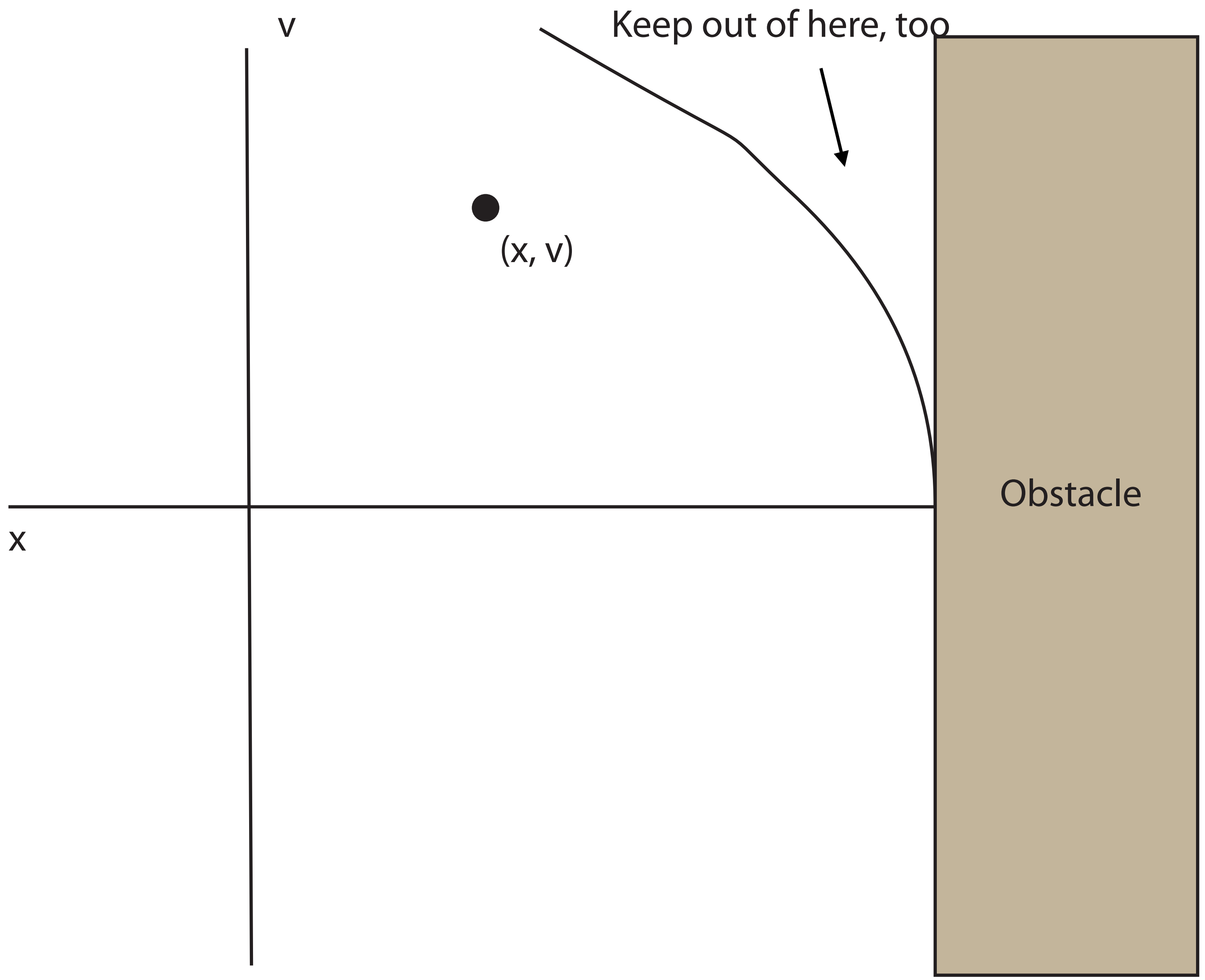
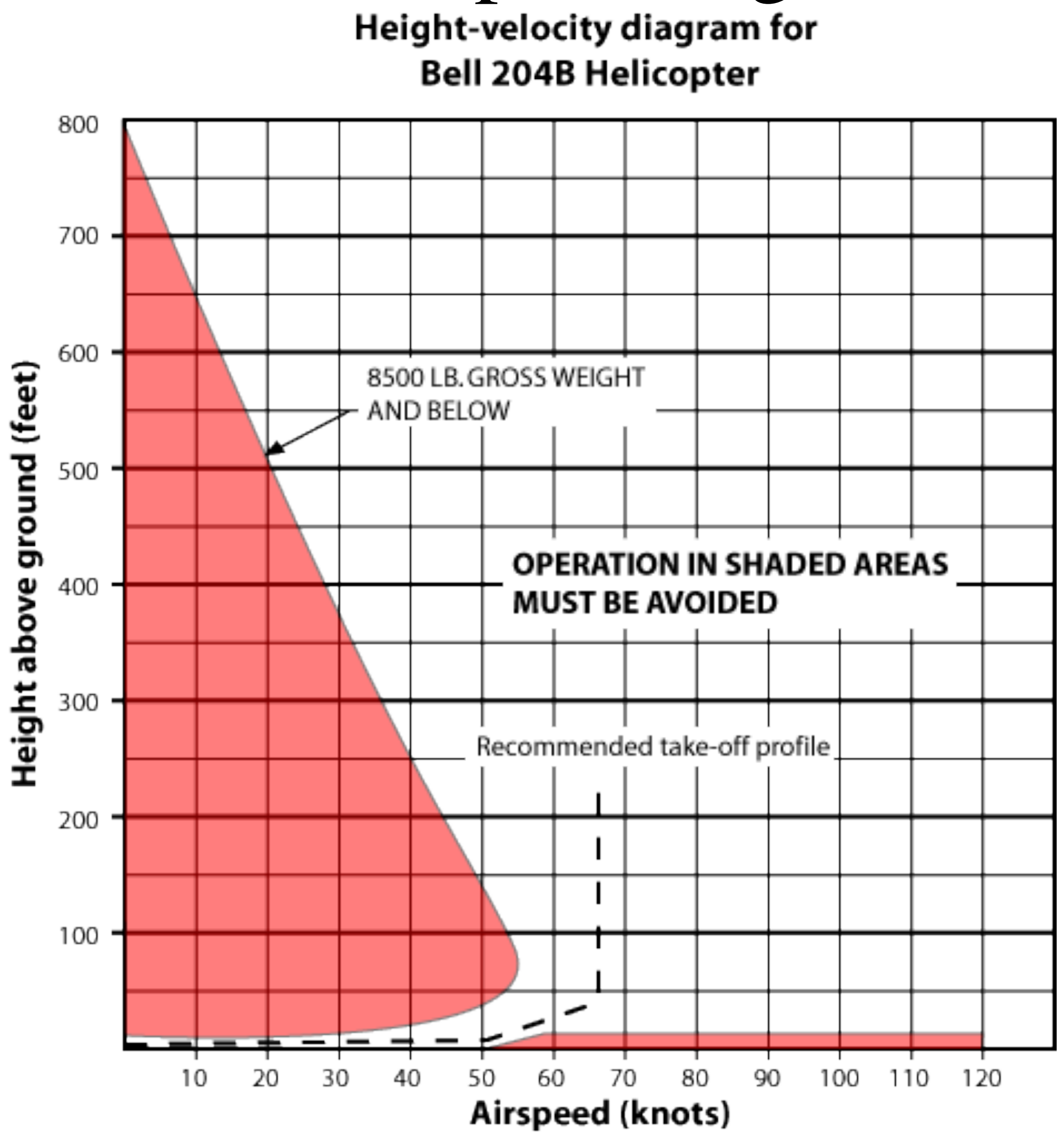
Planning, dynamics and motion graphs

D.A. Forsyth, UIUC


v
Keep out of here, too
(x, v)
Obstacle
x

Helicopter height-velocity diagram



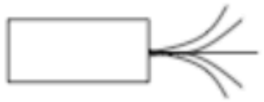
(colloquially, dead man's curve; from wikipedia; there are all sorts of operating limits to helicopters)

Key issue

- You have to think about control input as well
- Compare:
    - RRT in kinematic planning -
        - check is there no obstacle
    - RRT in dynamic planning
        - what feasible control gets you from qnear to qrand?
            - there might not be one
            - it might be hard to find

Taking actions into account

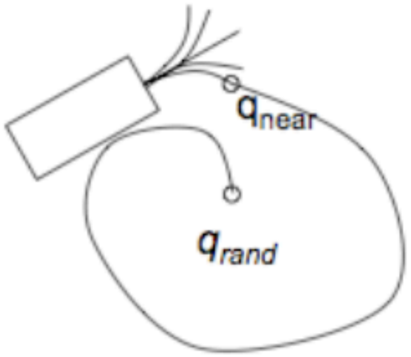
q_{near}



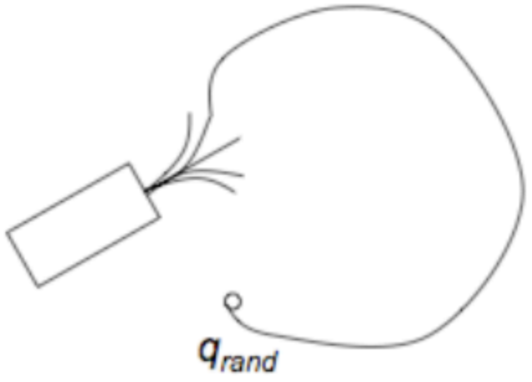
$q' = f(q,u)$ - - - use action u from q to arrive at q'

Notice this f isn't the f in slide 6 —
it maps initial state to final state given control u

chose $u_* = \arg\min(d(q_{rand}, q'))$

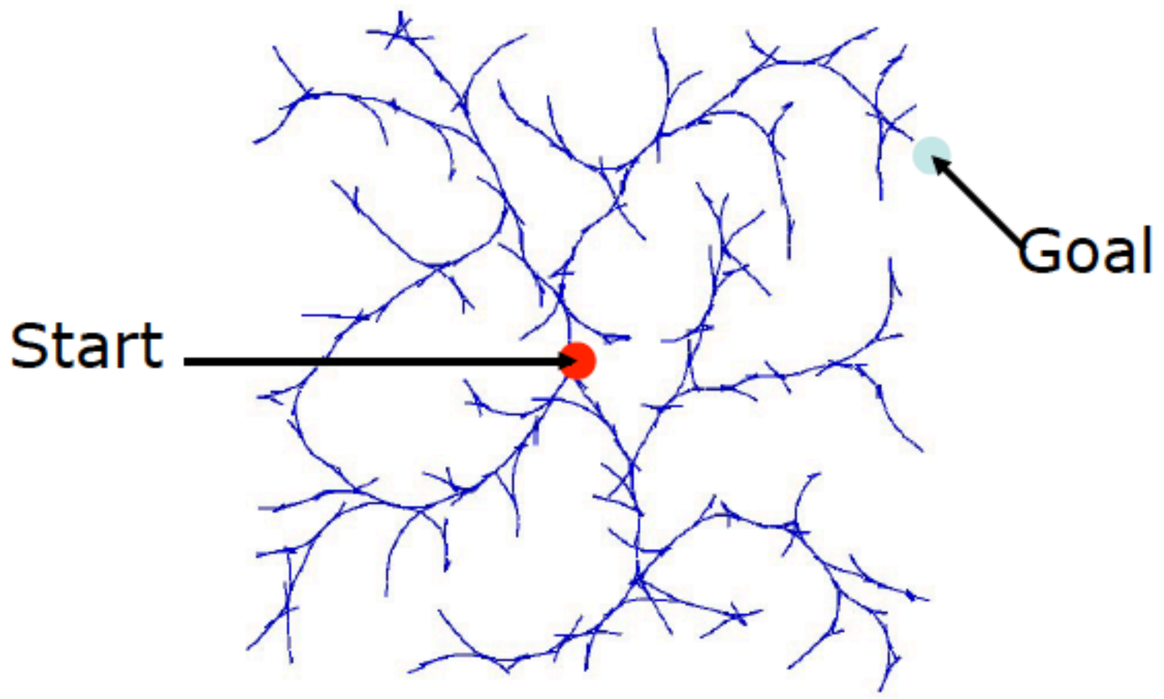


Is this the best?



