does not exist

Even More Radical: Rapidly Exploring Random Trees (RRT)




```
Algorithm BuildRRT
    Input: Initial configuration q_init, number of vertices in RRT K, incremental distance Δq)
    Output: RRT graph G

    G.init(q_init)
    for k = 1 to K do
        q_rand ← RAND_CONF()
        q_near ← NEAREST_VERTEX(q_rand, G)
        q_new ← NEW_CONF(q_near, q_rand, Δq)
        G.add_vertex(q_new)
        G.add_edge(q_near, q_new)
    return G
```

- "←" denotes assignment. For instance, "*largest* ← *item*" means that the value of *largest* changes to the value of *item*.
- "**return**" terminates the algorithm and outputs the following value.

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- "←" denotes assignment. For instance, "*largest* ← *item*" means that the value of *largest* changes to the value of *item*.
- "**return**" terminates the algorithm and outputs the following value.

- The sample qrand is drawn UAR from configuration space
  - or reject if inside obstacle
- Notice
  - node with big voronoi region of free space more likely to get expanded
    - the nearest neighbor step
  - so tree builds out into free space quickly
  - in different applications, one uses different epsilon
    - sometimes even add whole edge

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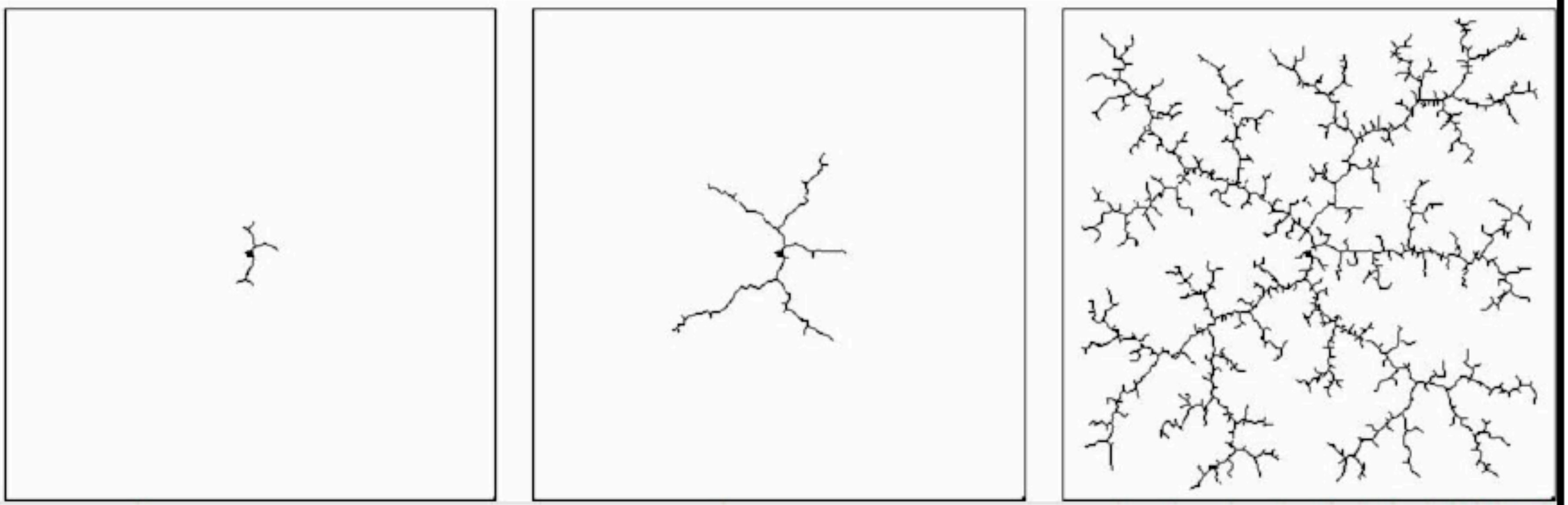
Properties



- Tends to explore the space rapidly in all directions
- Does not require extensive pre-processing
- Single query/multiple query problems
- Needs only collision detection test → No need to represent/pre-compute the entire C-space

