Inequality constraints: Working set methods.

Notice from KKT that some inequality constraints are $\begin{cases} \text{active} \\ \text{bound} \\ \text{blocking} \end{cases}$ at soln, others aren't. Active constraints behave like equalities — others don't count.

IDEA:

- Search for active constraints while searching for point.
- Strategy:
  - start w feasible point
  - improve, while keeping feasible.
  - Maintain model of working set.
- We will only consider linear constraints

- assume we are at x_k.
- our model is

$$\min \quad x^T \frac{M}{2} x + d^T x.$$

$$\text{s.t.} \quad \underline{a}_i^T \underline{x} + b_i = 0$$

all active constraints

- ~~the~~ we want $\underline{p}$ st $\underline{x}_{k+1} = \underline{x}_k + \underline{p}$
- so

$$\min_p \quad \underline{x}_{k+1}^T \frac{M}{2} \underline{x}_{k+1} + \underline{d}^T \underline{x}_{k+1}$$

$$\text{st} \quad \underline{a}_i^T \underline{x}_{k+1} + b_i = 0$$

substitute, get

$$\min_p \quad p^T \frac{G}{2} p + e^T p$$

$$\underline{\text{st}} \quad a_i^T p = 0$$

active constraints

Q: How do we solve this?

Notice that $x_{k+1} + \alpha p$ is feasible for all α and all working constraints (3)

Cases:

(A) x_{k+1} is feasible for all constraints
(easy check – they're linear)
$\longrightarrow$ accept it.

(B) it isn't. – then there is some $\alpha \in [0, 1)$ st
$\underline{x}_k + \alpha \underline{p}$ is feasible
(cause our working set is right –
x_k is feasible in W.S)

Case B :

- consider a constraint NOT in W.S.

if $\underline{a}_i^T p \geq 0$
then $\underline{a}_i^T(\underline{x}_k + \alpha p) \geq \underline{a}_i^T \underline{x}_k \geq b$

— ANY +ve α will work for this constraint.

- if $\underline{a}_i^T \underline{p} < 0$ then

$$\alpha_k \leq \frac{b_i - \underline{a}_i^T \underline{x}_k}{\underline{a}_i^T p_k}$$

Walk constraints to find smallest α_k.

- constraints that have smallest α_k are active/blocking/etc.

(α_k could be zero if we're missing a constraint in working set.)

Insert a blocking constraint into W_k to get W_k

- ~~Maint~~ Removing redundant constraints

Q: How do we know there is a constraint in working set that shouldn't be there?

A: Lagrange mult is -ve.

Q: Where did LM come from?

A: recipe for quadratic form s.t. linear constraints

Overall story

- start x_0 feasible
- Until finished
    - Until x_k is minimizer of QP over working set
        - Take a step, and adjust W.S.
        - Minimizer when $p = 0$
    - Any -ve Lagrange mults?
        - Yes: remove one from WS
        - No: finished.

Check:

- at finished, KKT are true.

Concern:

- How much work to adjust W.S.?
- Could this cycle?
- What if constraints aren't linear?