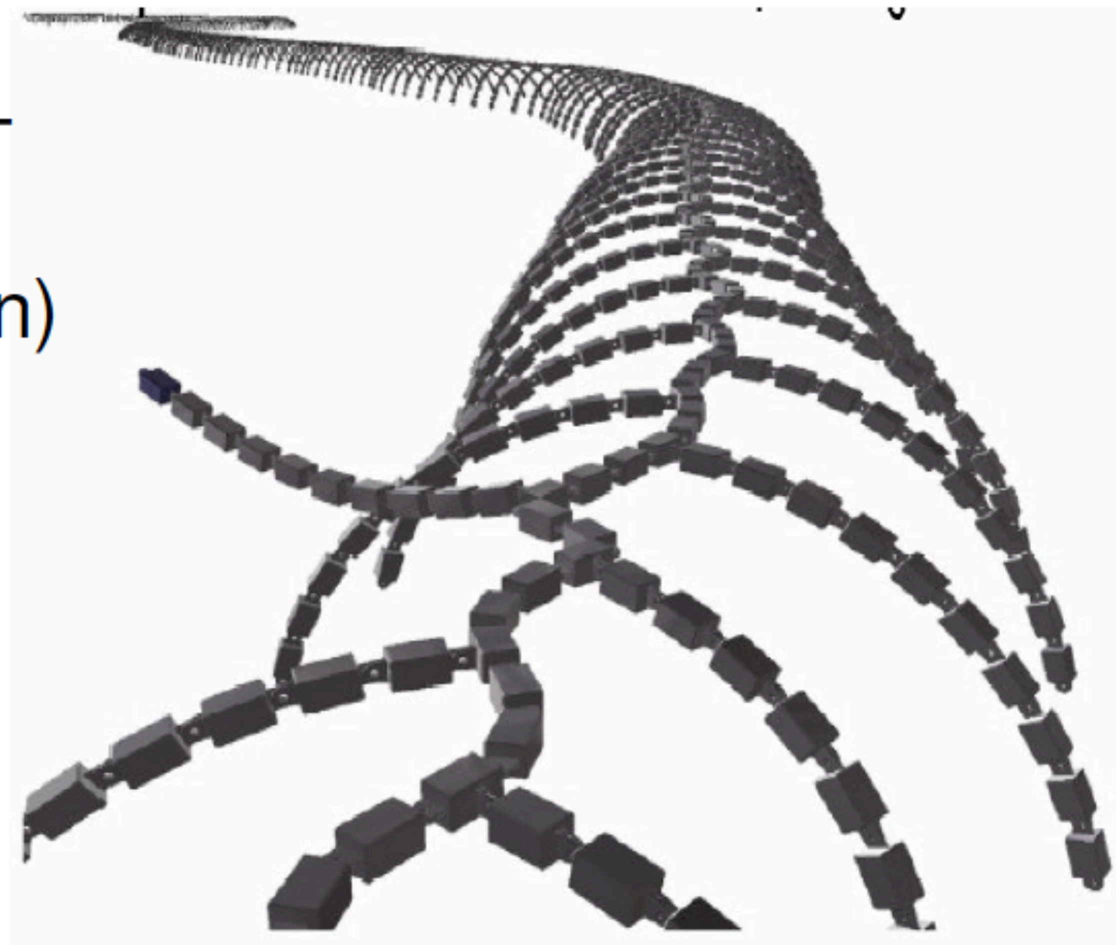
Motion Planning II

D.A. Forsyth
(with a lot of H. Choset, and some J. Li)

Large C-Space Dimension

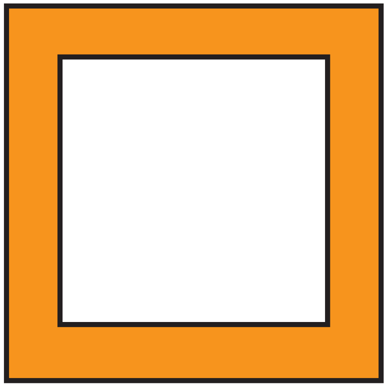
Millipede-like robot (S. Redon)



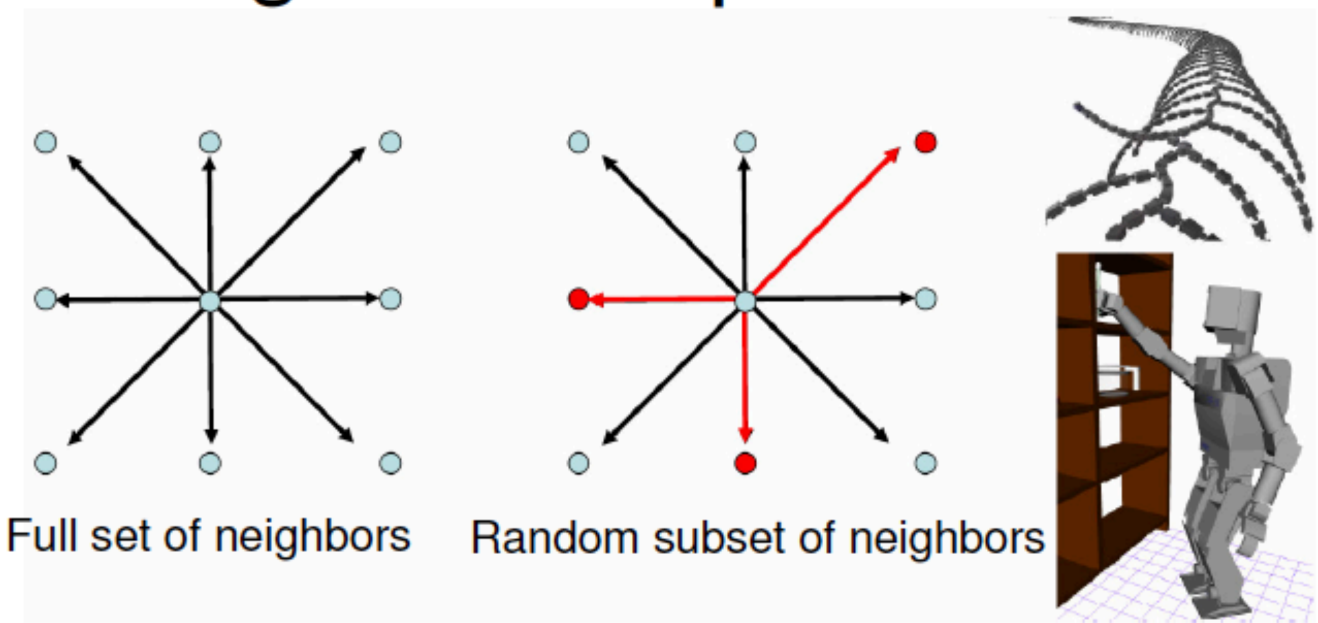
~13,000 DOFs !!!

Dimension and its nuisances

- Counting:
  - A d-dimensional cube has 2^d vertices
- Volume:
  - your intuitions about volume are wrong in high dimension
  - consider cubical “orange” in high d
    - skin depth e/2
    - pulp (1-e)
    - volume of pulp:
      - (1-e)^d
    - volume of skin:
      - 1-(1-e)^d
    - IT’S ALL SKIN!
  - Almost all the volume of high d objects is very close to surface



Dealing with C-Space Dimension



- We should evaluate all the neighbors of the current state, but:
- Size of neighborhood grows exponentially with dimension
- Very expensive in high dimension

Solution:

- Evaluate only a random subset of *K* of the neighbors
- Move to the lowest potential neighbor

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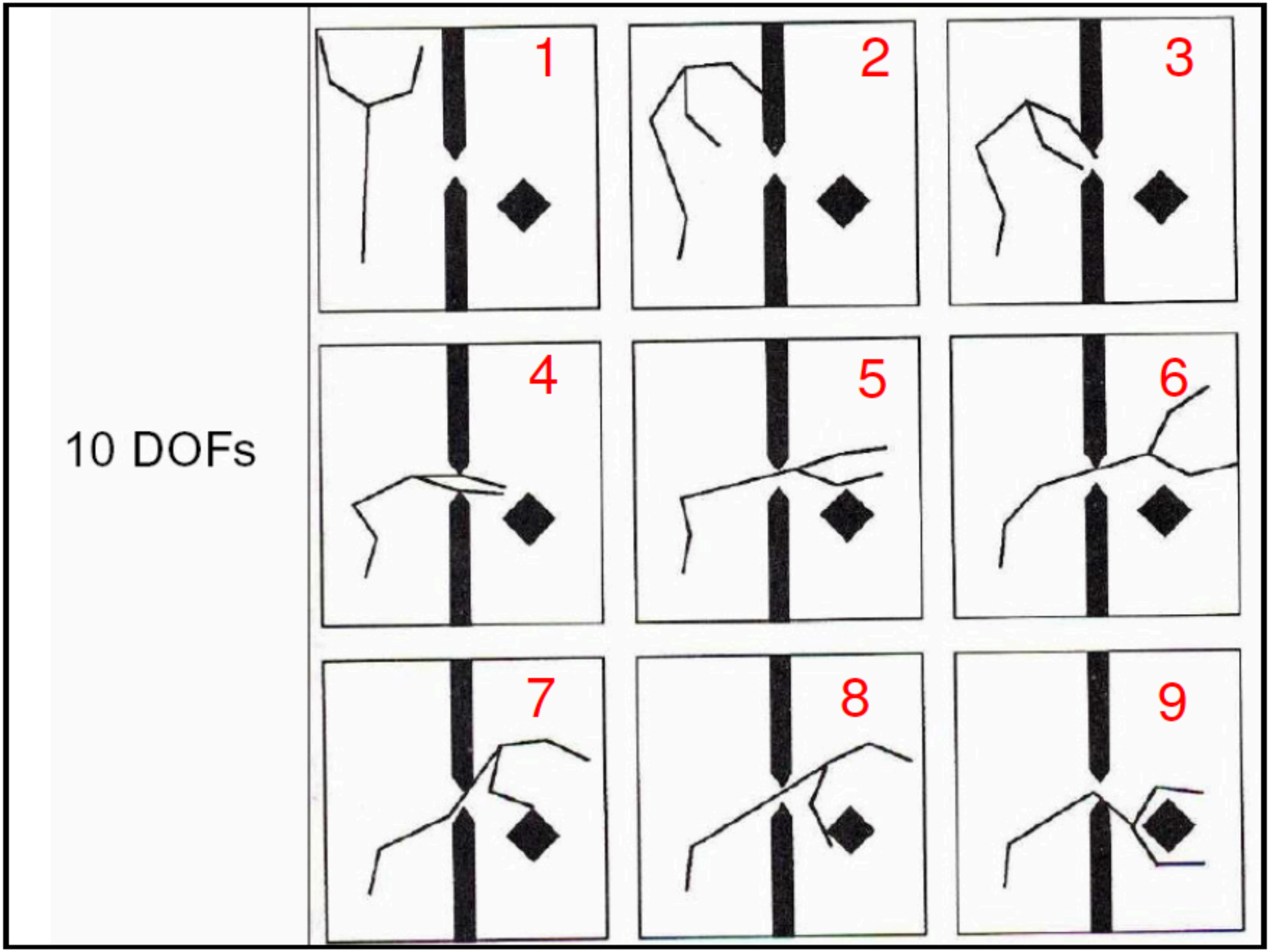

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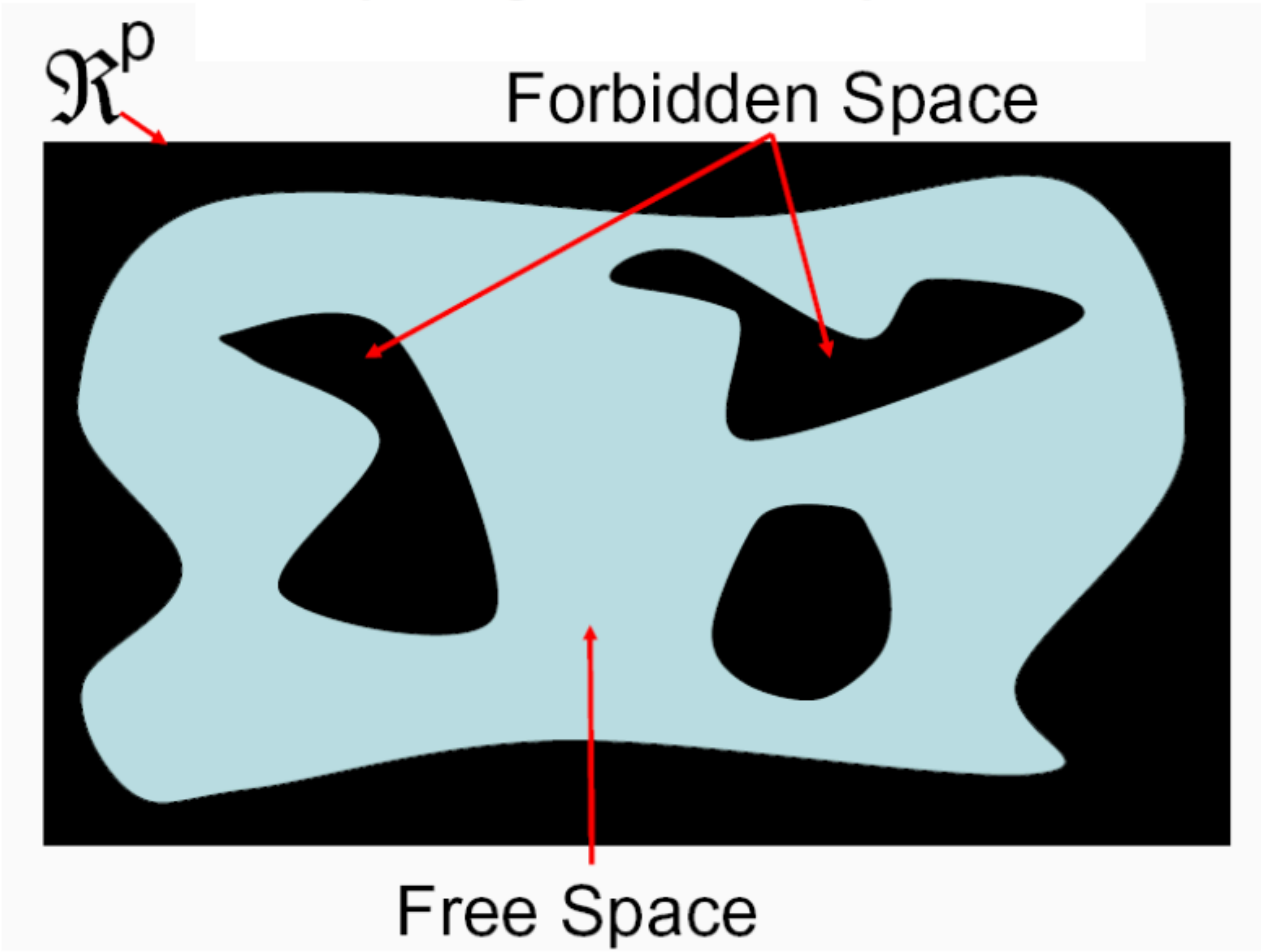
Sampling Techniques