You have to be able to draw the samples - this can get tricky

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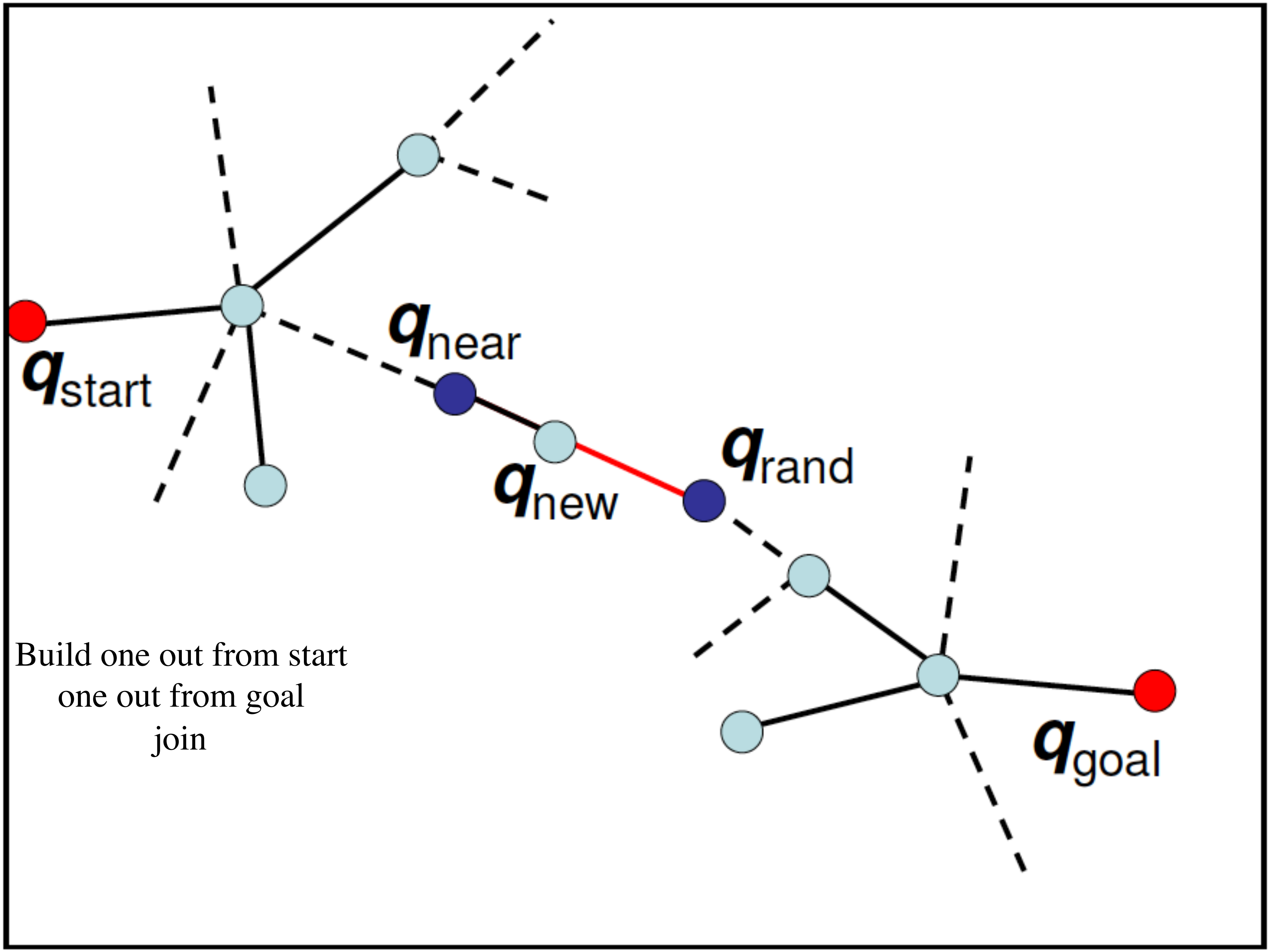