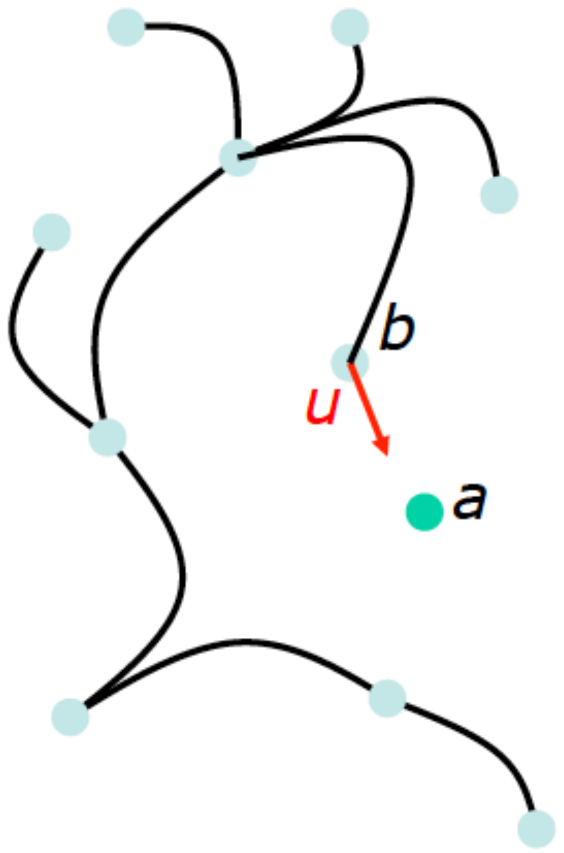
How it Works

- Build a rapidly-exploring random tree in state space (X), starting at s_{start}
- Stop when tree gets sufficiently close to s_{goal}



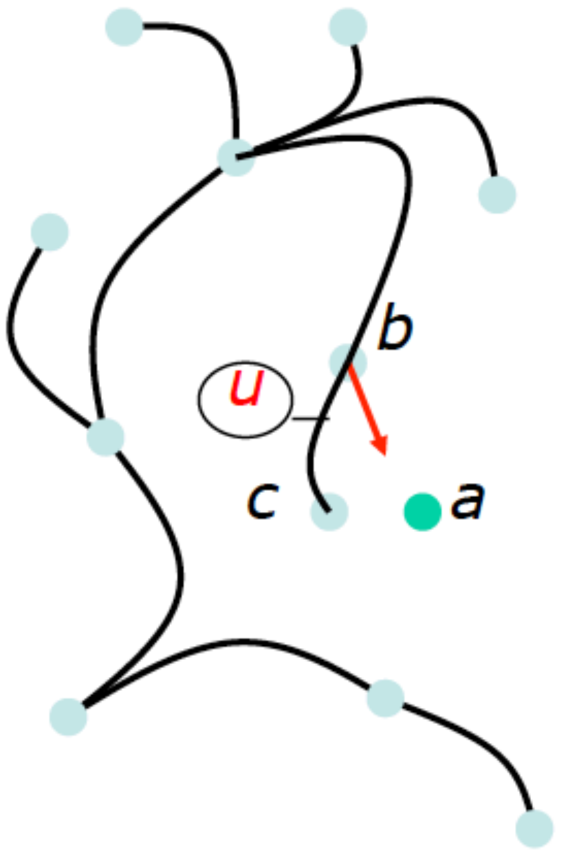
Building an RRT

- To extend an RRT:
  - Pick a random point *a* in *X*
  - Find *b*, the node of the tree closest to *a*
  - Find control inputs *u* to steer the robot from *b* to *a*



Building an RRT

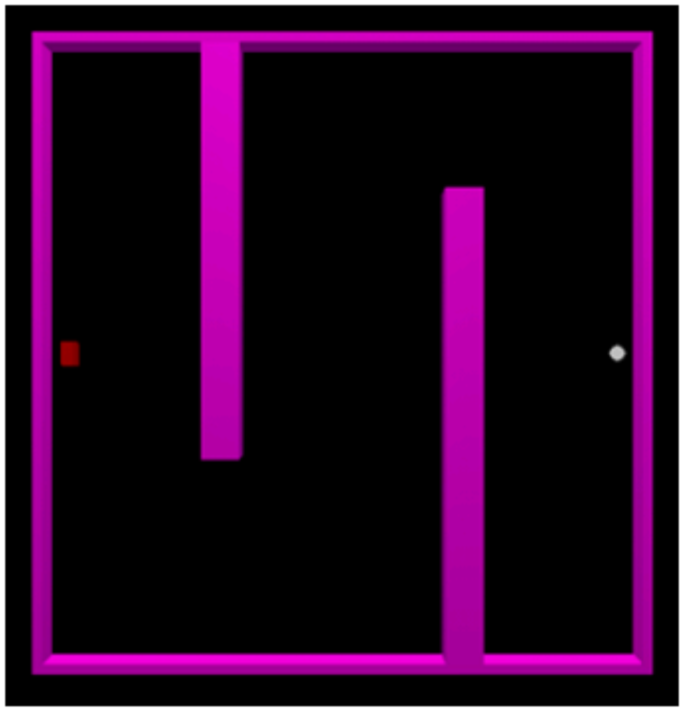
- To extend an RRT (cont.)
  - Apply control inputs *u* for time δ, so robot reaches *c*
  - If no collisions occur in getting from *a* to *c*, add *c* to RRT and record *u* with new edge

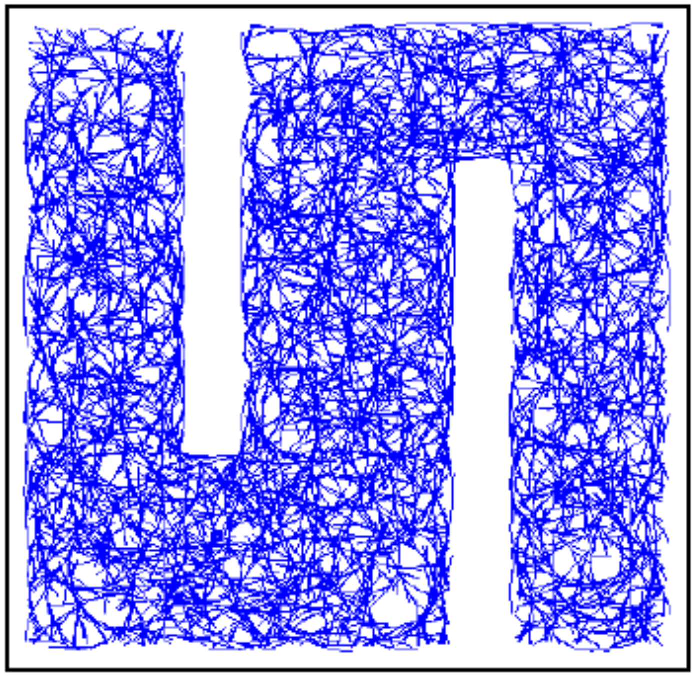


Executing the Path

- Once the RRT reaches s_{goal}
  - Backtrack along tree to identify edges that lead from s_{start} to s_{goal}
  - Drive robot using control inputs stored along edges in the tree

Problem of Simple RRT Planner

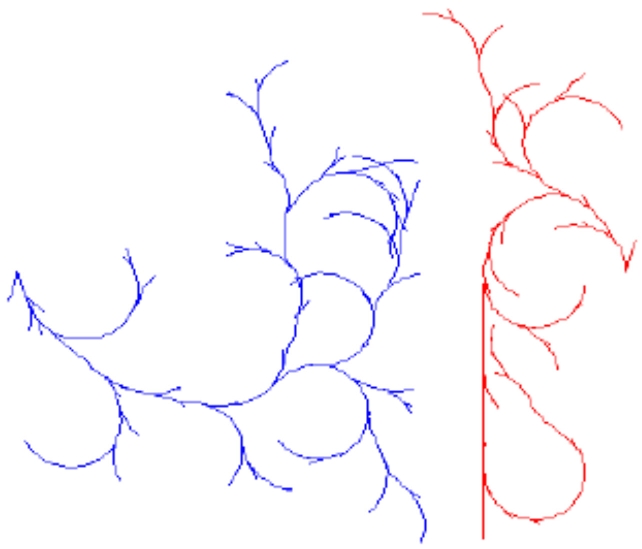




- Problem: ordinary RRT explores *X* uniformly
  → slow convergence
- Solution: bias distribution towards the goal – once in a while choose goal as new random configuration (5-10%)
- If goal is choose 100% time then it is randomized potential planner

Bidirectional Planners

- Build two RRTs, from start and goal state



- Complication: need to connect two RRTs
  - local planner will not work (dynamic constraints)
  - **bias** the distribution, so that the trees meet

RRT’s

- Link
- http://msl.cs.uiuc.edu/rrt/gallery.html

- Issues/problems
- Metric sensitivity
- Nearest neighbour efficiency
- Optimal sampling strategy
- Balance between greedy search and exploration

- Applications in mobile robotics, manipulation, humanoids, biology, drug design, areo-space, animation
- Extensions – real-time RRT’s, anytime RRT’s dynamic domains RRT’sm deterministic RRTs, hybrid RRT’s

Building an RRT

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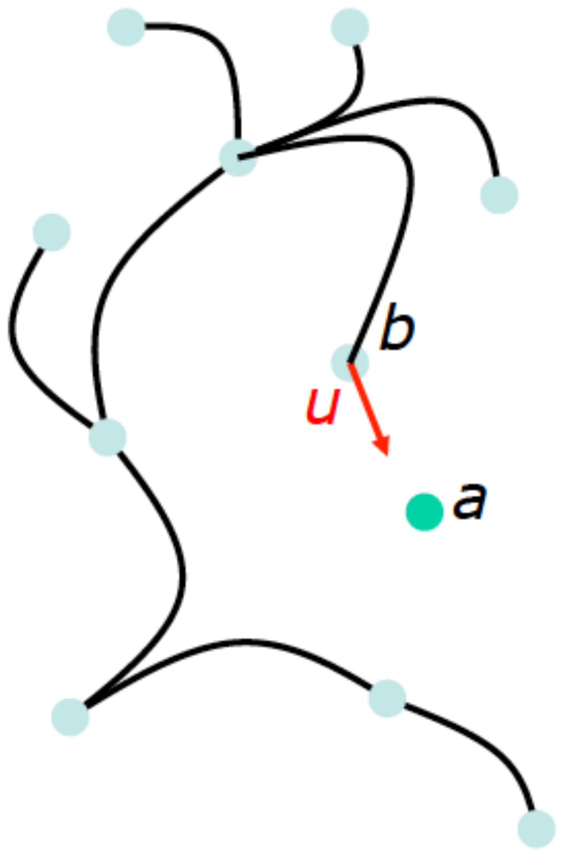
↑

HOW?