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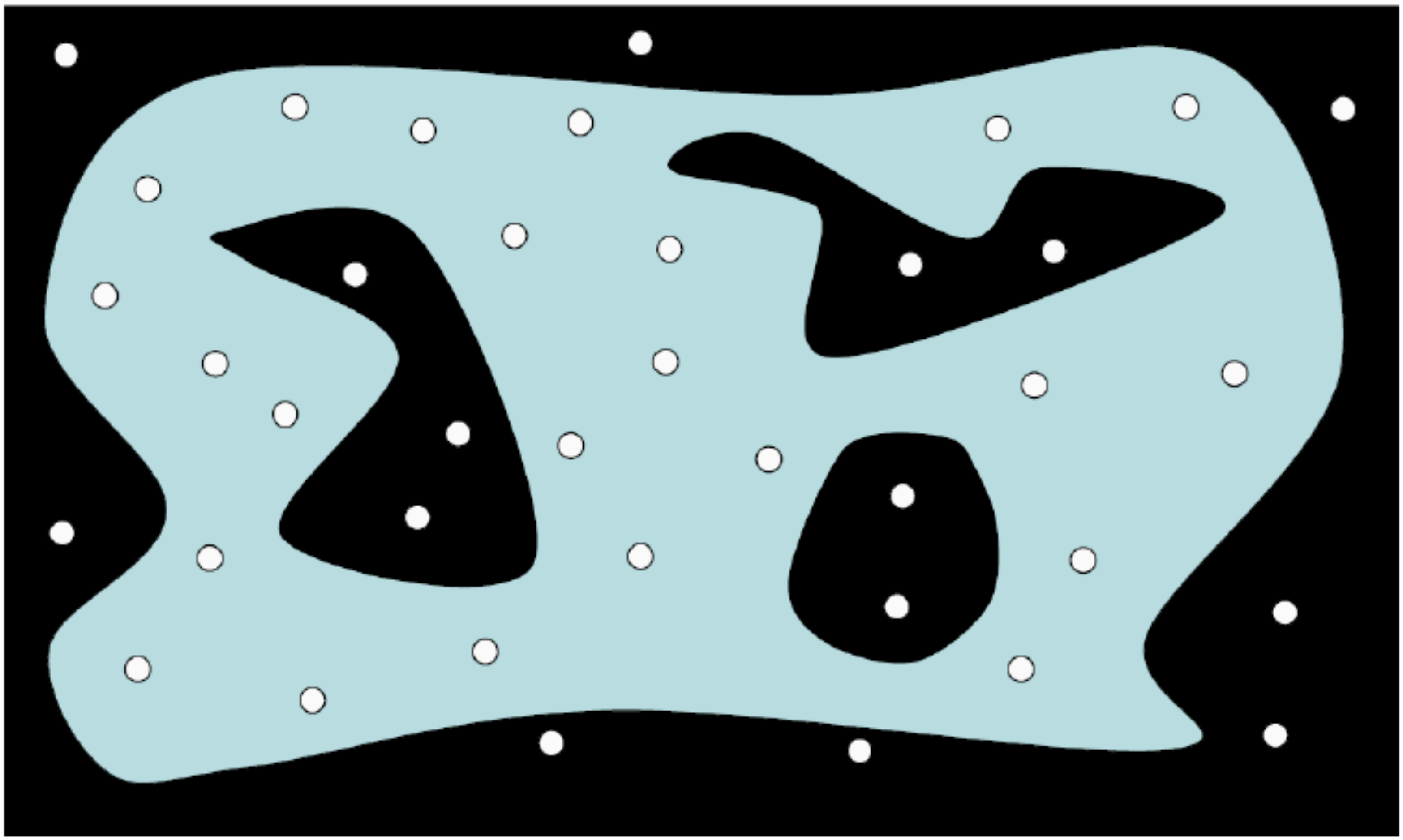
Sampling Techniques

Sample random locations



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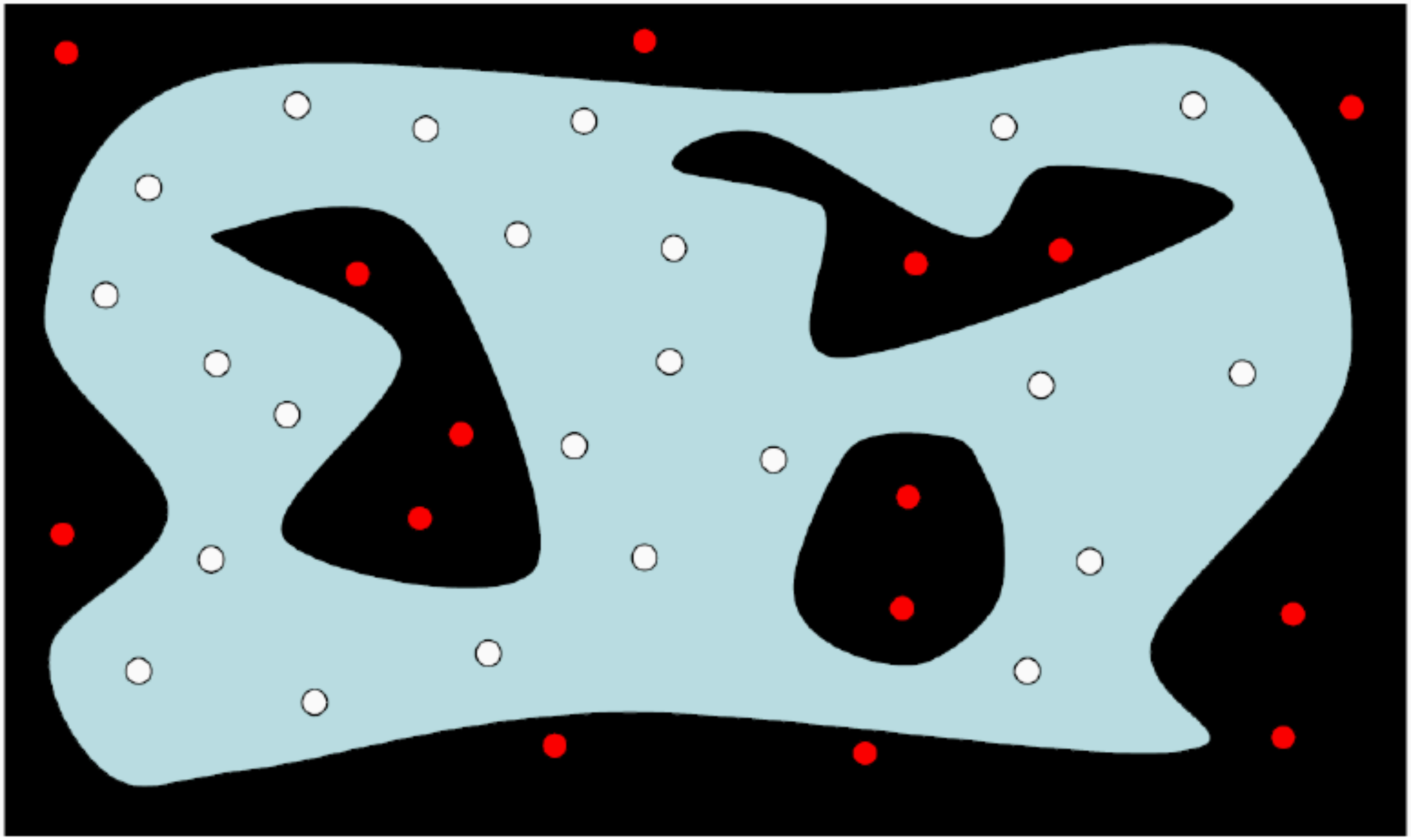
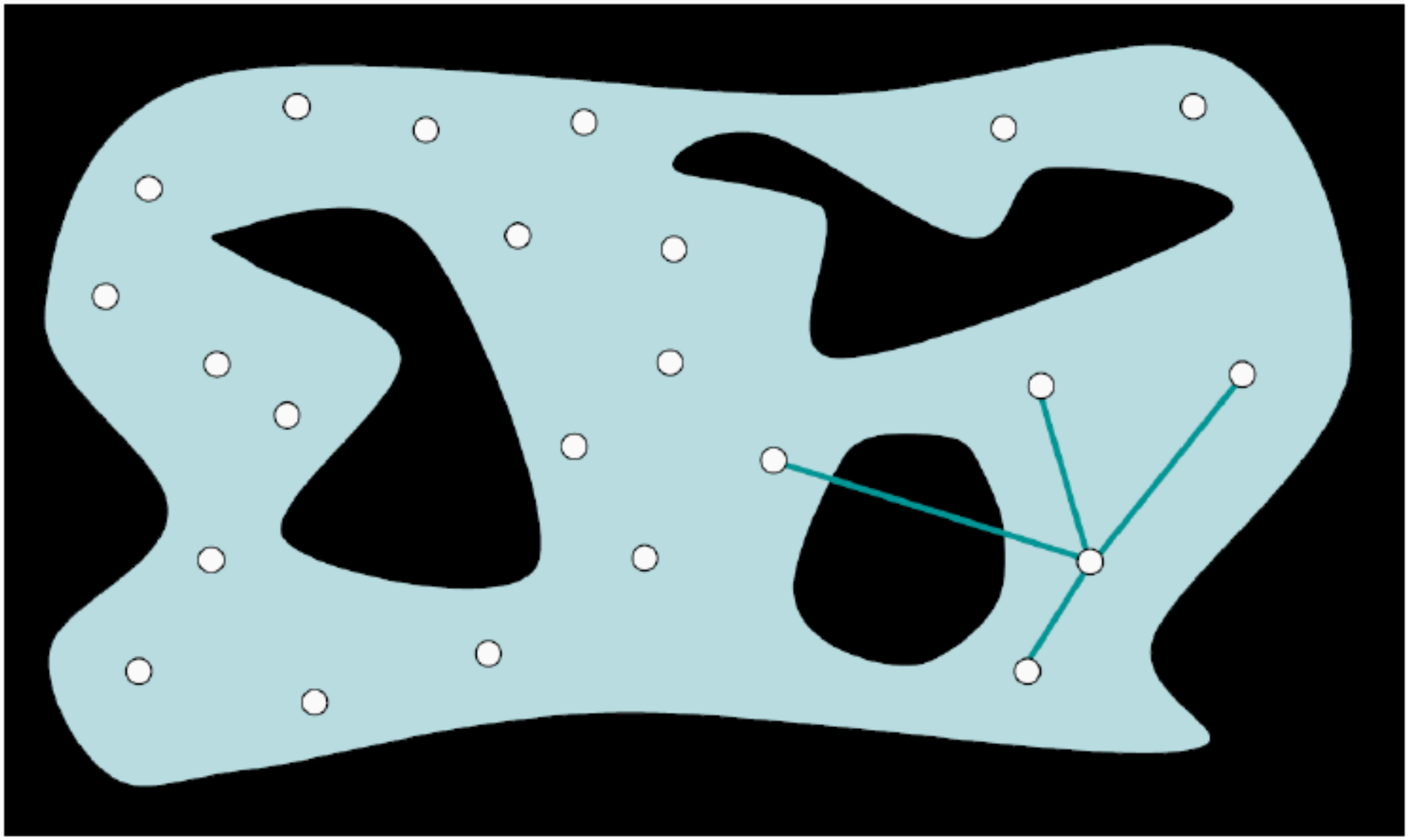
Sampling Techniques