Build one out from start
one out from goal
join

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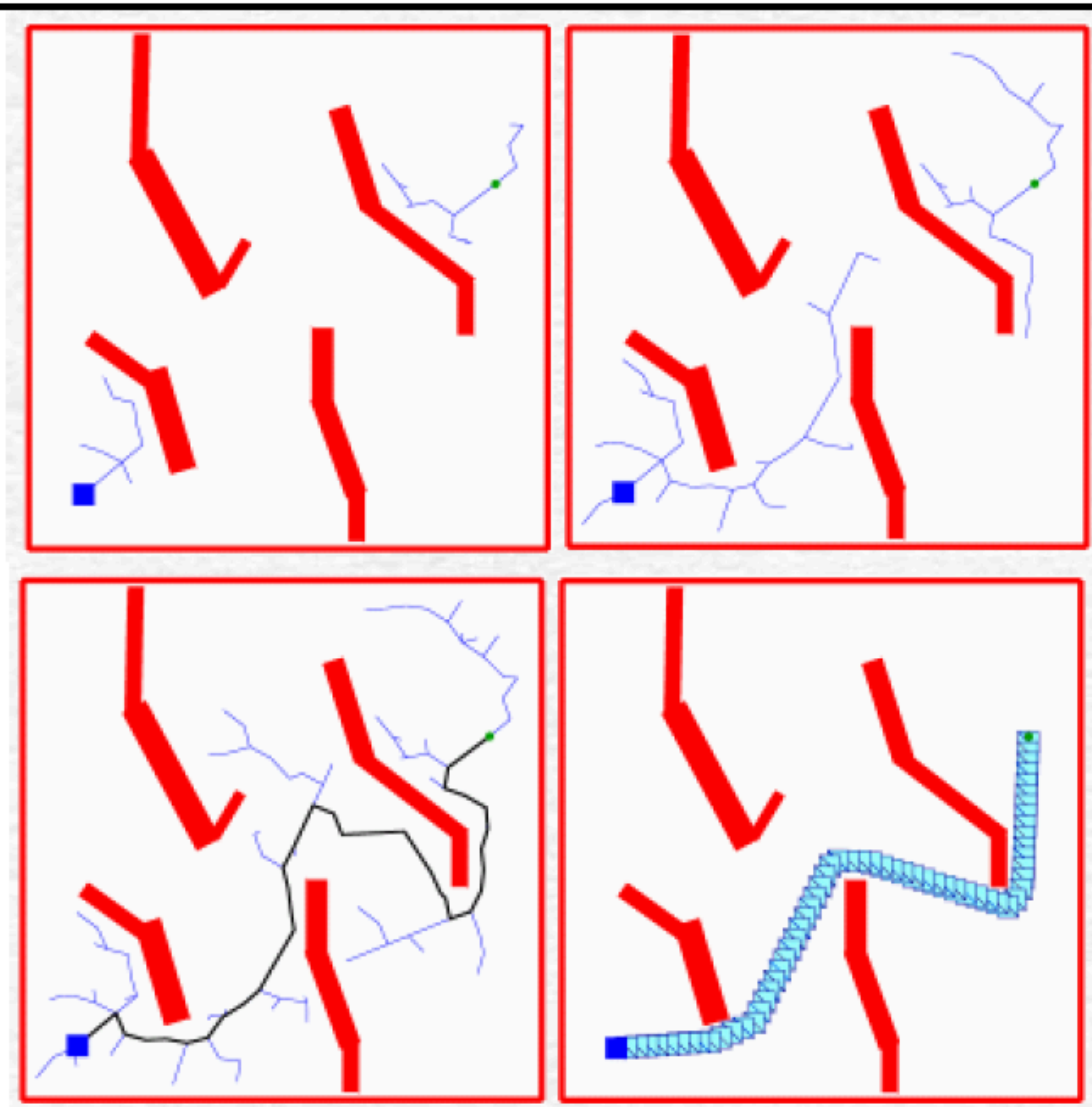