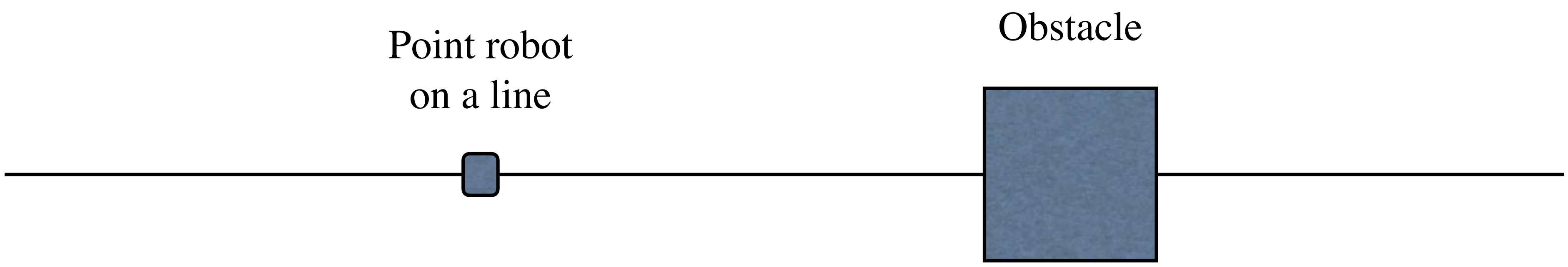
Strategies:

Optimization
with simulator

Discretize



Boundary value problems



$$\begin{pmatrix} \dot{x} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} v \\ u \end{pmatrix}$$

- Our simple system
  - start at x=0, v=0
  - obstacle starts at x=1
  - plan to go to x=0.9, v=-2

Control constraint: $-1 \leq u \leq 1$

Boundary value problems - II

- Our simple system
  - start at x=0, v=0
  - obstacle starts at x=1
  - plan to go to x=0.9, v=-2
- Write this system
  - so general solution is:

$$\frac{d^2x}{dt^2} = u$$

$$c_0 + c_1 t + \frac{u}{2}t^2$$

$$\begin{pmatrix} x \\ v \end{pmatrix} = \begin{pmatrix} c_0 + c_1 t + \frac{u}{2}t^2 \\ c_1 + ut \end{pmatrix}$$

What does this look like in (x, v) space?

Control constraint: $-1 \leq u \leq 1$

Boundary value problems - III

- Imagine we're at $\begin{pmatrix} a \\ b \end{pmatrix}$
- and want to go to $\begin{pmatrix} c \\ d \end{pmatrix}$
- What u do we use?
  - and can we do it?

$$\begin{pmatrix} x \\ v \end{pmatrix} = \begin{pmatrix} c_0 + c_1 t + \frac{u}{2} t^2 \\ c_1 + ut \end{pmatrix}$$

- Notice we have four constraints, four unknowns
  - didn't specify arrival time!

Boundary value problems - IV

- Start at t=0 $$\begin{pmatrix} x(0) \\ v(0) \end{pmatrix} = \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$
- arrive at time t=s $$\begin{pmatrix} x(s) \\ v(s) \end{pmatrix} = \begin{pmatrix} a + bs + \frac{u}{2}s^2 \\ b + us \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}$$

$$s = \frac{d-b}{u} \qquad (a-c)u = -b(d-b) - \frac{1}{2}(d-b)^2$$

$$w = \frac{-b(d-b) - \frac{1}{2}(d-b)^2}{(a-c)}$$

$$u = \begin{cases} 1 & \text{if } w > 1 \\ w & \text{if } -1 \le w \le 1 \\ -1 & \text{otherwise} \end{cases}$$

Building an RRT

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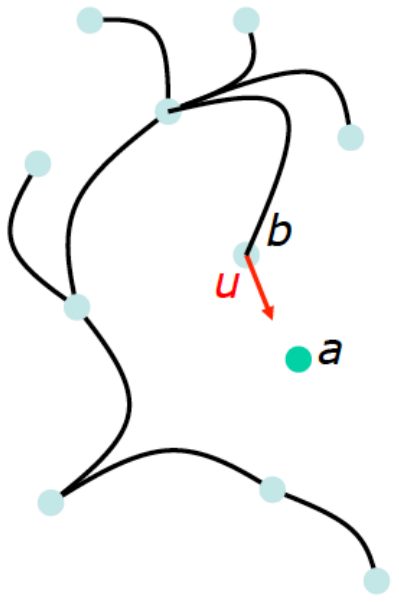
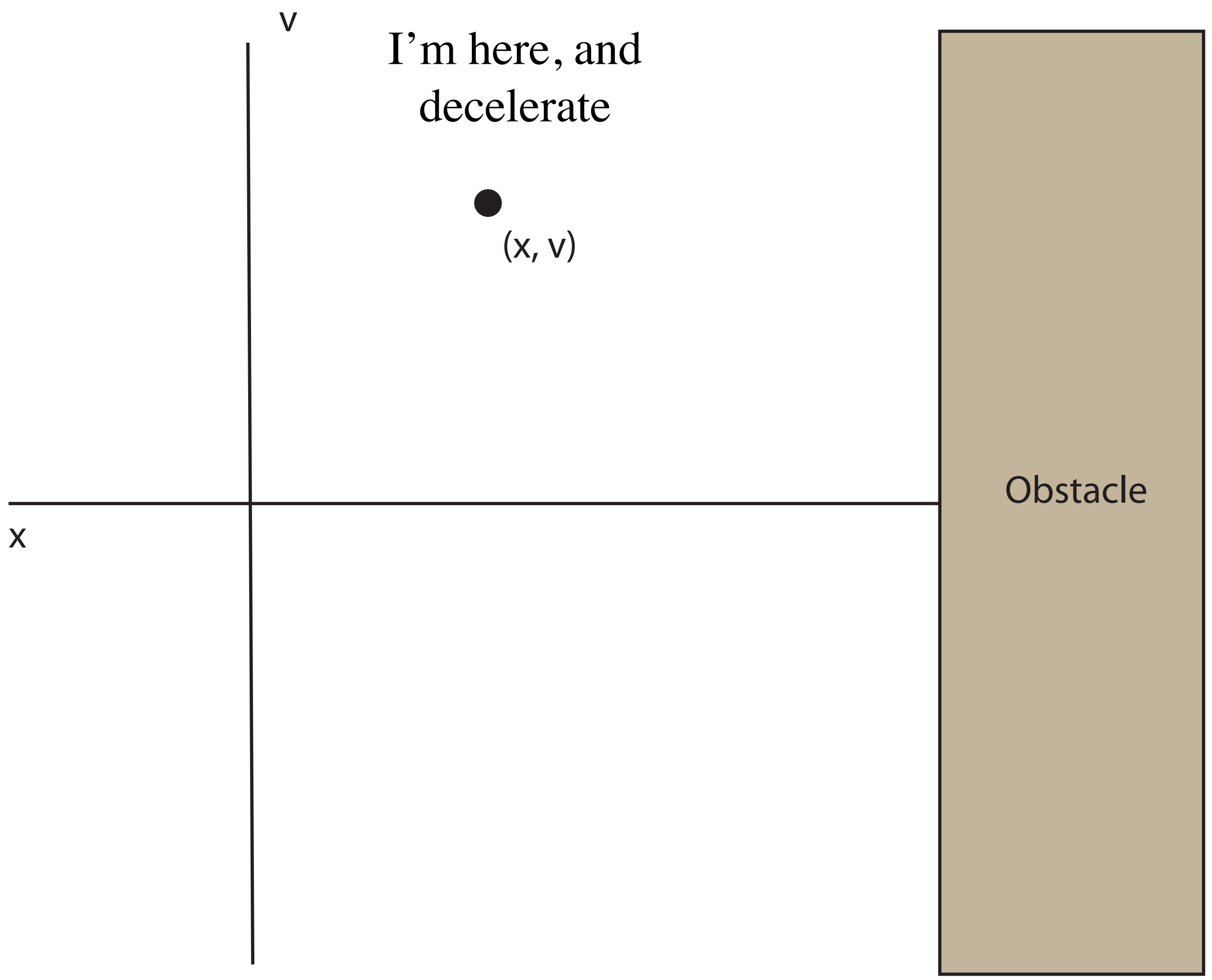


- Can now compute u, s
  - that takes us from closest point to sample
    - or nearby (recall constraints on u)
  - does that path intersect physical obstacle?
    - yes - either drop, or reduce s
    - no - add sample, recording u, s as well
- We now can build a tree!



Distance travelled until stationary at acceleration = -1

$$\Delta x = \frac{v^2}{2}$$

Boundary value problems - V

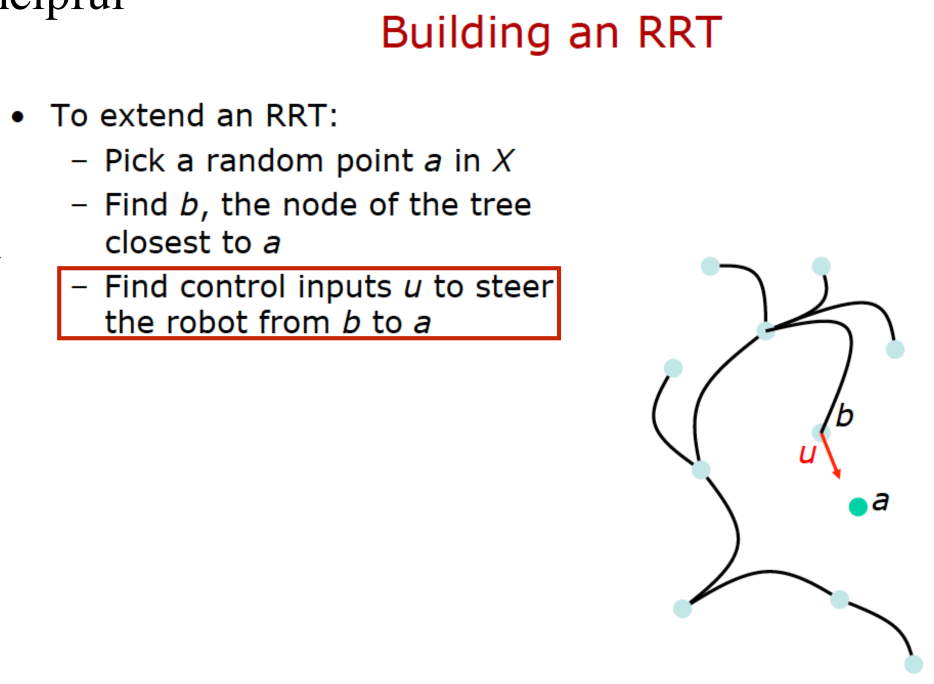
- This should strike you as being hard to generalize
  - most odes don’t have easy closed form solutions
  - why use constant acceleration?
    - I got an easy answer
- Strategies
  - use a simulator
  - search a discrete set of control inputs
- Particularly important idea
  - Do many path segments in advance; cache the results (motion primitives)
  - Search that set for something that gets “close” to the endpoint
- Q:
  - what primitives? search how?

