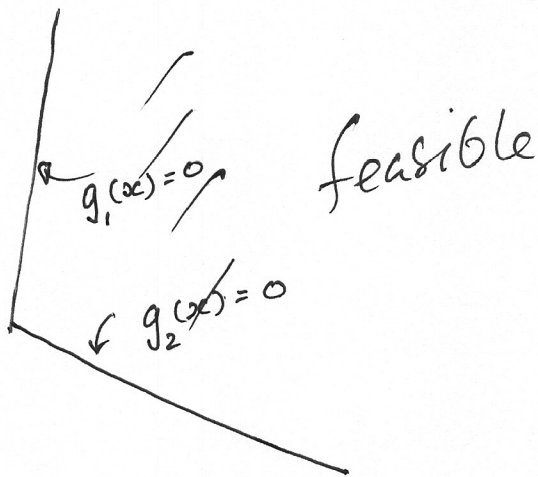


Lagrangians and inequalities

(1)



consider $\max f(x)$
st $g(x) \geq 0$

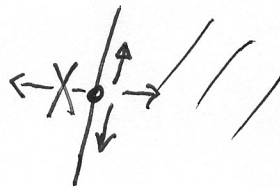
Cases:

interior —

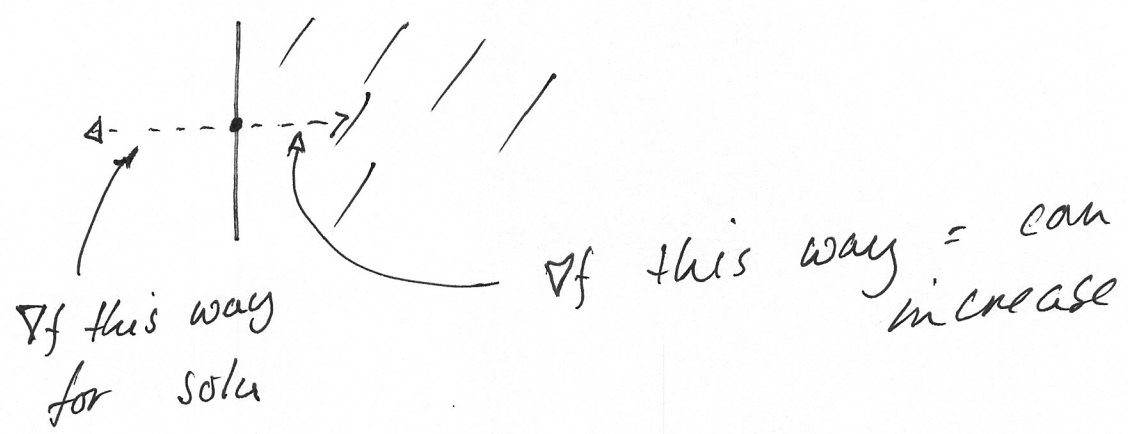
constraints don't operate

boundary —

one can move along, into interior, but not across



max f st g ≥ 0



i.e. at solu

$$\nabla f = \lambda \nabla g$$

$$\lambda \geq 0$$

and $g \geq 0$

and if $g = 0, \lambda > 0$

$g > 0, \lambda = 0$

and this applies for multiple constraints

Lagrangians for mixed probs

(3)

$$\max f \quad \text{s.t.} \quad g \geq 0 \\ h = 0$$

$$L: \quad f(x) + \lambda^T g + \mu^T h$$

Conditions:

$$\begin{array}{l} \nabla_x L = 0 \\ g \geq 0 \\ h = 0 \\ \lambda_i \cdot g_i = 0 \\ \lambda \geq 0 \end{array}$$

Karush - Kuhn - Tucker
Conditions or KKT.

complementarity
condition - source
of much mischief
VICIOUSLY non-linear
 $\equiv \left\{ \begin{array}{l} g=0, \lambda > 0 \\ \text{OR} \\ g > 0, \lambda = 0 \end{array} \right\}$

(5)

~~Q1~~

$$L(x, \lambda, \mu) = C^T x - \lambda^T x - \mu^T (Ax - b)$$

$$Q(\lambda, \nu) = \sup_x L(x, \lambda, \nu)$$

this is infinite unless the coeff of x in $L = 0$

$$\therefore \boxed{C - A^T \mu - \lambda = 0}$$

↑ domain where dual exists

In this domain, $L(x, \lambda, \nu) = \mu^T b$
and $\lambda \geq 0$ (from original).

So dual is
min $\mu^T b$

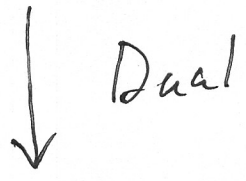
$$\text{s.t. } C - A^T \mu - \lambda = 0$$

$$\lambda \geq 0$$

Notice different forms have different forms of dual

eg.

$$\begin{array}{ll} \max & c^T x \\ \text{st} & Ax \leq b \\ & x \geq 0 \end{array}$$



$$\begin{array}{ll} \min & b^T \mu \\ \text{st} & A^T \mu \geq c \\ & \mu \geq 0 \end{array}$$

KKT link primal and dual

e.g. L.P. case.

- (x, λ, μ) is
- (a) Primal feasible
 - (b) Dual feasible
 - (c) Complementarity

|||

(x, λ, μ) satisfy KKT

|||

(x, λ, μ) are soln.

This is just book keeping, but it's powerful.

This opens a new world of
algorithmic possibilities

9

Simplex - work in primal.

New - A - work with primal feasible x ,
dual infeasible λ, μ
complementary.

B : - work w/ primal ~~in~~feasible x
dual feasible λ, μ
complementary.

C : - work w/ primal feasible x
dual feasible λ, μ
NOT complementary.

In each, adjust working point to
meet missing properties better.