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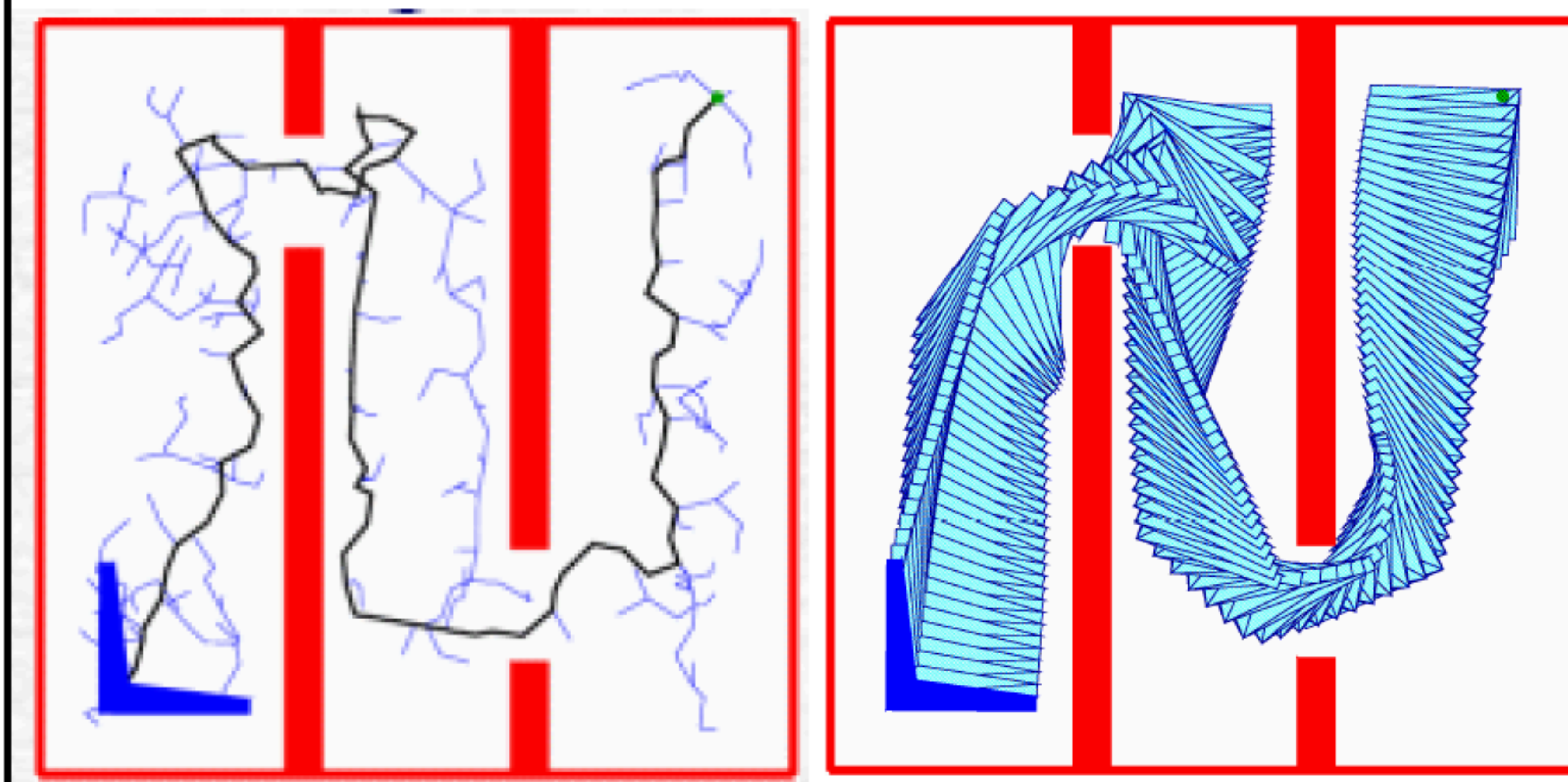

From Kuffner et al.