Motion Primitives

- Discretization
    - build a set of motion primitives
        - start state, control input -> path, end state
    - procedures for composing them
        - what primitives can be applied in what state?
            - there is translation, rotation covariance
- Search this set of primitives
    - various options

Searching the primitives - I

- Build an RRT whose edges are primitives
  - Recall in RRT, we must build an edge from a state to a state
    - or partway
  - Do this by finding a primitive that is “helpful”
    - blank search
    - some form of hashing
  - “helpful”
    - new state is “close to” desired state



Building an RRT

- To extend an RRT:
  - Pick a random point a in X
  - Find b, the node of the tree closest to a
  - Find control inputs u to steer the robot from b to a

Searching the primitives - II

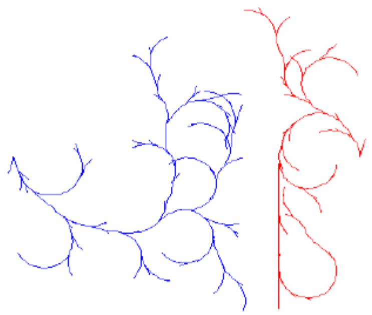
- Some form of randomized search
    - to assemble primitives into path from start to goal
    - (more under motion graphs)

Joining up

- Generic issue: how to adjust path?

Bidirectional Planners

- Build two RRTs, from start and goal state



- Complication: need to connect two RRTs
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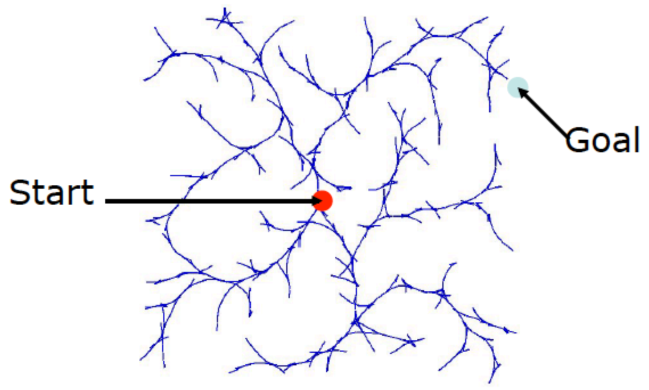


Dynamics make planning harder

- Dynamics introduce differential constraints

$$\underset{\substack{\uparrow\\\text{Derivative of state}}}{\dot{x}} = \underset{\substack{\uparrow\\\text{Quite possibly nasty}}}{f}(\underset{\substack{\uparrow\\\text{State}}}{x}, \underset{\substack{\uparrow\\\text{Control input -}\\\text{there might be}\\\text{constraints on this,}\\\text{too}}}{u})$$

Boundary value problems - I

$$\underset{\substack{\uparrow\\ \text{Derivative of state}}}{\dot{x}} = \underset{\substack{\uparrow\\ \text{Quite possibly nasty}}}{f}(\overset{\substack{\text{State}\\ \downarrow}}{x}, \underset{\substack{\uparrow\\ \text{Control input -}\\ \text{there might be}\\ \text{constraints on this,}\\ \text{too}}}{u})$$

$$x(0) = a$$ Start

$$x(1) = g$$ Goal

Notice over here we've assumed an arrival time at goal

Constraints on controls
Constraints on state

Boundary value problems - II

Generally, find u such that:

$$\dot{x} = f(x, u)$$

$$x(0) = a$$

$$x(1) = g$$

Constraints on controls
Constraints on state

We already have u such that:

$$\dot{x} = f(x, u)$$

$$x(0) = a$$

$$x(1) = b$$

Constraints on controls
Constraints on state

BVP's - III

$$\min_{\delta u} \ \text{Cost}(\delta u)$$

$$\begin{aligned}\text{such that}\\ \dot{x} = f(x; u + \delta u)\\ x(0) = a\\ x(1; u + \delta u) = g\\ \text{Constraints on state}\\ \text{Constraints on controls}\end{aligned}$$

BVP's - IV

$$\min_{\delta u} \ \mathrm{Cost}(\delta u)$$

$$\text{such that}$$

$$\dot{x} = f(x; u + \delta u)$$

$$x(0) = a$$

$$x(1; u + \delta u) = g$$

$$\text{Constraints on state}$$

$$\text{Constraints on controls}$$

- Simplify by
  - making \delta u finite dimensional
  - assuming a solution exists
    - and is a “reasonable” function of \delta u
    - not trivial!

- becomes:
  - f-d constrained optimization problem

BVP's - IV

- Simplify by
  - making \delta u finite dimensional
  - assuming a solution exists
    - and is a "reasonable" function of \delta u
    - not trivial!

Search for these

$$\delta_u = \sum_i \overset{\downarrow}{a_i} \underset{\uparrow}{\phi_i}(t)$$

Choose these

$$x(1; a) = G(a)$$

- becomes:
  - f-d constrained optimization problem

BVP's - V

$$\min_a \; a^T a$$

$$\text{subject to}$$

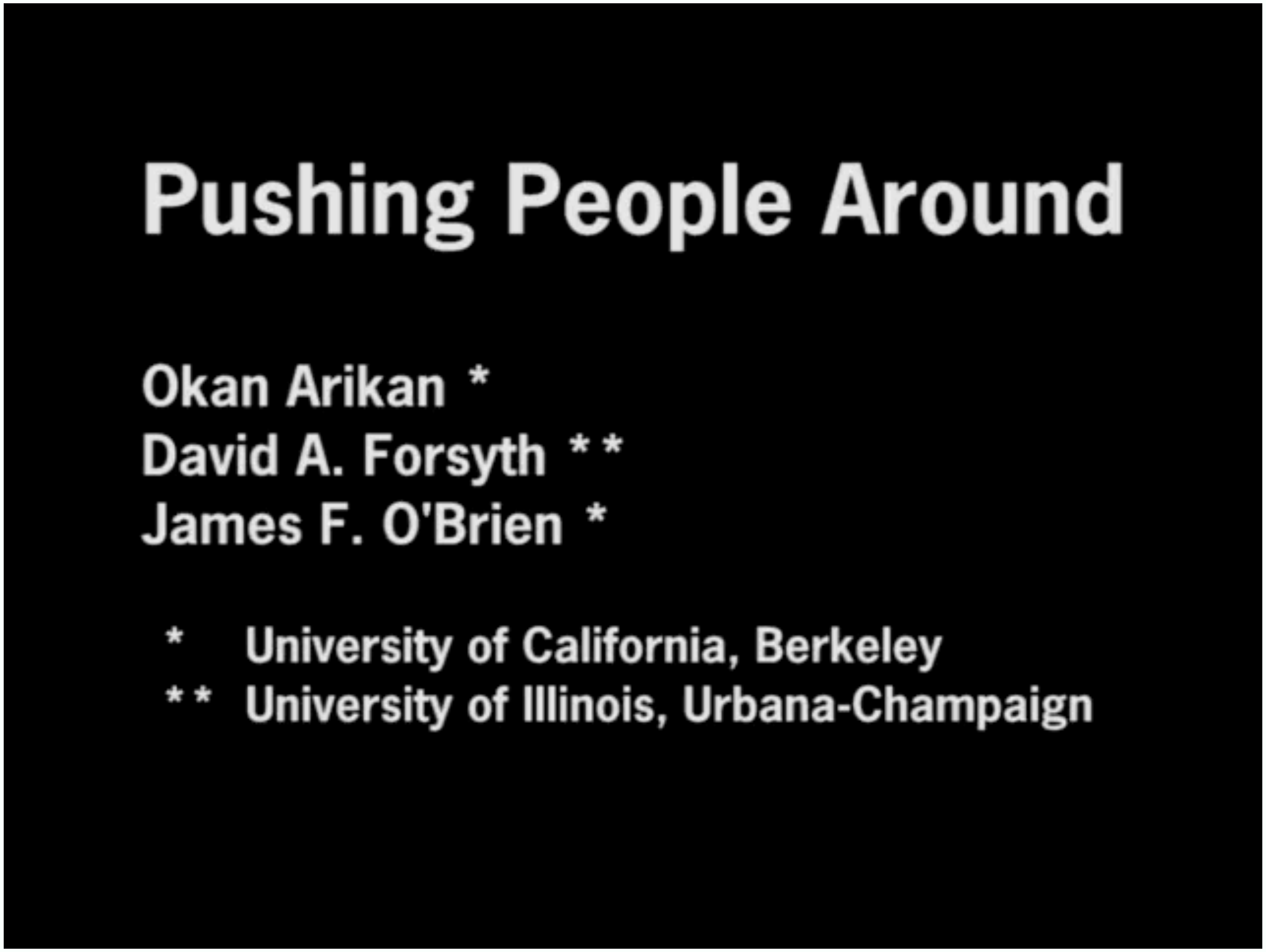
$$G(a) = g$$

$$\text{Constraints on state}$$

$$\text{Constraints on controls}$$

- Could use a numerical method
- For example
  - Augmented Lagrangian method
  - Q: gradient of G wrt a?
    - possibly numerical
- With a reasonable hope of success

Motion capture


Pushing People Around

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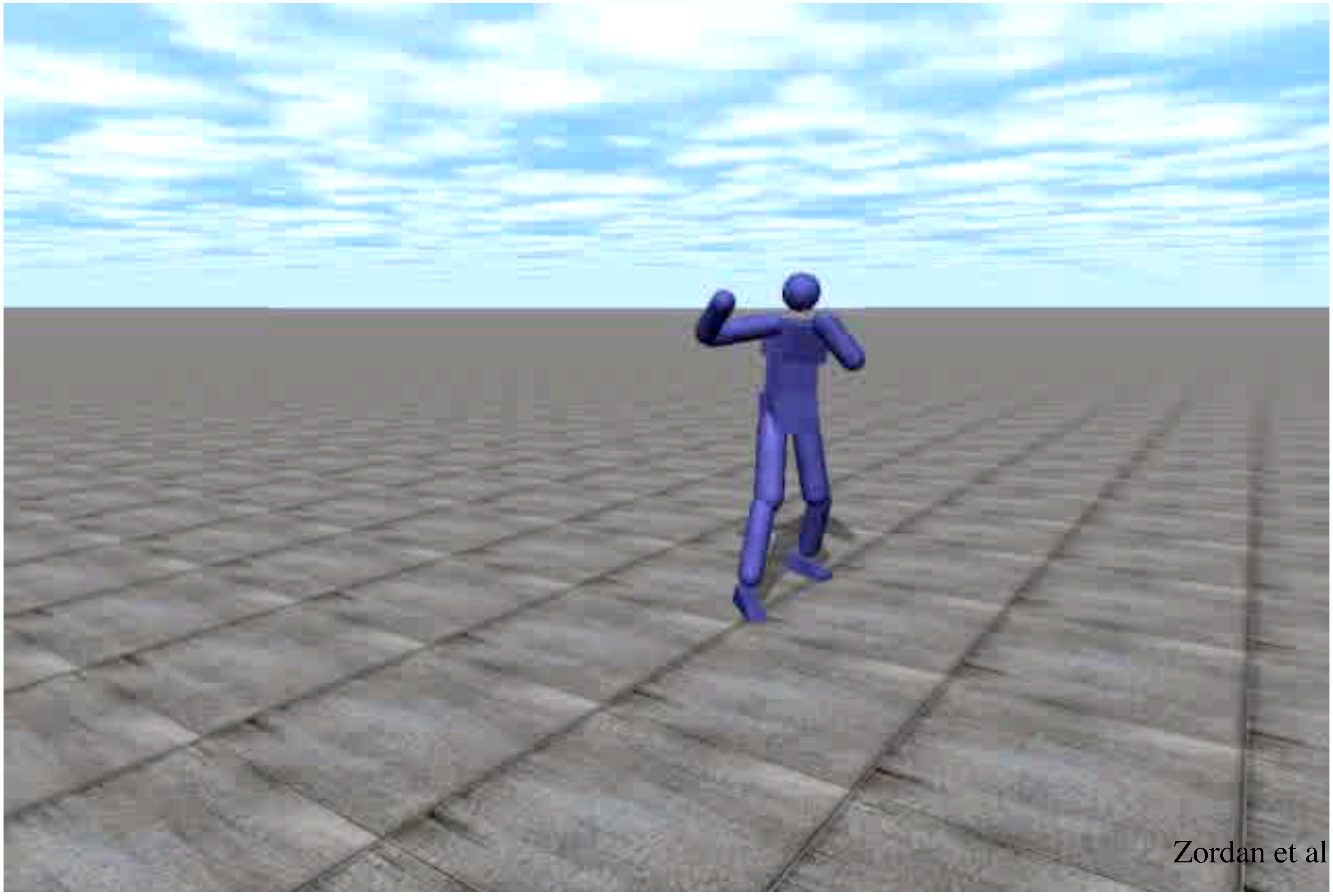
James F. O'Brien *