Remove the samples in the forbidden regions



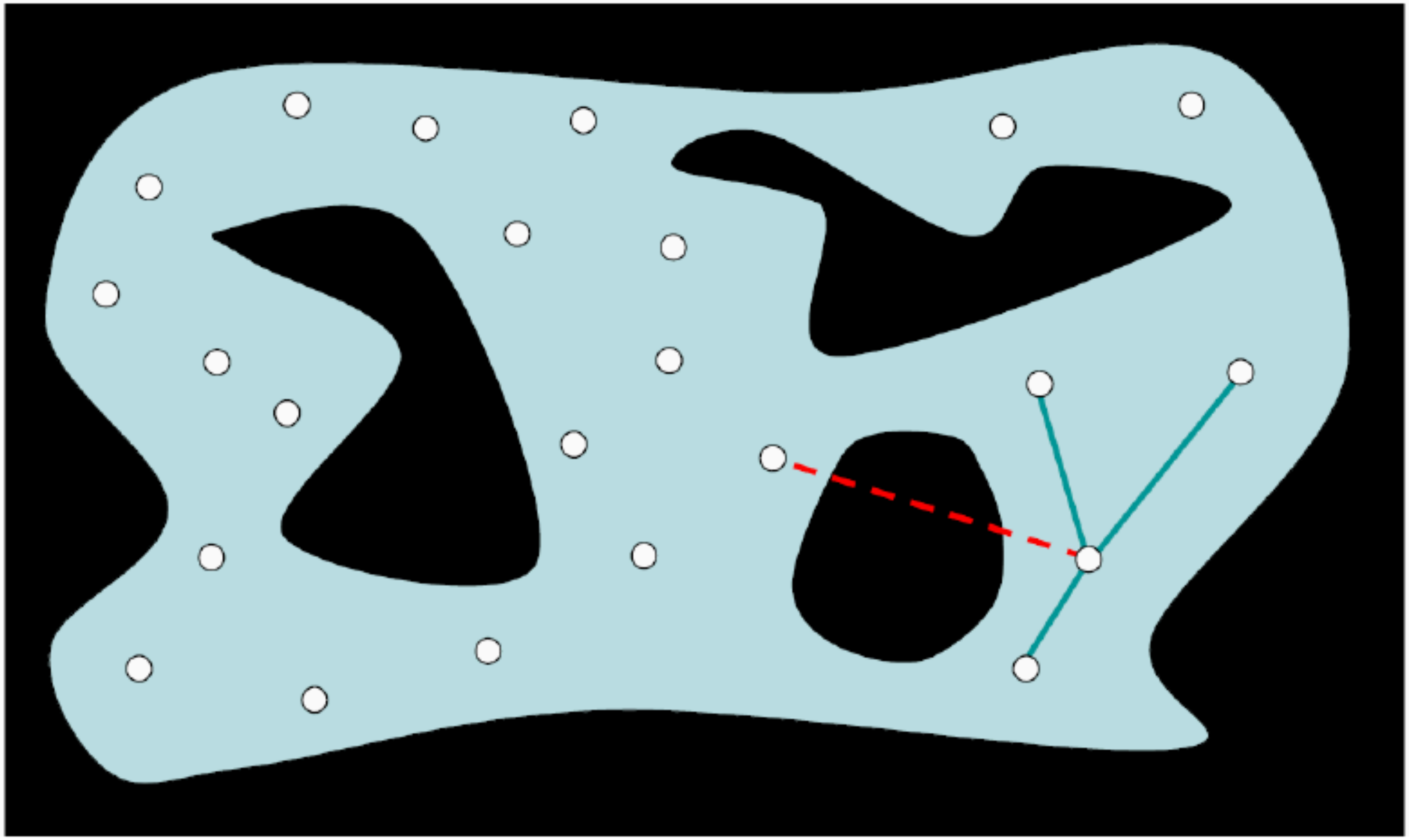
Sampling Techniques

Link each sample to its *K* nearest neighbors



Sampling Techniques

Remove the links that cross forbidden regions



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Sampling Techniques

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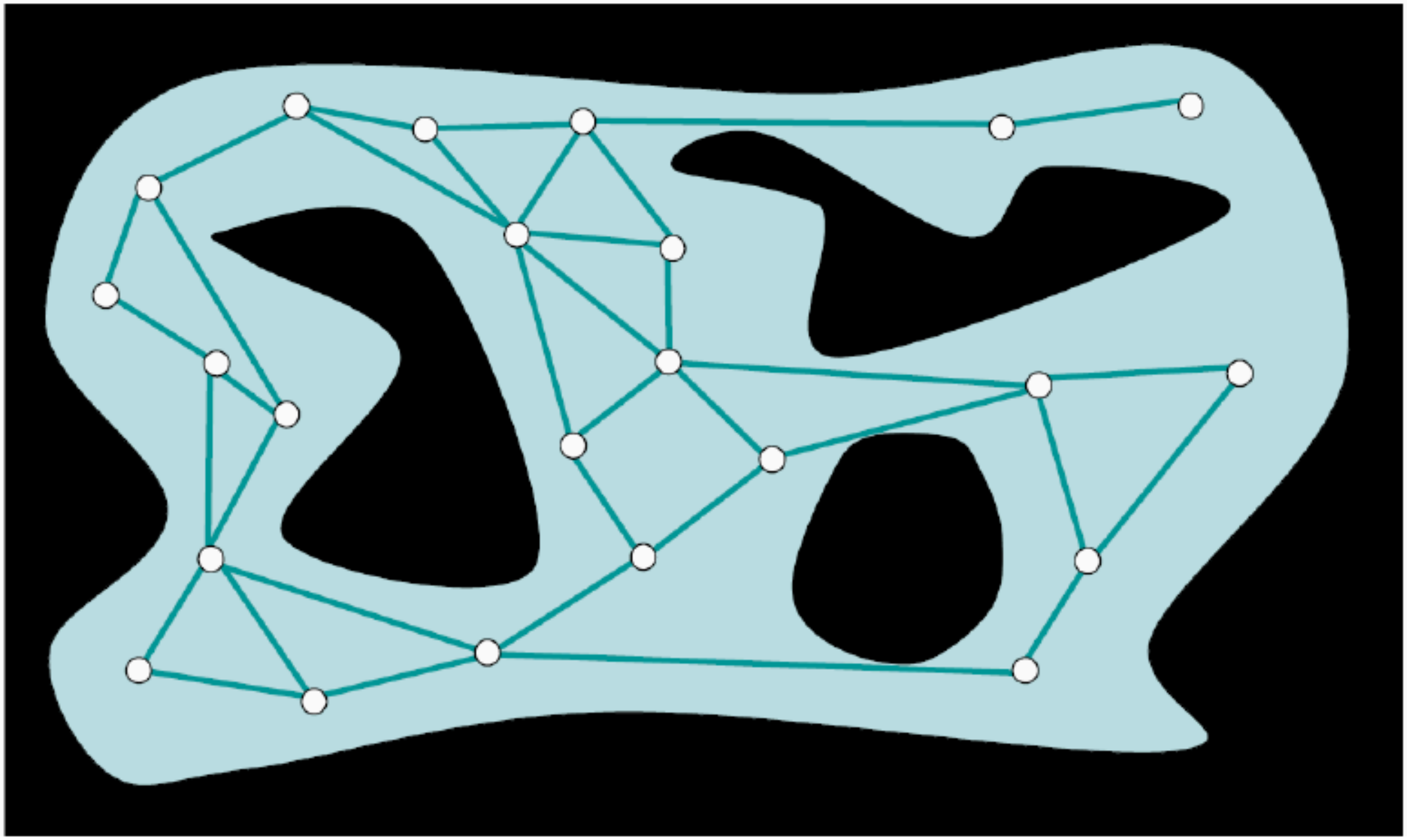

The resulting graph is a *probabilistic roadmap (PRM)*

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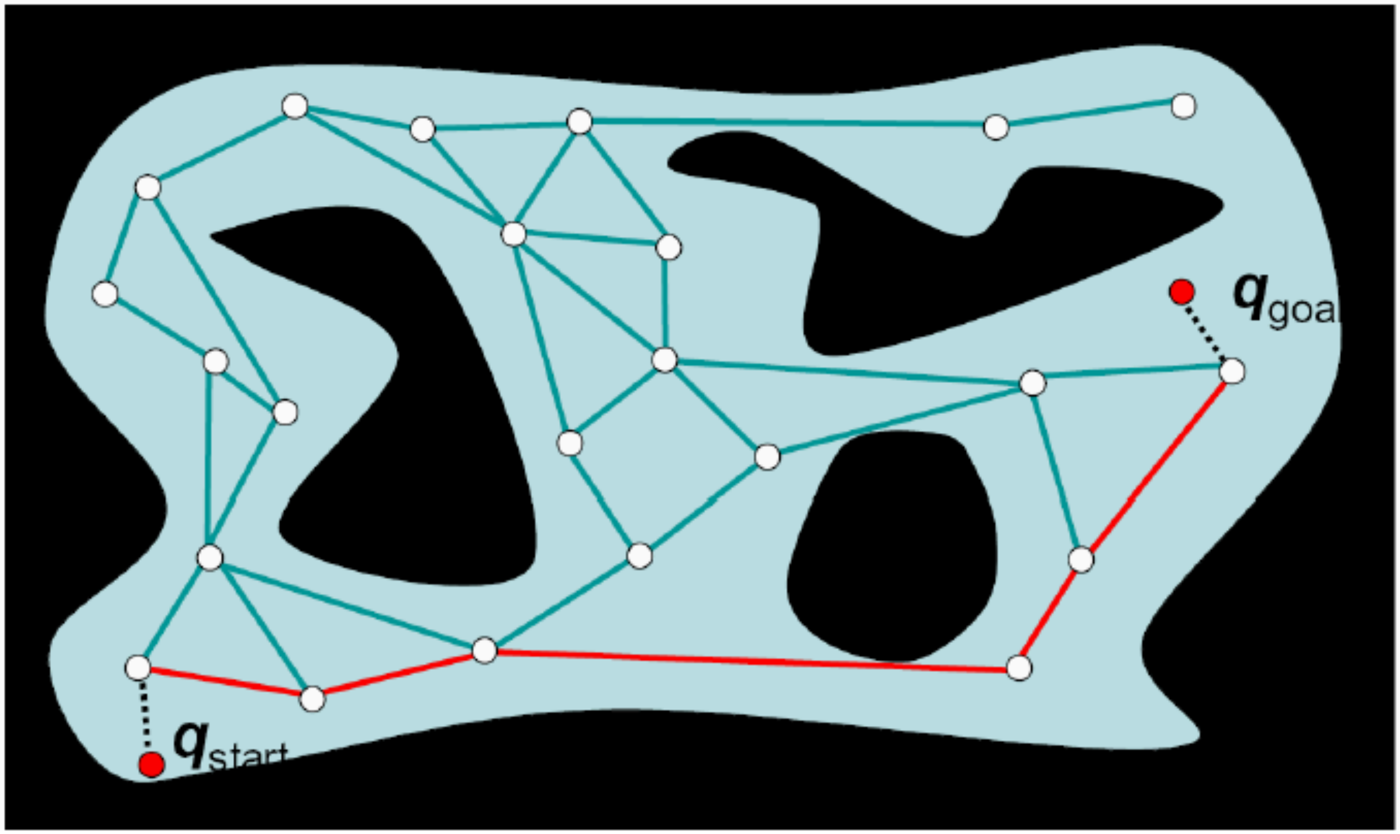
Sampling Techniques

Link the start and goal to the PRM and search using A*



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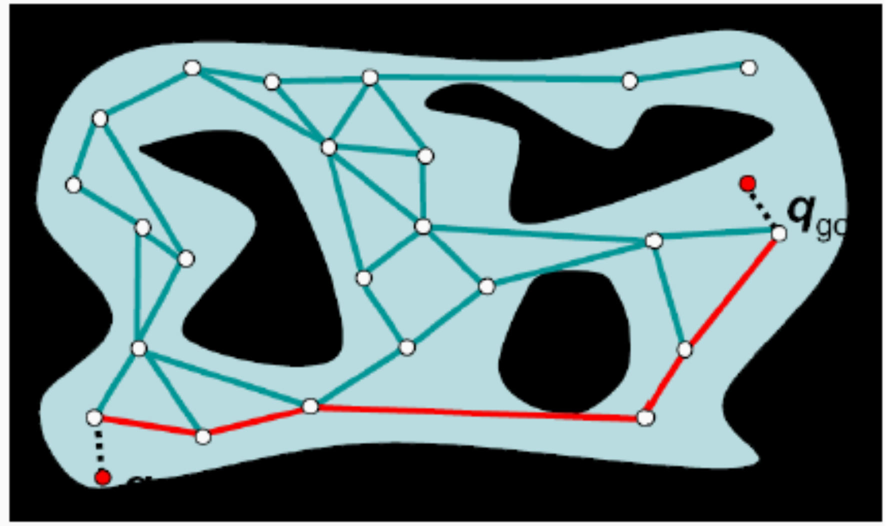
Sampling Techniques

Continuous Space

Discretization

A* Search



- "Good" sampling strategies are important:
  - Uniform sampling
  - Sample more near points with few neighbors
  - Sample more close to the obstacles
  - Use pre-computed sequence of samples

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Sampling Techniques

- Remarkably, we can find a solution by using *relatively few randomly* sampled points.
- In most problems, a relatively small number of samples is sufficient to cover most of the feasible space with probability 1
- For a large class of problems:
  – Prob(finding a path) $\rightarrow$ 1 exponentially with the number of samples
- *But*, cannot detect that a path does not exist

Random trees

- Notice how randomized roadmap is for “any plan”
  - but we may not need that
  - plan for a specific start, a specific goal
- For the moment, focus on start
  - grow a tree with start at root
  - join tree to goal
    - perhaps by growing backward from goal, and linking
- Q: how to grow the tree?