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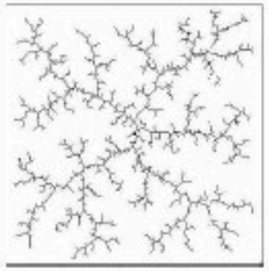
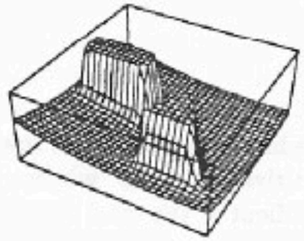
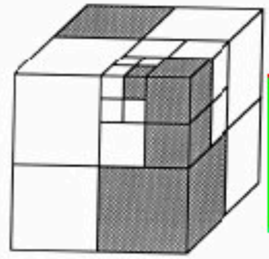
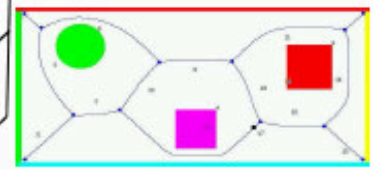
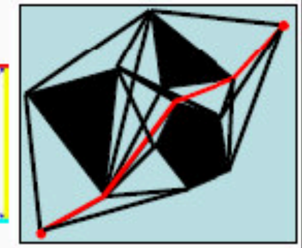

	Sampling	Potential Fields	Approx. Cell Decomposition	Voronoi	Visibility
Practical in ~2-D or 3-D	Y	Y	Y	Y	Y
Practical in >> 2-D or 3-D	Y	Y (using randomized version)	??	N	N
Fast	Y	Y	Y	In low dim.	In 2-D
Online Extensions	Y	Y	??	??	N
Complete?	Probabilistically complete	Probabilistically-resolution complete	Resolution-Complete	Y	Y

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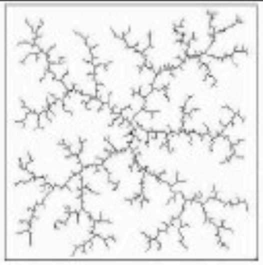

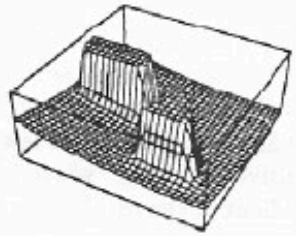

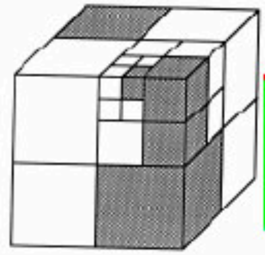

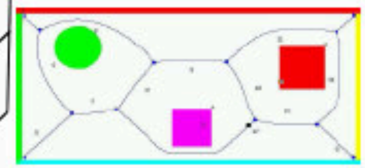

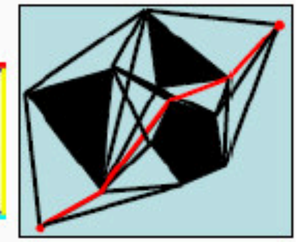

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Practical in >> 2-D or 3-D	Y	Y (using randomized version)	??	N	N
Fast	Y	Y	Y	In low dim.	In 2-D
Online Extensions					N
Complete?	Probabilistically complete	Probabilistically-resolution complete	Resolution-Complete	Y	Y

More exact/Complete

Faster/More practical in high dim.

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- (Limited) background in Russell&Norvig Chapter 25
- Two main books:
  - J-C. Latombe. Robot Motion Planning. Kluwer. 1991.
  - S. Lavalle. Planning Algorithms. 2006. http://msl.cs.uiuc.edu/planning/
  - H. Choset et al., Principles of Robot Motion: Theory, Algorithms, and Implementations. 2006.
- Other demos/examples:
  - http://voronoi.sbp.ri.cmu.edu/~choset/
  - http://www.kuffner.org/james/research.html
  - http://msl.cs.uiuc.edu/rrt/