* University of California, Berkeley
** University of Illinois, Urbana-Champaign

Arikan ea 06

However, modifying motion is dangerous


Zordan et al 03

The motion graph

- Old idea in human animation
  - Essentially, build a roadmap of what people can do by
    - joining up animation sequences
  - Control by
    - searching these sequences

Motion graph

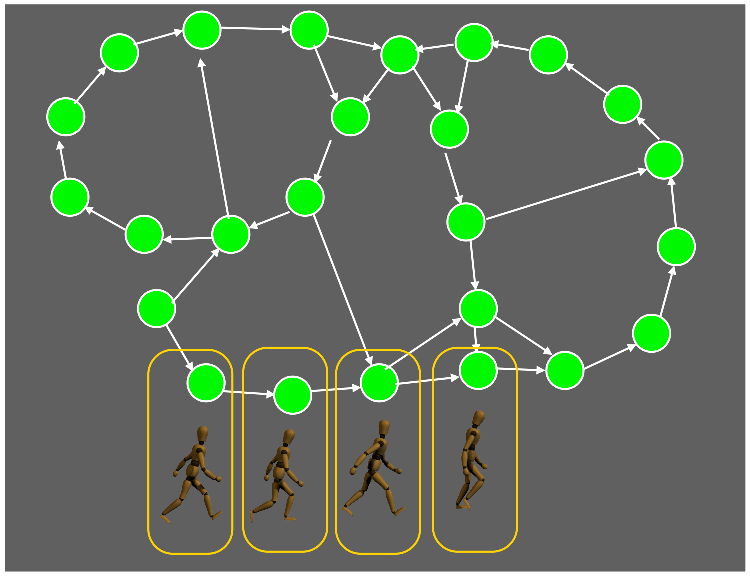
- Motion graph: by analogy with
  - text synthesis, texture synthesis, video textures
- Take measured frames of motion as nodes
  - from motion capture, given us by our friends
- Edge from frame to any that could succeed it
  - decide by dynamical similarity criterion
  - see also (Kovar et al 02; Lee et al 02)
- A path is a motion
- Search with constraints
  - like root position+orientation, etc.
  - In various ways
    - Local (Kovar et al 02)
    - Lee et al 02; Ikemoto, Arikan+Forsyth 05

Motion Graph:

Nodes = Frames

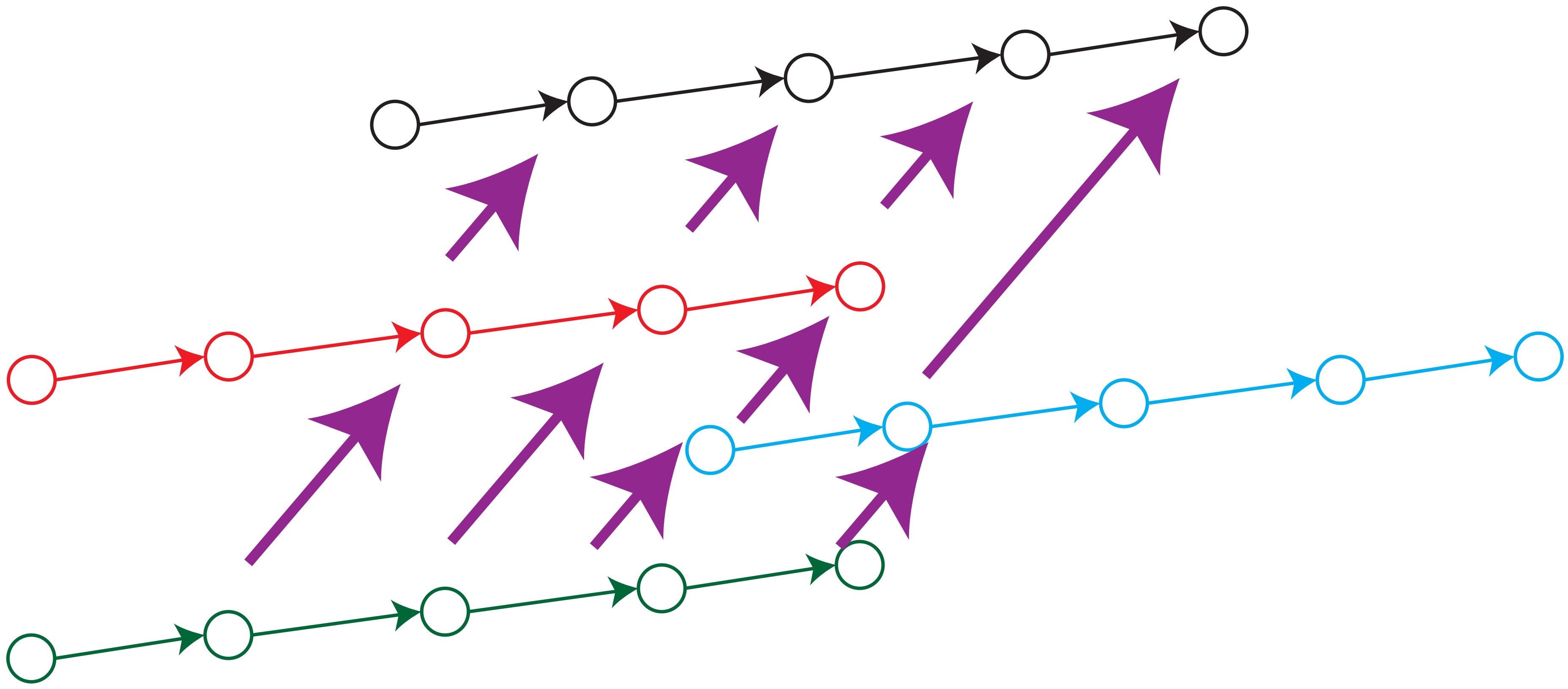
Edges = Transition

A path = A motion



Inserting edges

- Given a reasonable matching function, we can insert edges



Matching frames

- Q: when can I cut from one sequence to another?
    - without problems?
- A: when the next frames are "like one another"
    - configuration is similar
        - up to rotation and translation of the torso
            - rotate and translate f_i to f_j's frame
            - check joints
    - velocity is the same
        - in torso frame
            - rotate and translate f_i to f_j's frame
            - check joint velocities
- Generally, you expect a lot of matches

Why a motion graph?

- No reliable method for generating novel motions
    - at the time (current learned methods quite good)
    - some special cases work OK

    - Keys for special cases
        - data driven methods work well for temporal composition
        - Some motions can be blended successfully
        - Contacts create special problems
        - There are complex, cross-body correlations

- Easy to build complex movements by search

Car motion graph

- Analogy with car
  - drive around “at random”
    - record position, velocity, controls
  - we now have a set of sequences of observed configurations
    - and the controls required
  - build motion graph out of sequences
    - NOTE: should have a very large number of edges
      - basically, check that velocities are similar

  - search this for a path
    - controls? read off from frames in path
    - problems? replan by searching again

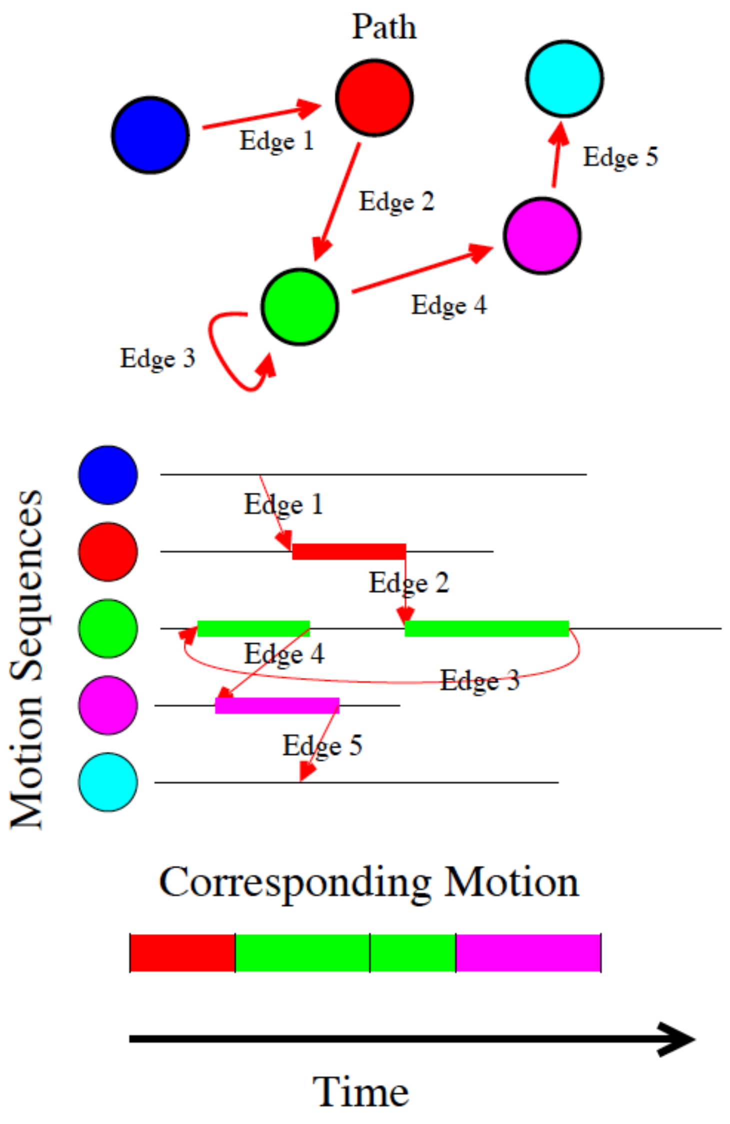


Figure 1: We wish to synthesize human motions by splicing together pieces of existing motion capture data. This can be done by representing the collection of motion sequences by a directed graph (**top**). Each sequence becomes a node; there is an edge between nodes for every frame in one sequence that can be spliced to a frame in another sequence or itself. A valid path in this graph represents a collection of splices between sequences, as the **middle** shows. We now synthesize constrained motion sequences by searching appropriate paths in this graph using a randomized search method.