Naïve Random Tree

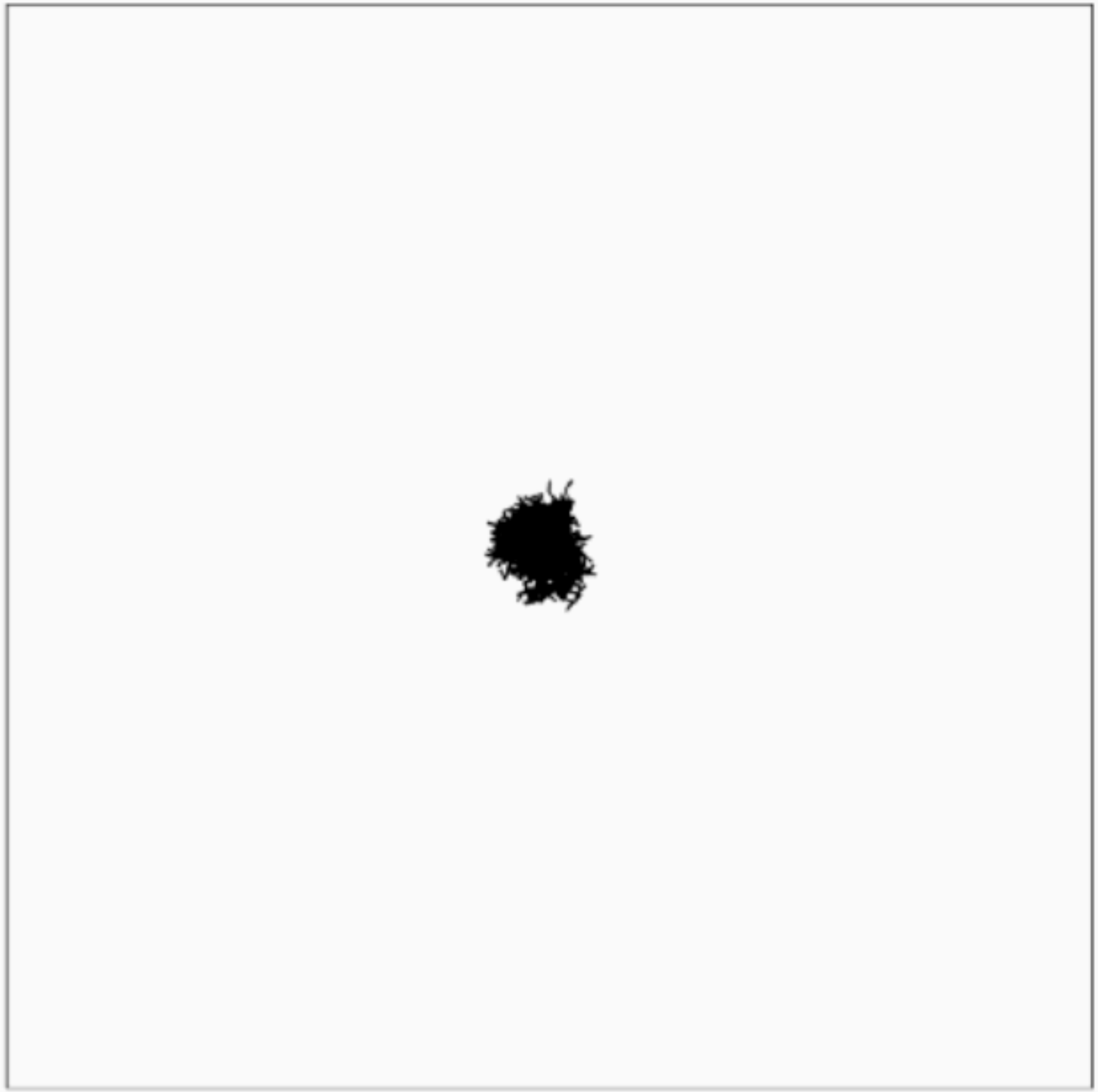
Start with middle

Sample near this node

Then pick a node at random in tree

Sample near it

End up Staying in middle



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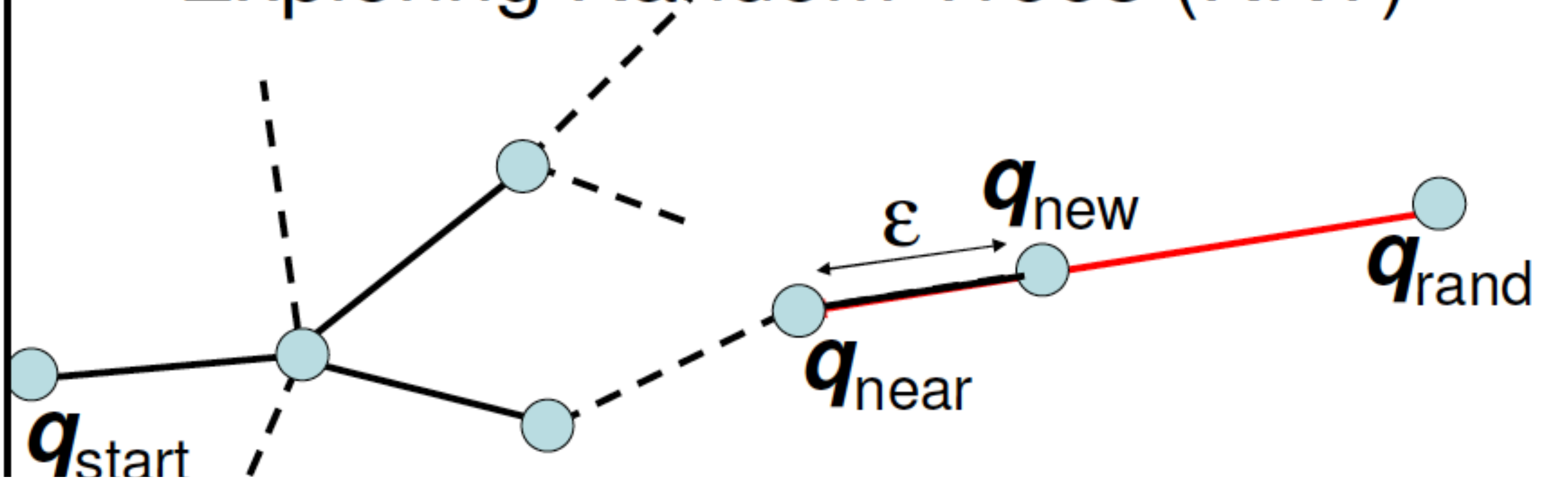
Even More Radical: Rapidly Exploring Random Trees (RRT)




```
Algorithm BuildRRT
    Input: Initial configuration q_init, number of vertices in RRT K, incremental distance Δq)
    Output: RRT graph G

    G.init(q_init)
    for k = 1 to K do
        q_rand ← RAND_CONF()
        q_near ← NEAREST_VERTEX(q_rand, G)
        q_new ← NEW_CONF(q_near, q_rand, Δq)
        G.add_vertex(q_new)
        G.add_edge(q_near, q_new)
    return G
```

- "←" denotes assignment. For instance, "*largest* ← *item*" means that the value of *largest* changes to the value of *item*.
- "**return**" terminates the algorithm and outputs the following value.

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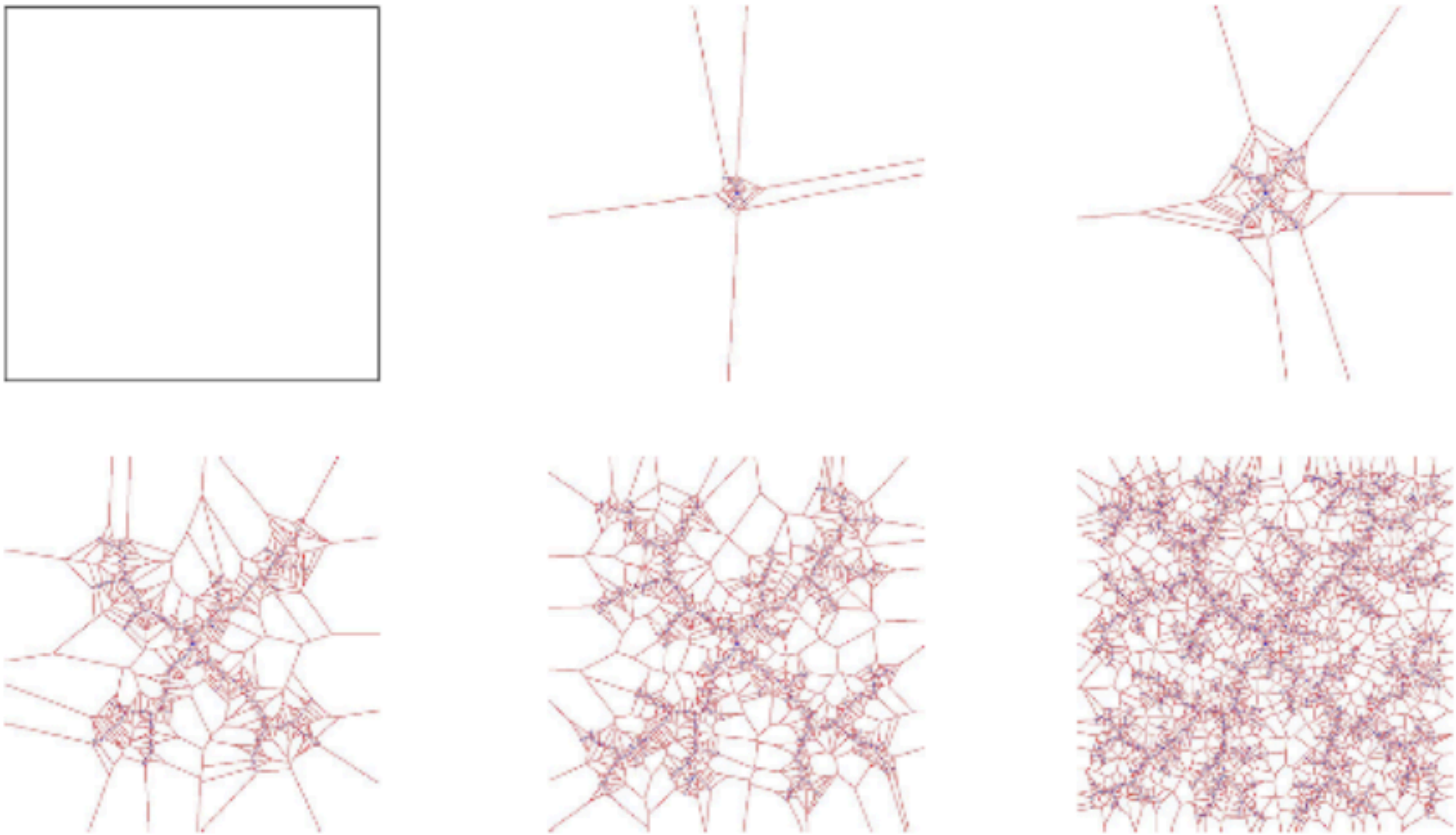
RRT’s are biased towards large Voronoi cells



The nodes most likely to be closest to a randomly chosen point in state space are those with the largest Voronoi regions. The largest Voronoi regions belong to nodes along the frontier of the tree, so these frontier nodes are automatically favored when choosing which node to expand.

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RRT's expand (another way)

- The nodes of the tree are
  - mostly on the boundary
  - of a "blob" of nodes
    - because that's where the volume is
- Draw a sample in c-space
  - if the blob is spread out in c-space, it's "inside", but we're OK
  - otherwise, sample is likely "outside"
    - so nearest node is very likely on boundary

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- "←" denotes assignment. For instance, "*largest* ← *item*" means that the value of *largest* changes to the value of *item*.
- "**return**" terminates the algorithm and outputs the following value.

- The sample qrand is drawn UAR from configuration space
  - or reject if inside obstacle
  - this could be tricky
- Notice
  - tree builds out into free space quickly
  - in different applications, one uses different epsilon
    - sometimes even add whole edge

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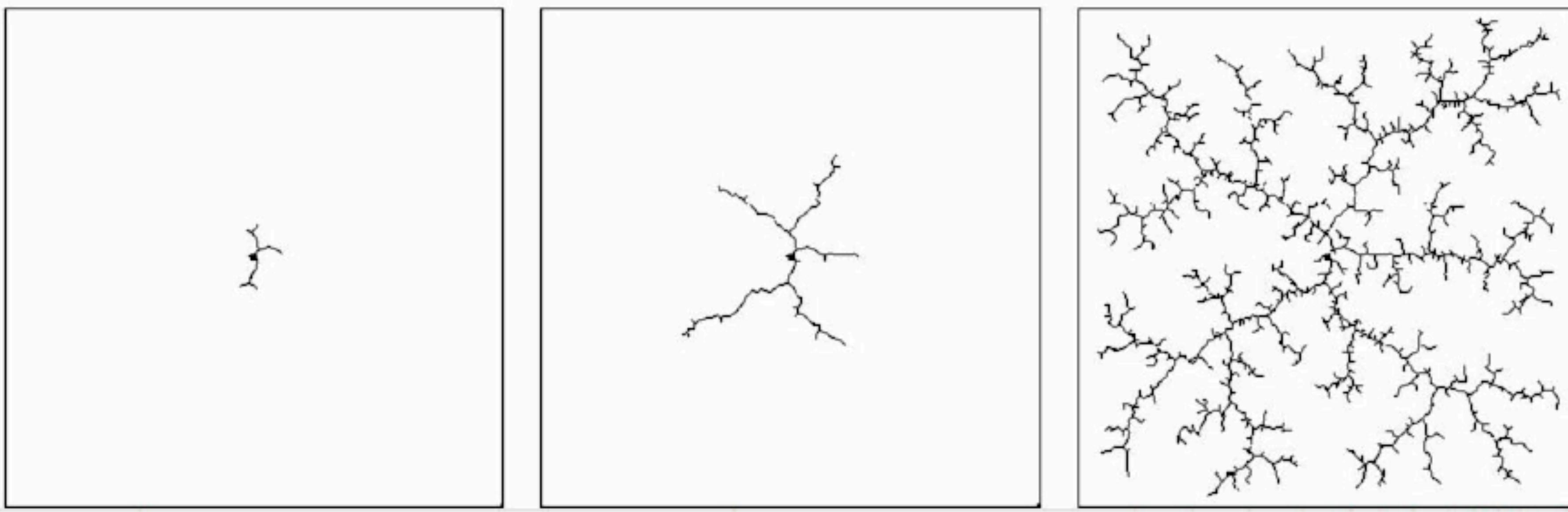
Properties