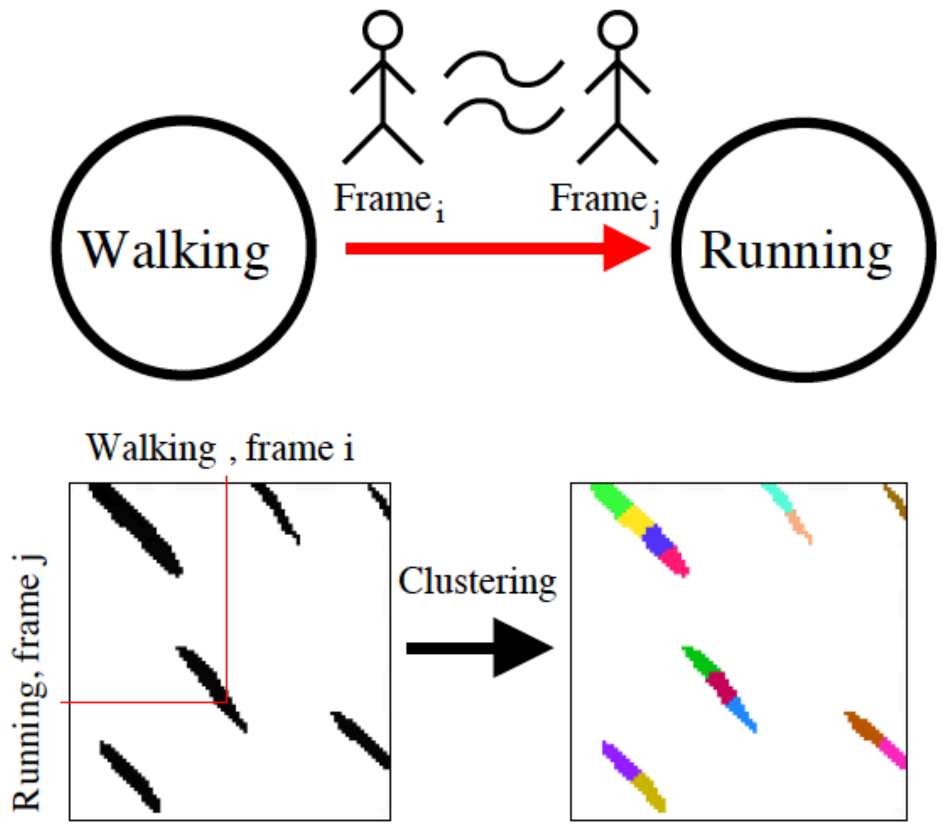


Figure 2: Every edge between two nodes representing different motion clips can be represented as a matrix where the entries correspond to edges. Typically, if there is one edge between two nodes in our graph, there will be several, because if it is legal to cut from one frame in the first sequence to another in the second, it will usually also be legal to cut between neighbors of these frames. This means that, for each pair of nodes in the graph, there is a matrix representing the weights of edges between the nodes. The i, j'th entry in this matrix represents the weight for a cut from the i'th frame in the first sequence to the j'th frame in the second sequence. The weight matrix for the whole graph is composed as a collection of blocks of this form. Summarizing the graph involves compressing these blocks using clustering.

1. Start with a set of n valid random “seed” paths in the graph G'
2. Score each path and score all possible mutations
3. Where possible mutations are:
    (a) Delete some portion of the path and replace it with 0 or 1 hops.
    (b) Delete some edges of the path and replace them with their children
4. Accept the mutations that are better than the original paths
5. Include a few new valid random “seed” paths
6. Repeat until no better path can be generated through mutations

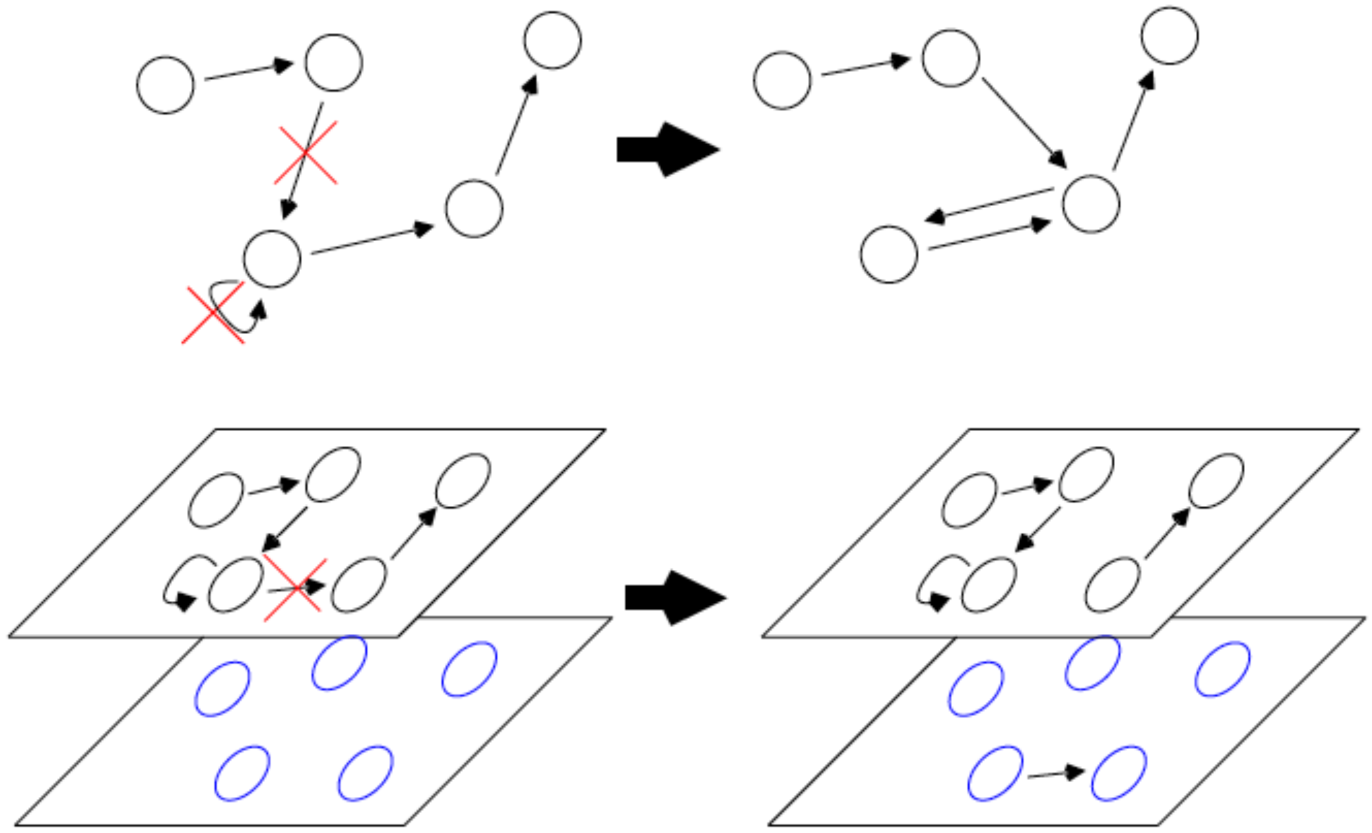


Figure 3: The two mutations are: deleting some portion of the path (top-left, crossed out in red) and replacing that part with another set of edges (top-right), and deleting some edges in the path (bottom-left) and replacing deleted edges with their children in our hierarchy (bottom-right)