- Tends to explore the space rapidly in all directions
- Does not require extensive pre-processing
- Single query/multiple query problems
- Needs only collision detection test → No need to represent/pre-compute the entire C-space

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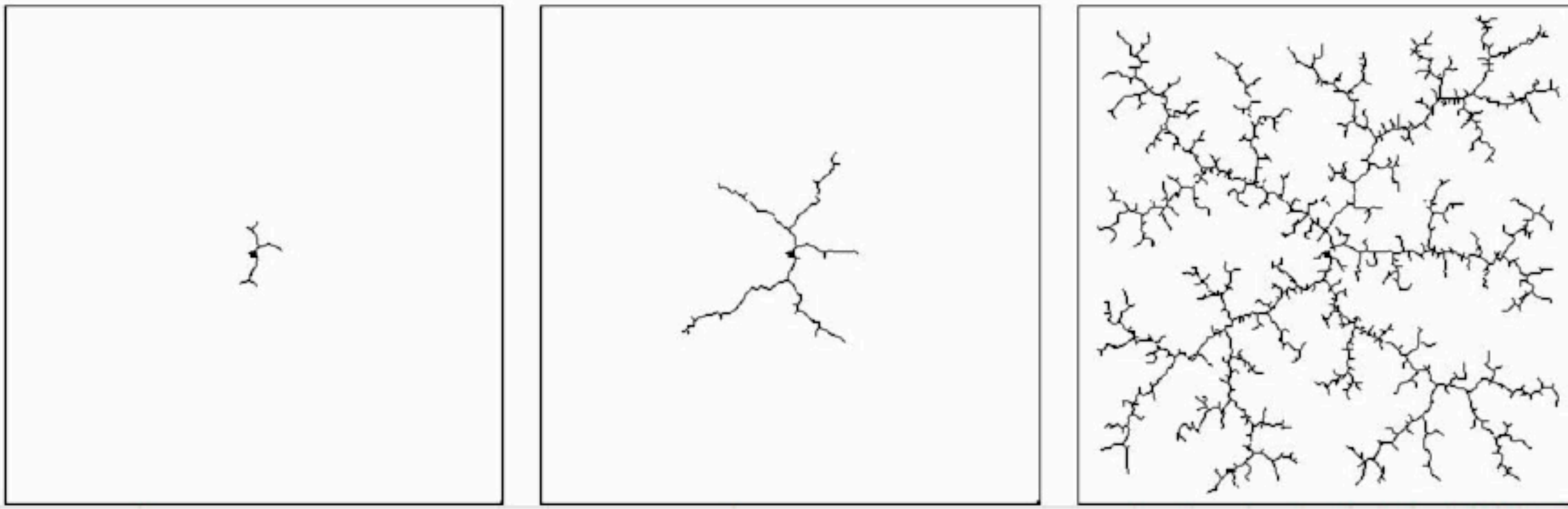
Properties



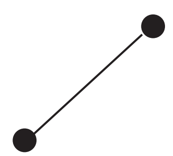
- Notice
  - Drawing the sample could get tricky
  - You need to be able to do the collision detection for the edge
    - BUT
      - many/most edges should be easy if there is a lot of free space

A quick conservative test - I

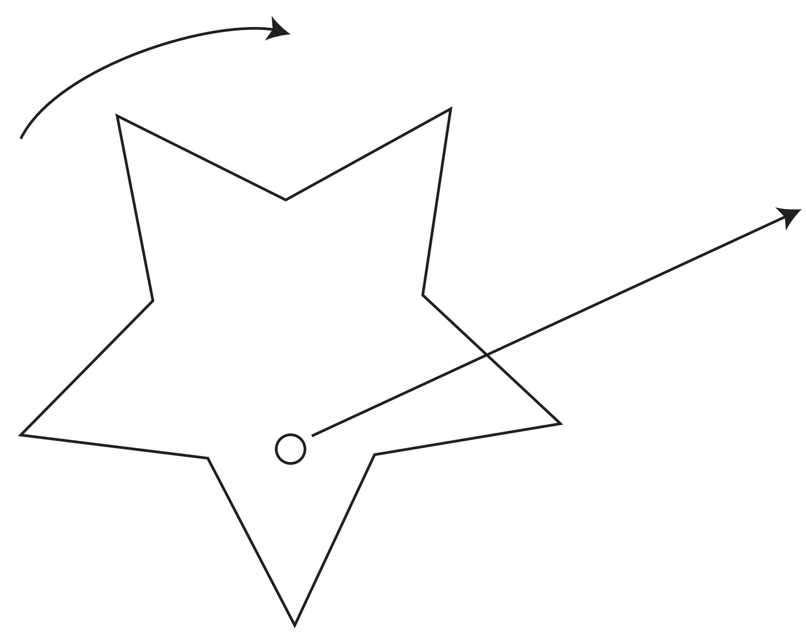
- Construct an axis aligned bounding box in 3-space
  - containing all configurations on the edge segment
  - how? below
- Test this box against objects
  - no intersection? edge is OK
  - intersection? more detailed test

A quick conservative test - II

- Building a box for robot rotating and translating
  - Robot rotates about origin in its own coordinate system
    - this origin translates



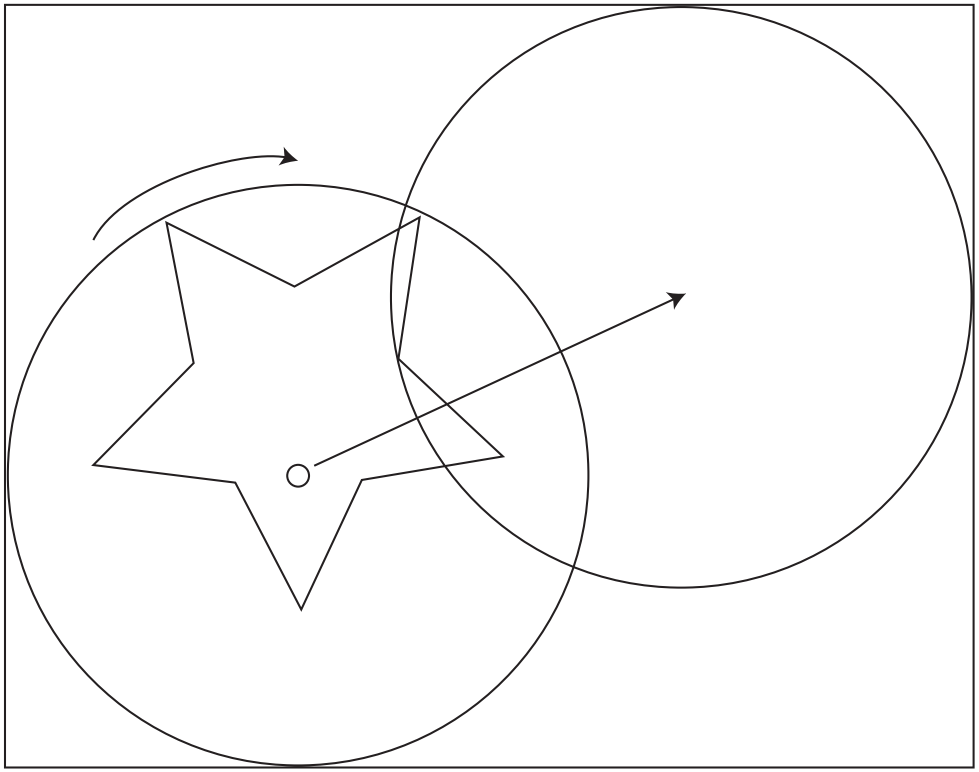
In c-space



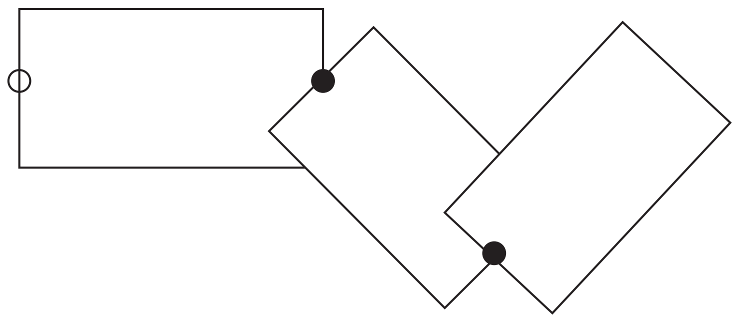
In 3D

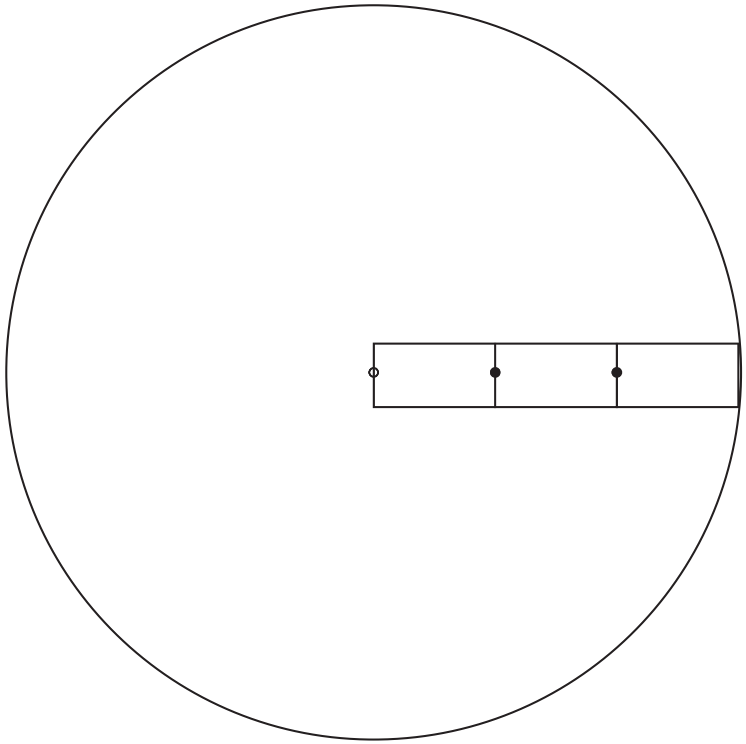
A quick conservative test - II

- Building a box for robot rotating and translating
  - Robot rotates about origin in its own coordinate system
    - this origin translates
  - Build bounding sphere, centered on origin, in advance
    - Translate this sphere's center - yields box
- Loose, quick bound
  - Loose
    - if segment intersects by this test
      - subdivide and go again



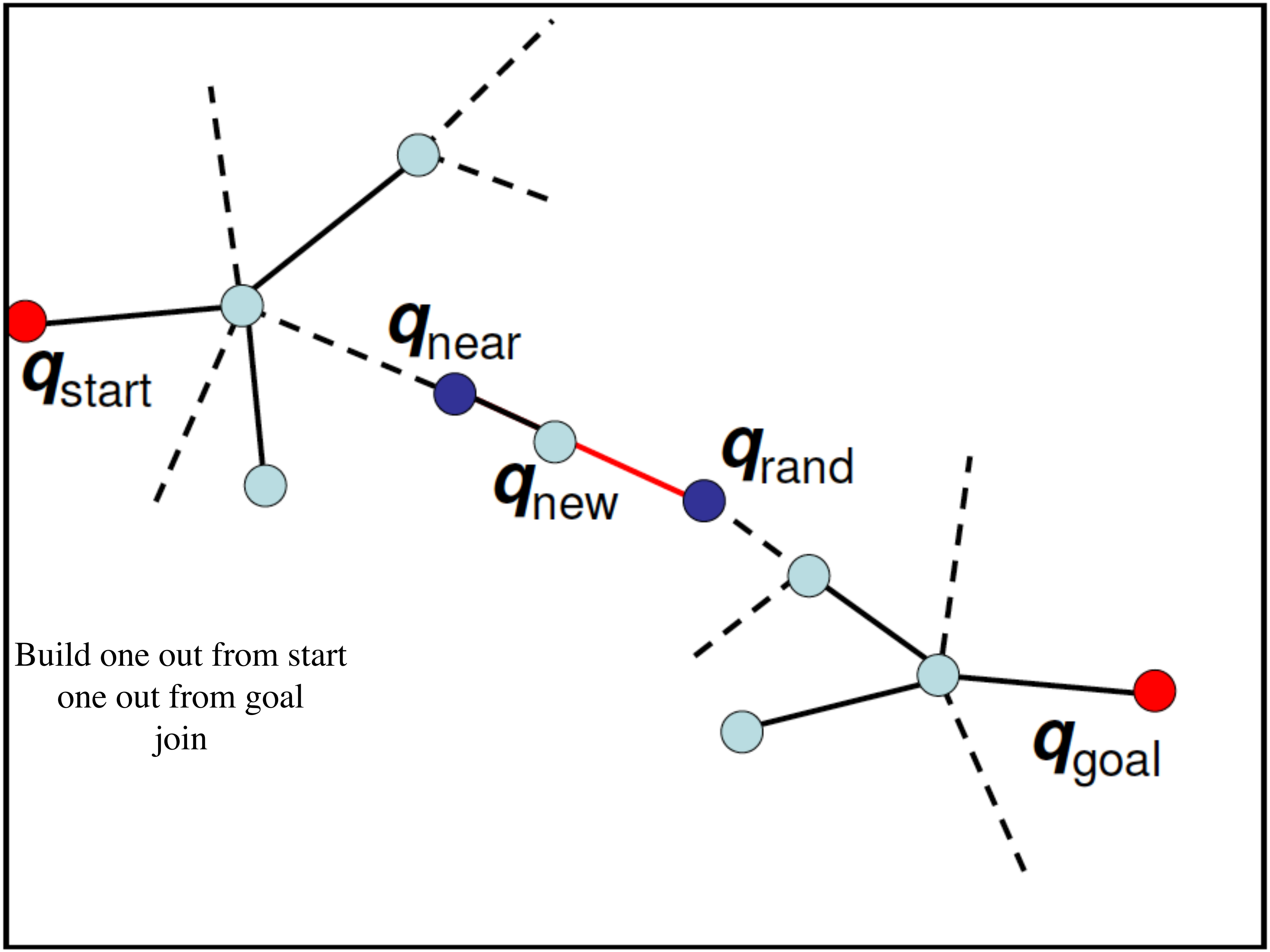
Bad for kinematic chains





Bad for kinematic chains - II

- Specialized techniques
  - typically per segment bounds
  - see Lavalle chapter, on website



Build one out from start
one out from goal
join

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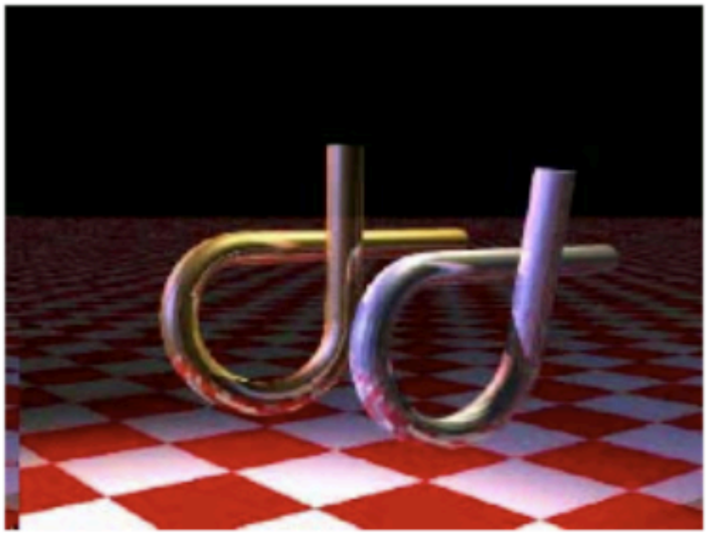
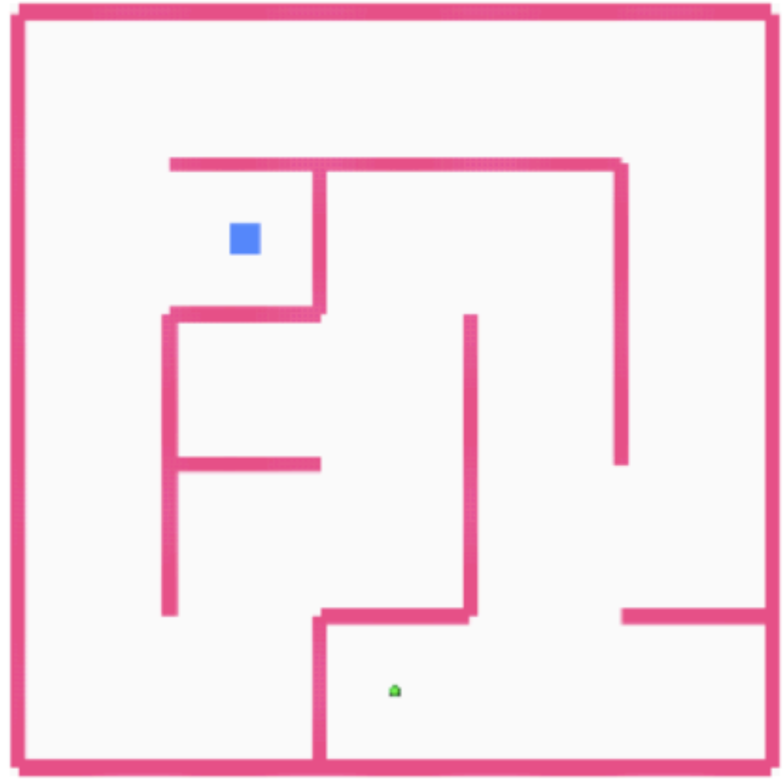
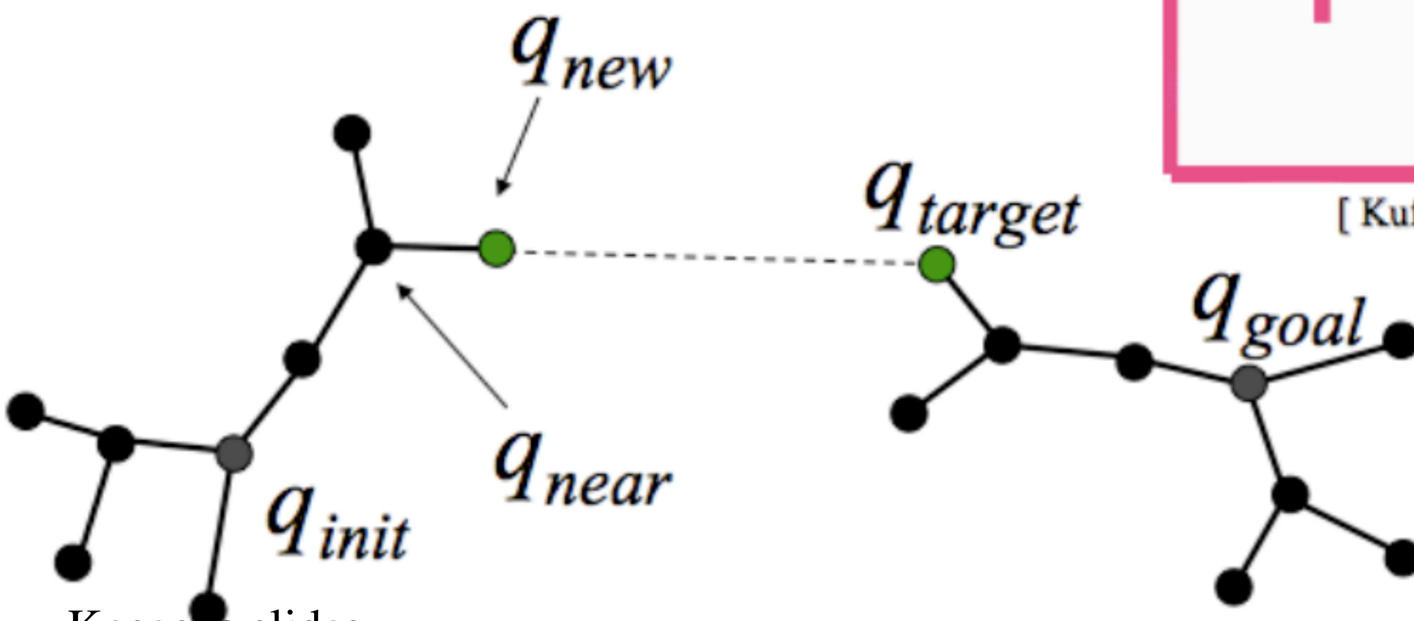
Grow two RRT's together





[Kuffner, LaValle ICRA '00]



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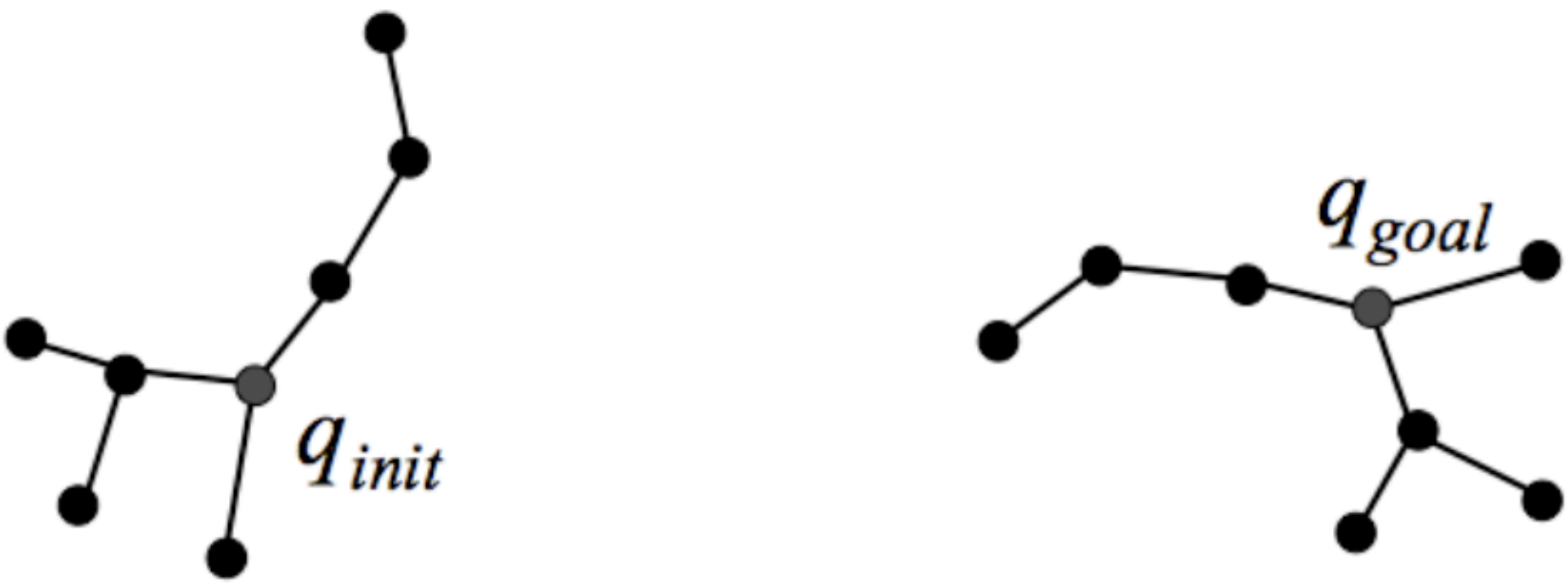
Two RRT's

A single RRT-Connect iteration...



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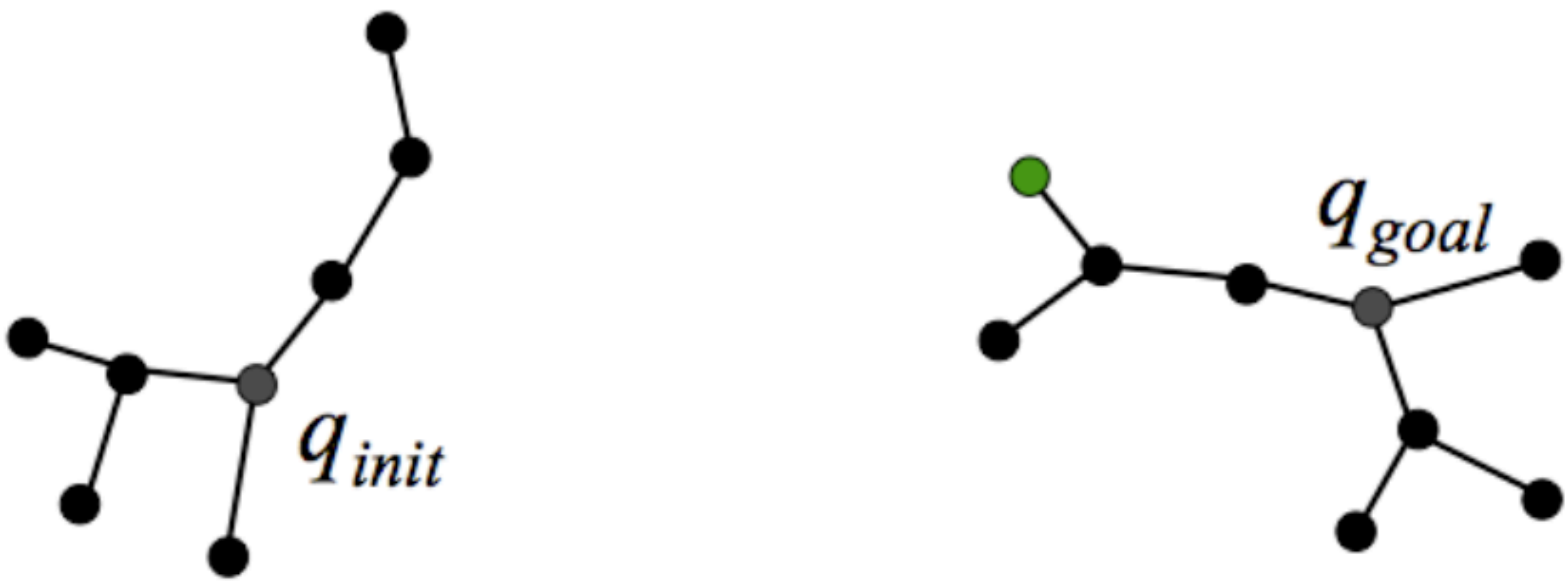
Two RRT's