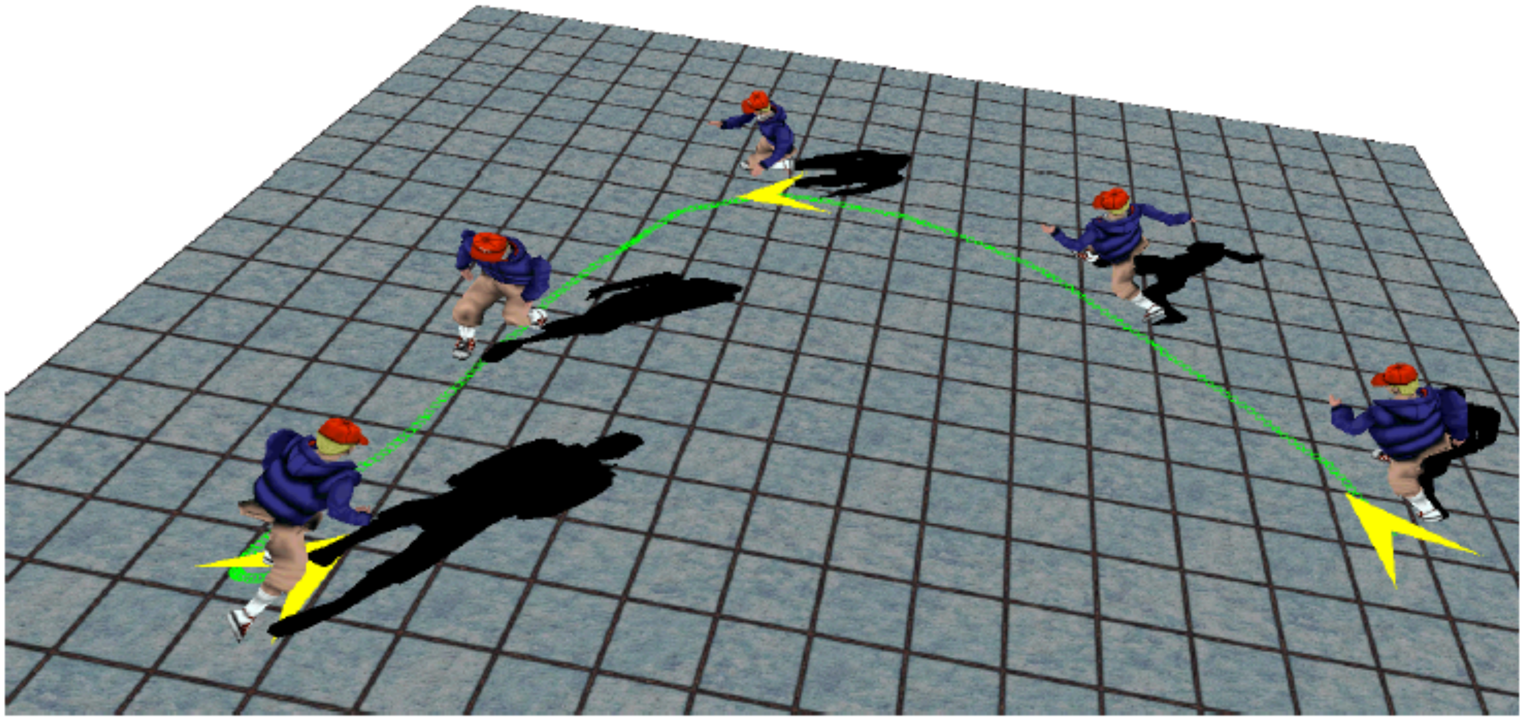
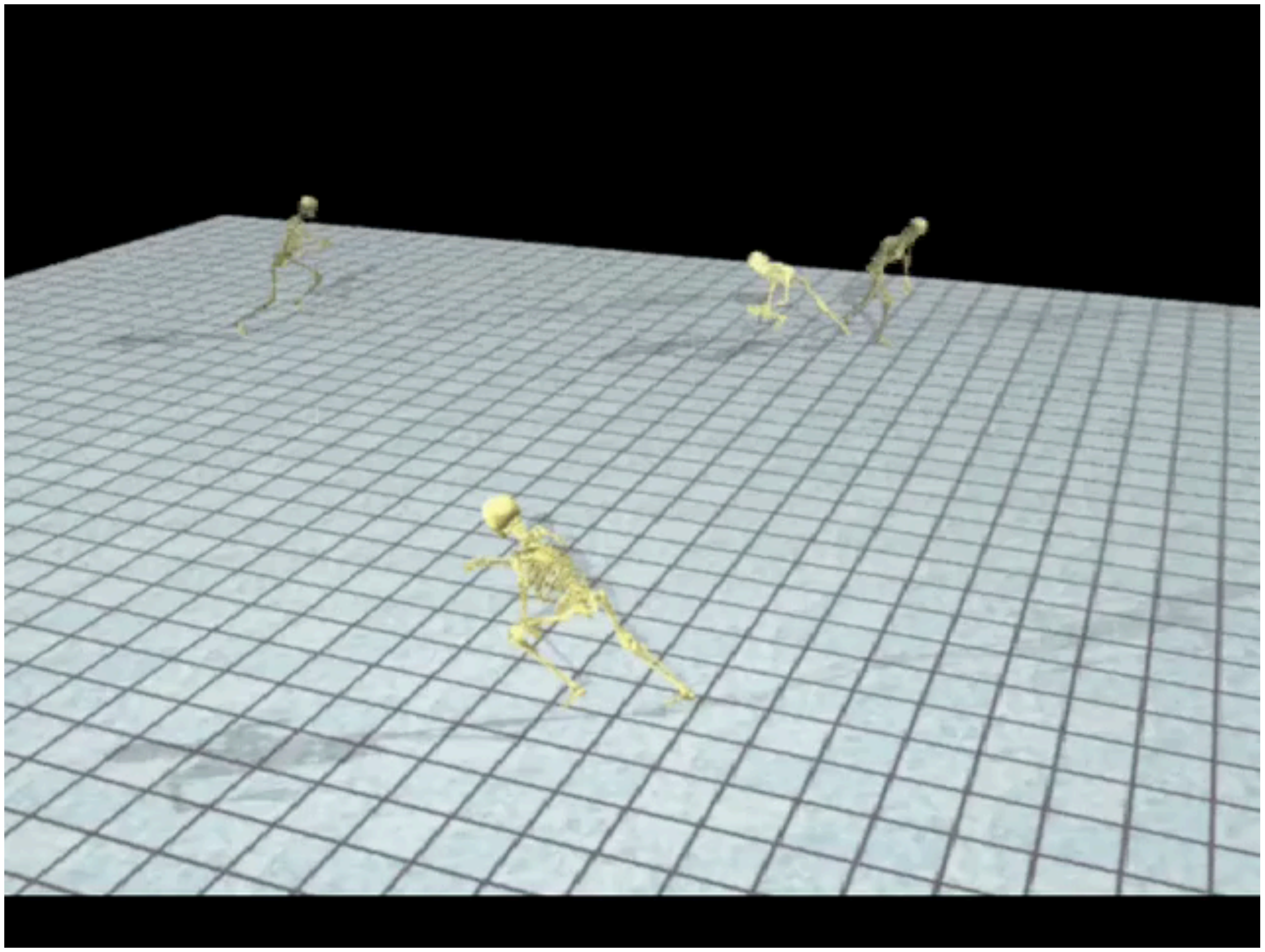
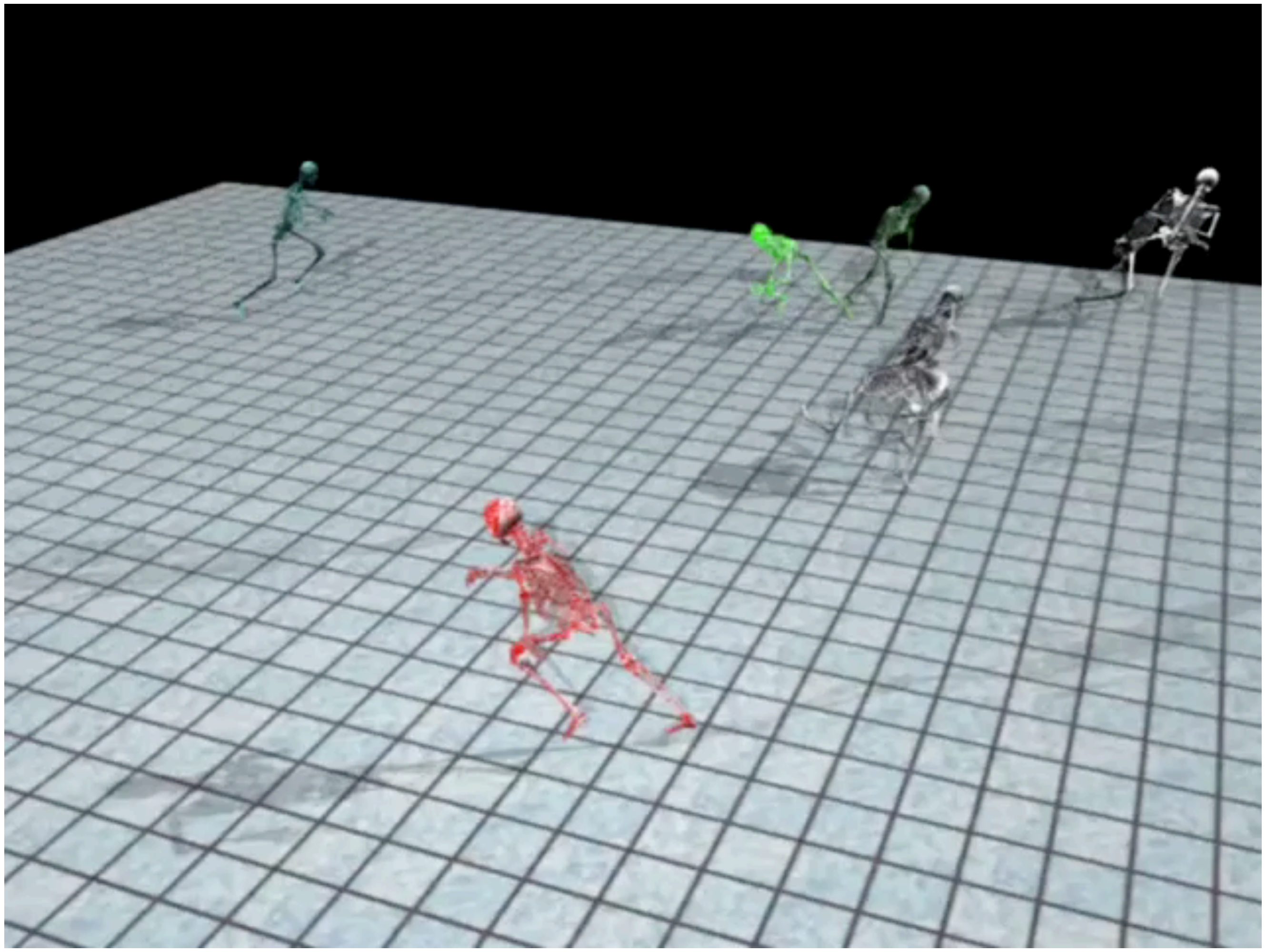
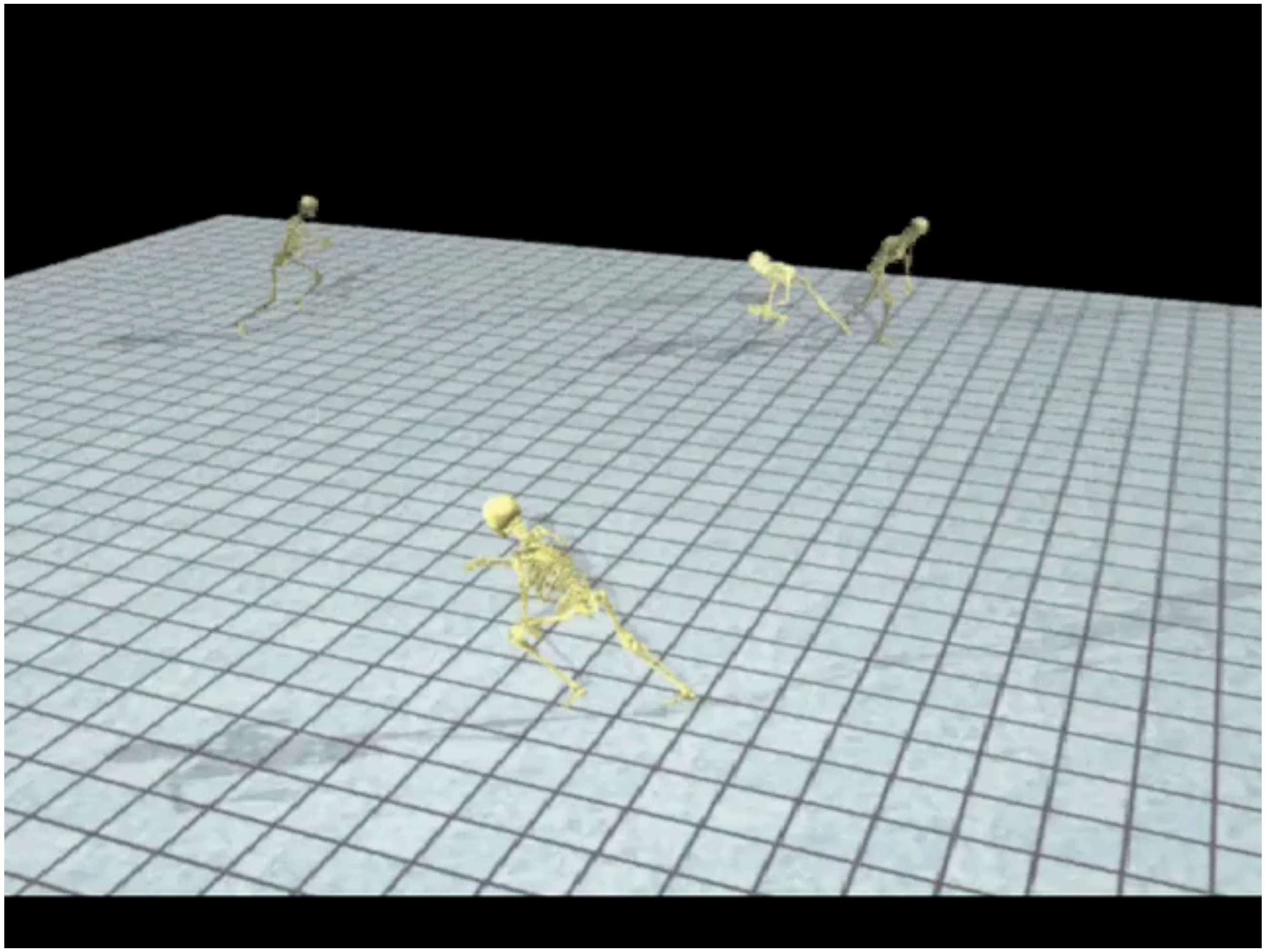
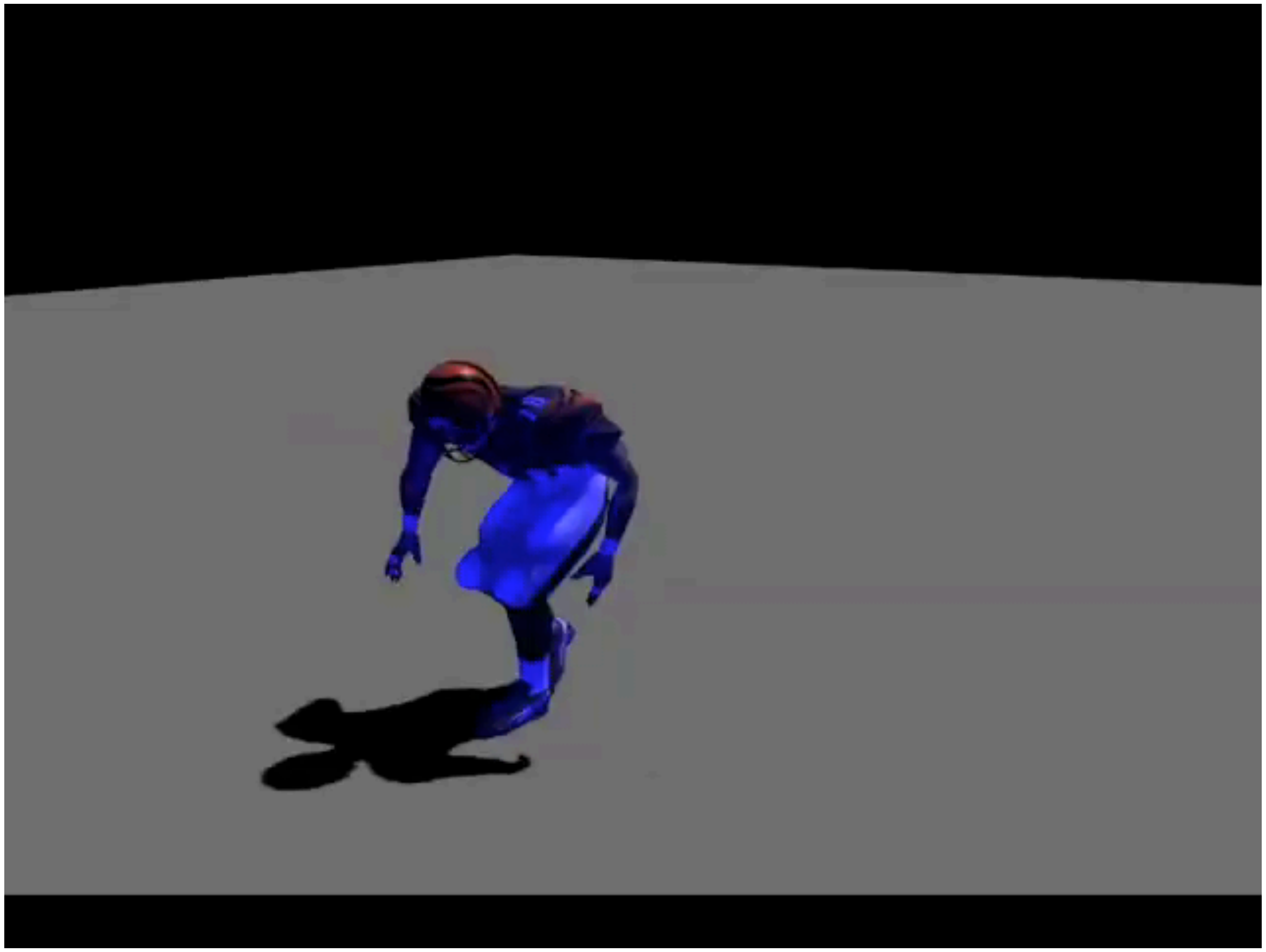