1) One tree grown using random target



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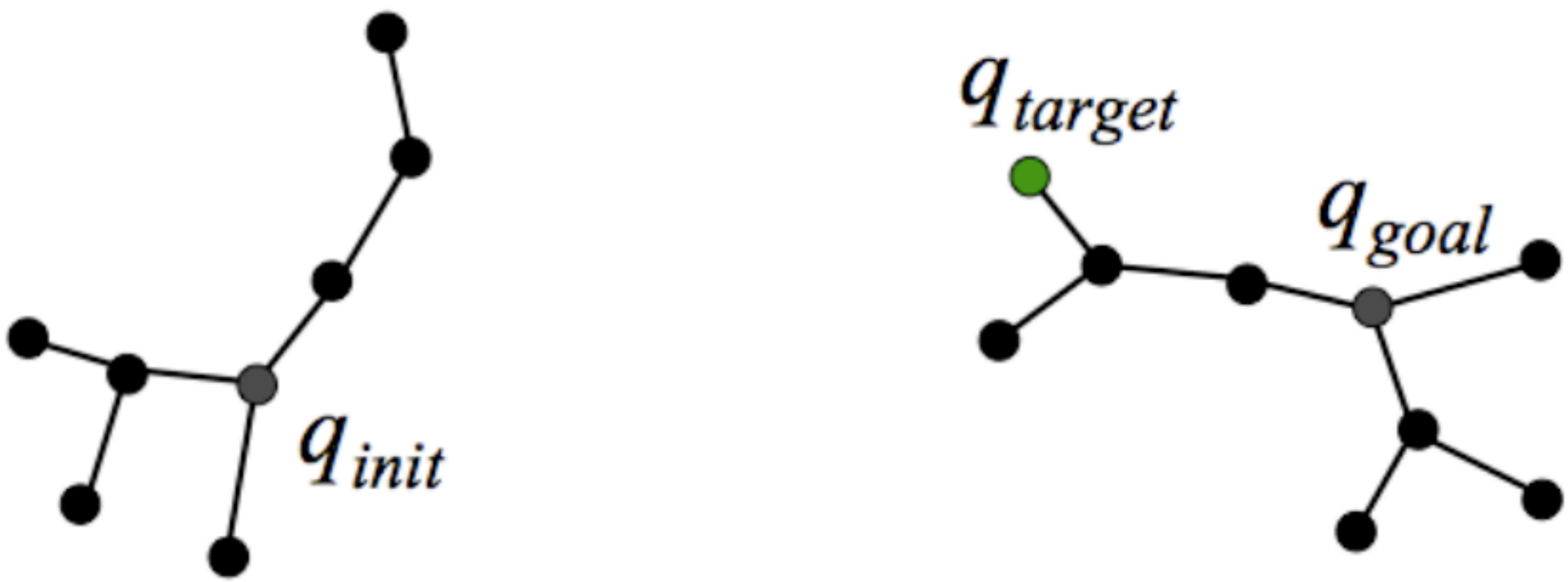
Two RRT's

2) New node becomes target for other tree



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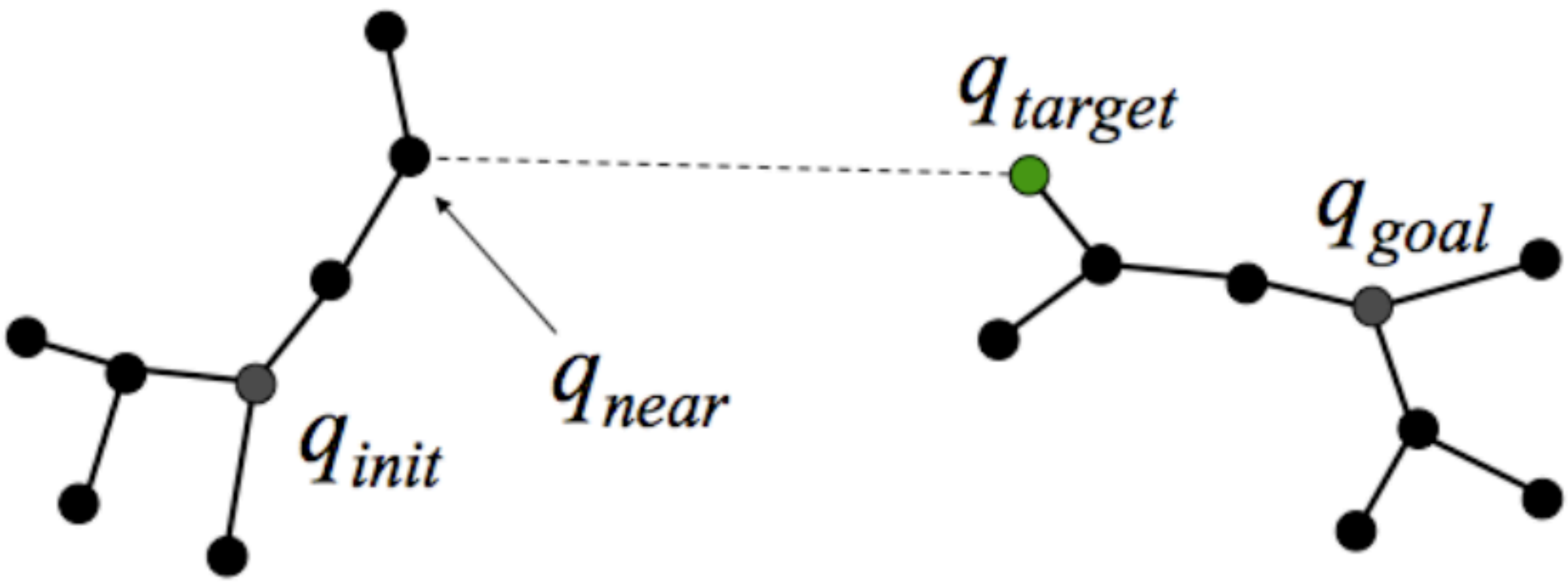
Two RRT's

3) Calculate node "nearest" to target



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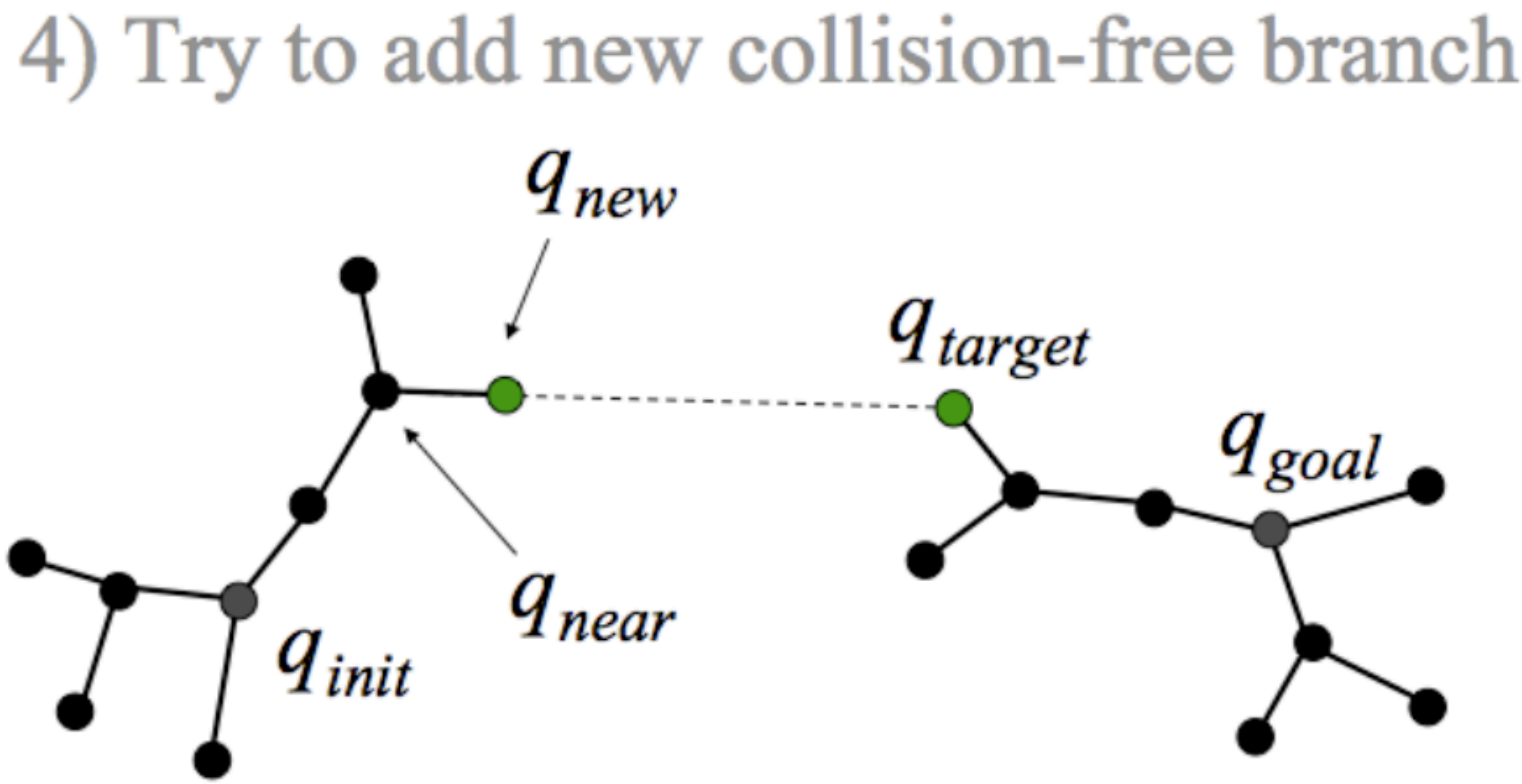
Two RRT's



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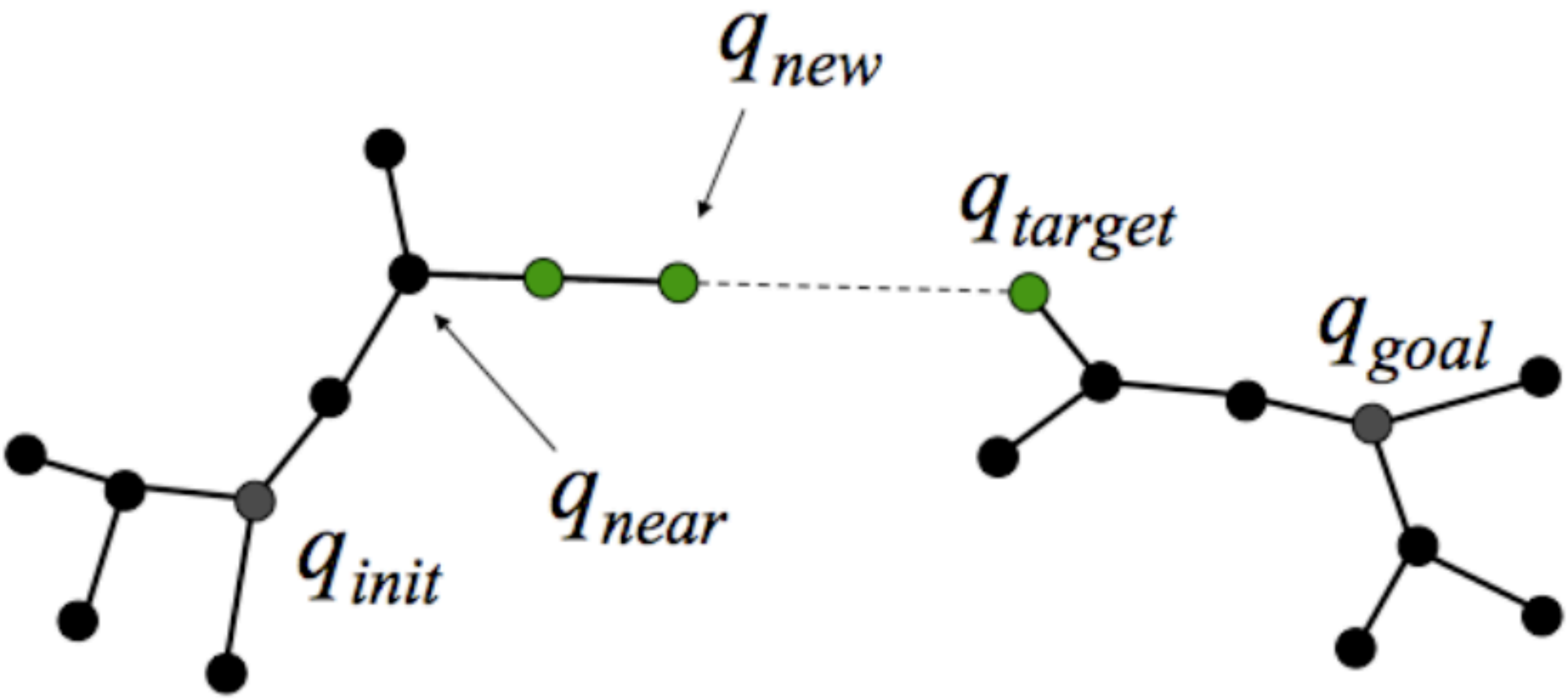
Two RRT's

5) If successful, keep extending branch



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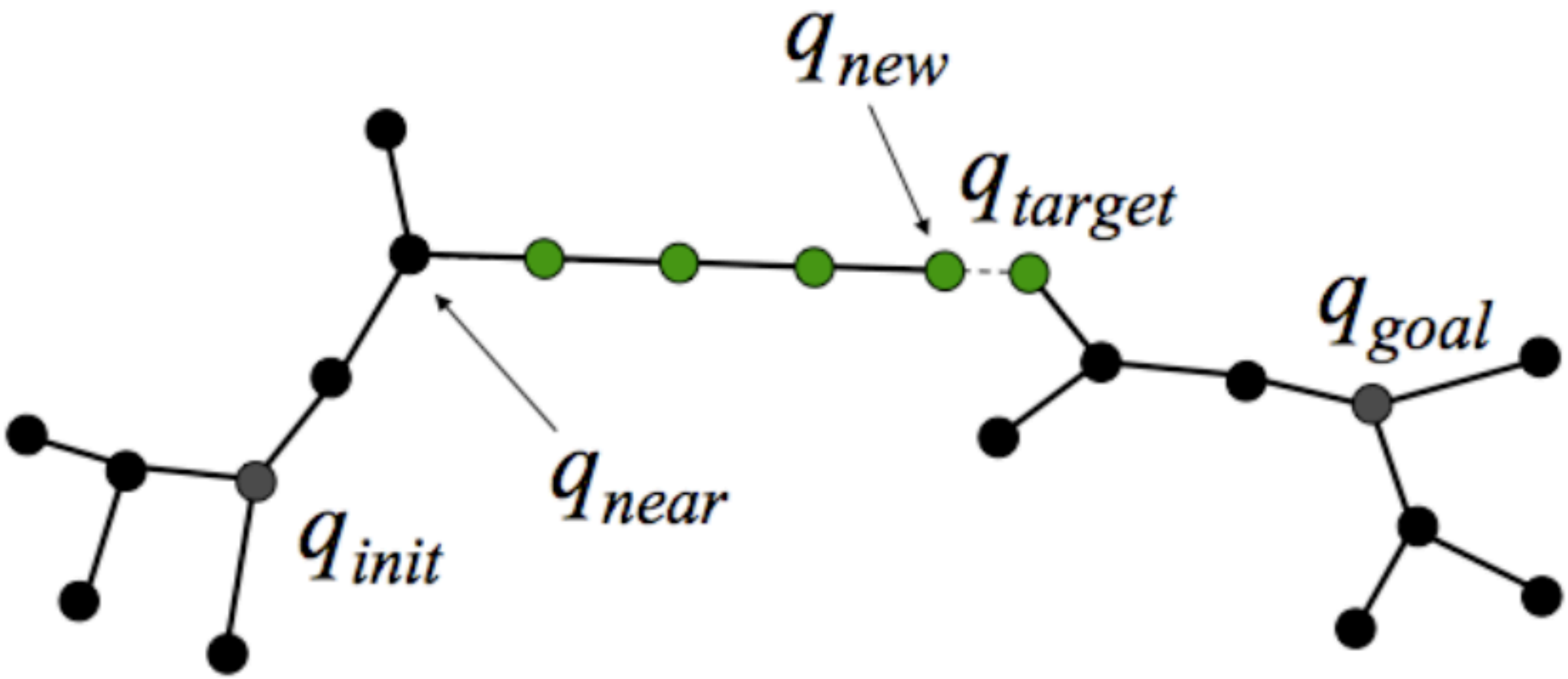
Two RRT's

5) If successful, keep extending branch



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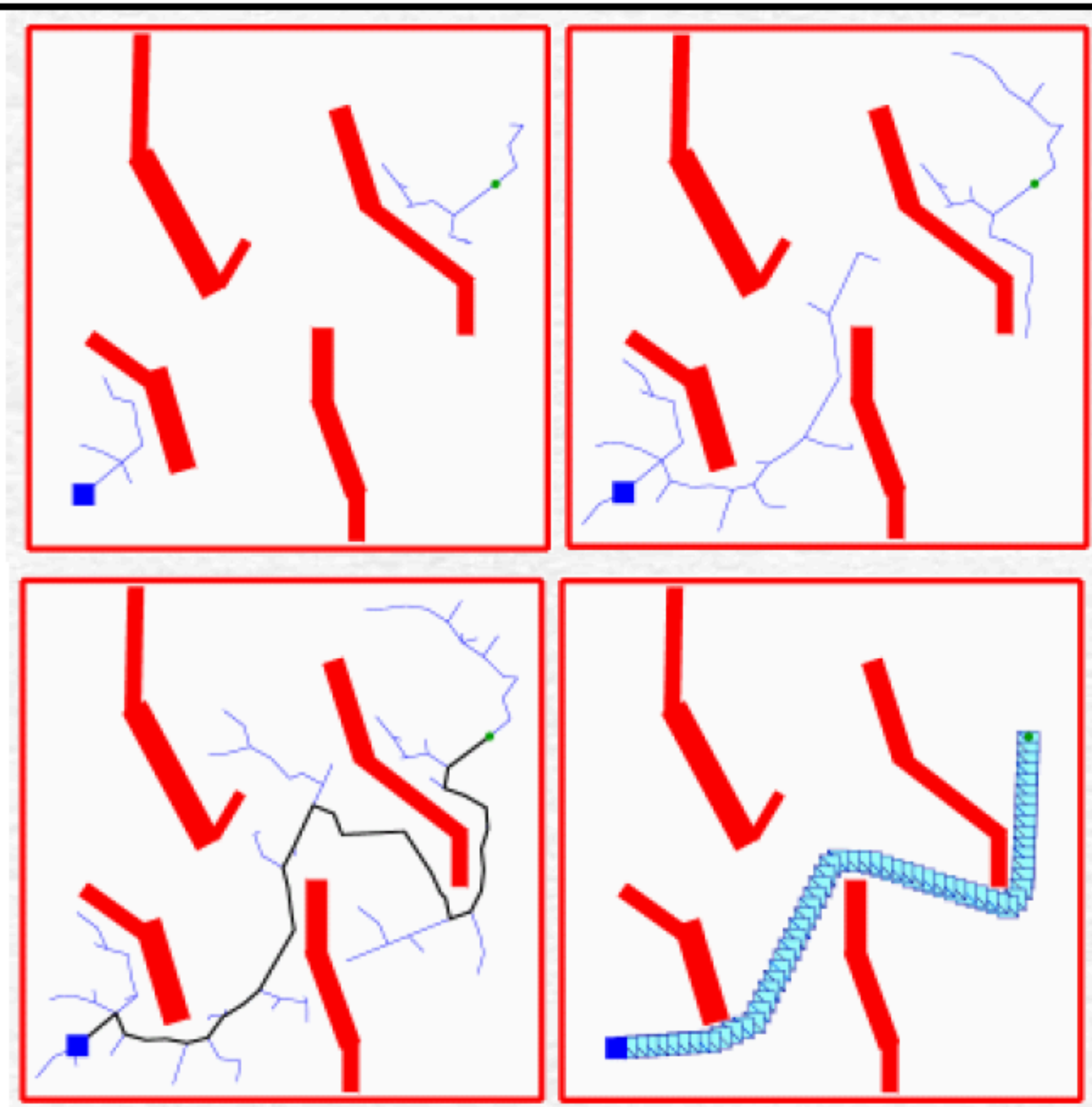

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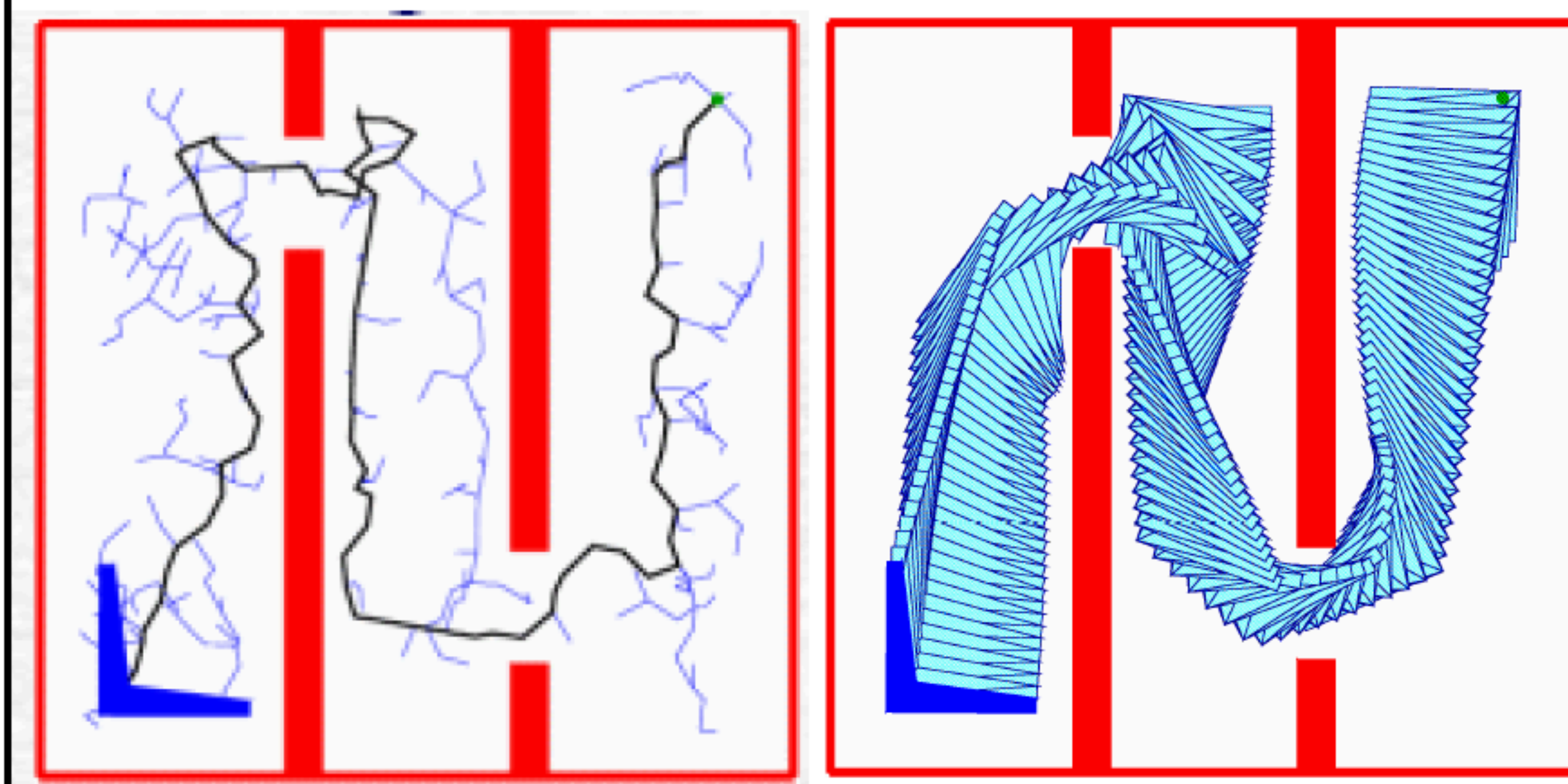

From Kuffner et al.

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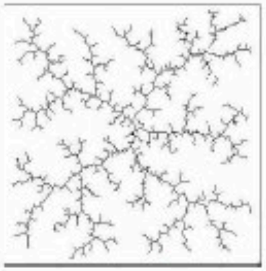 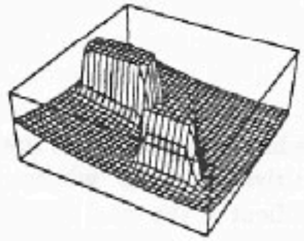 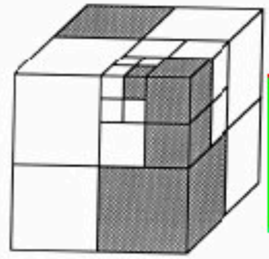 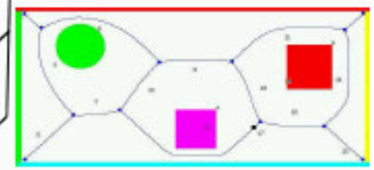 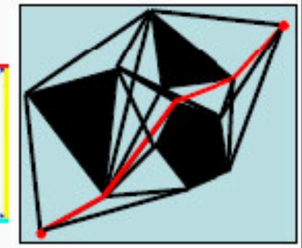

	Sampling	Potential Fields	Approx. Cell Decomposition	Voronoi	Visibility
Practical in ~2-D or 3-D	Y	Y	Y	Y	Y
Practical in >> 2-D or 3-D	Y	Y (using randomized version)	??	N	N
Fast	Y	Y	Y	In low dim.	In 2-D
Online Extensions	Y	Y	??	??	N
Complete?	Probabilistically complete	Probabilistically-resolution complete	Resolution-Complete	Y	Y

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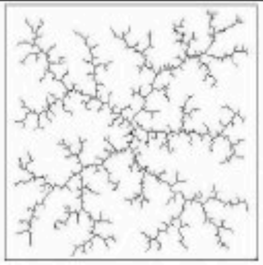
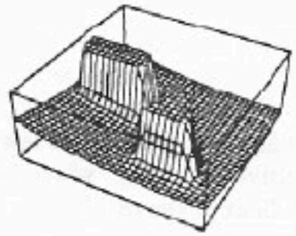
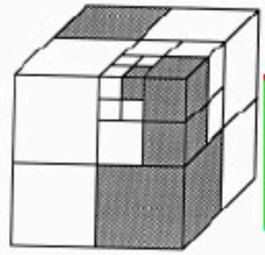
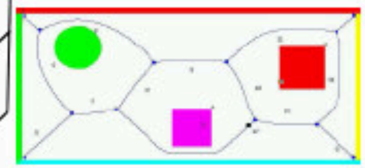
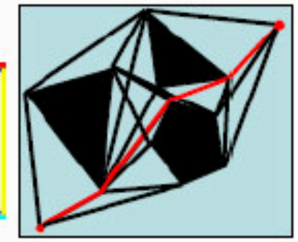

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Practical in >> 2-D or 3-D	Y	Y (using randomized version)	??	N	N
Fast	Y	Y	Y	In low dim.	In 2-D
Online Extensions					N
Complete?	Probabilistically complete	Probabilistically-resolution complete	Resolution-Complete	Y	Y

More exact/Complete

Faster/More practical in high dim.

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- (Limited) background in Russell&Norvig Chapter 25
- Two main books:
  - J-C. Latombe. Robot Motion Planning. Kluwer. 1991.
  - S. Lavalle. Planning Algorithms. 2006. http://msl.cs.uiuc.edu/planning/
  - H. Choset et al., Principles of Robot Motion: Theory, Algorithms, and Implementations. 2006.
- Other demos/examples:
  - http://voronoi.sbp.ri.cmu.edu/~choset/
  - http://www.kuffner.org/james/research.html
  - http://msl.cs.uiuc.edu/rrt/

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