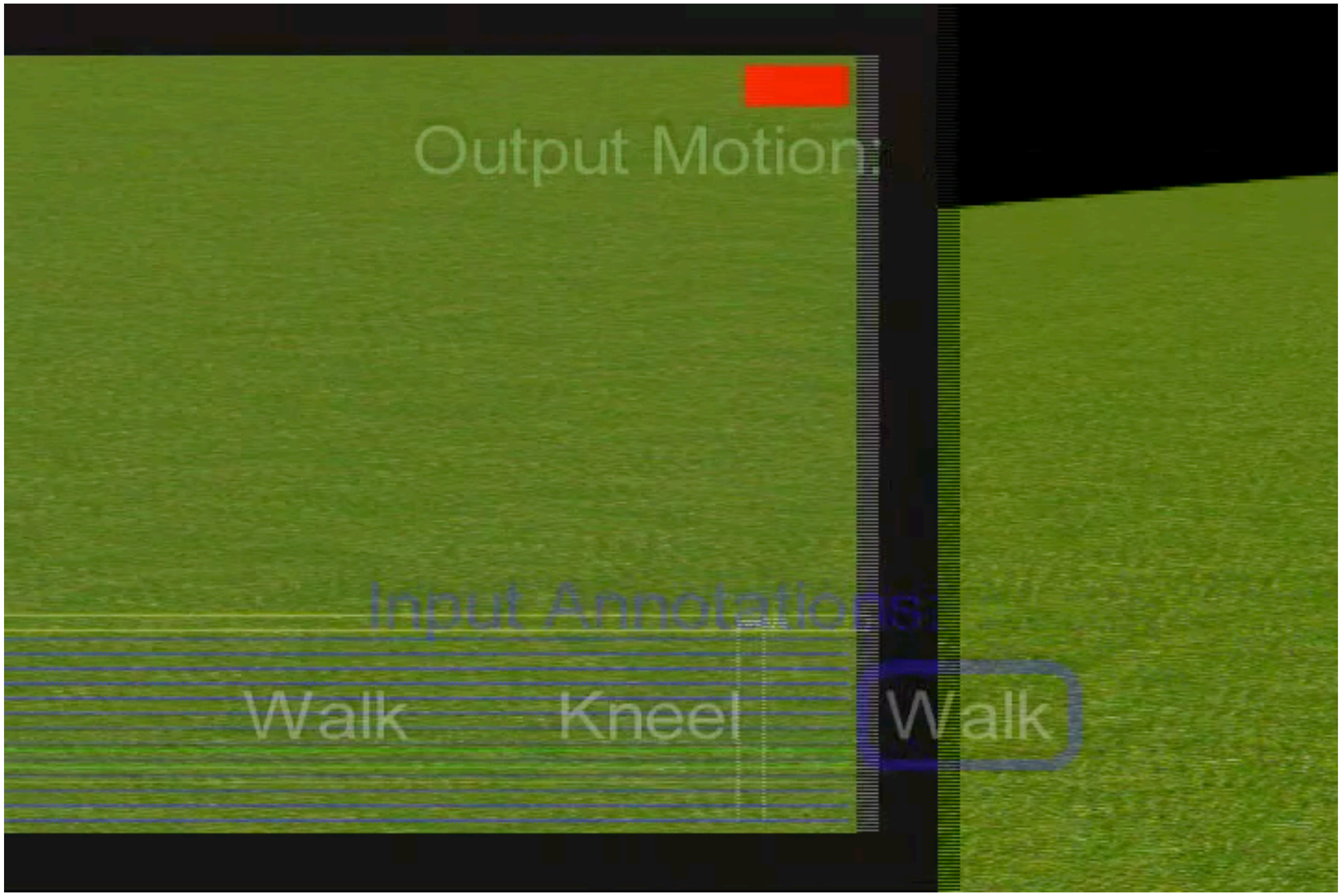
Figure 6: We can use multiple "checkpoints" in a motion. In this figure, the motion is required to pass through the arrow (body constraint) in the middle on the way from the right arrow to the left.

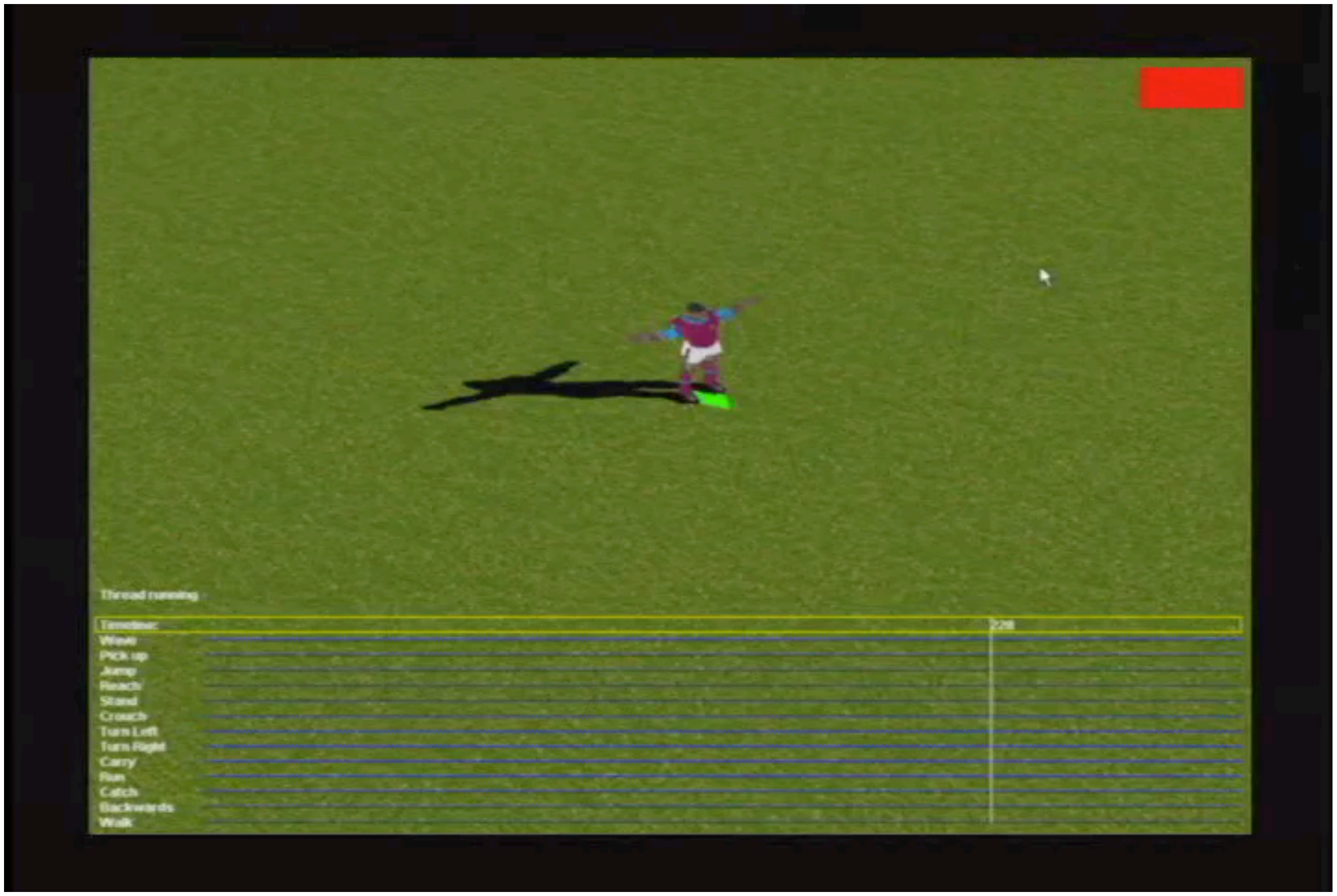









Output Motion:
Input Annotations:
Walk
Kneel
Walk


Thread running
Timeline:
228
Wave
Pick up
Jump
Reach
Stand
Crouch
Turn Left
Turn Right
Carry
Run
Catch
Backwards
Walk