Some equivalences

Discrete world :

$$\min_{x} \quad \frac{x^T A x}{2} + b^T x \qquad \leftarrow \text{D1}$$

$$x \in \{1, \dots r\}^N$$

III

$$\min_{\hat{x}_i} \quad \sum_{ij \in \mathcal{E}} \hat{x}_i^T M_{ij} \hat{x}_j + \sum_{i \in V} \hat{b}_i^T \hat{x}_i \qquad \leftarrow \text{D2}$$

$$\text{s.t} : \quad 1^T \hat{x}_i = 1$$

$$\hat{x}_i \in \{0,1\}^*$$

(here $\hat{x}_i$ are 1-hot vectors)

Graphical model world

$$\log P(X_1 \cdots X_n) = \sum_{ij \in \mathcal{E}} \theta_{ij}(X_i, X_j) + \sum_{i \in V} \theta_i(X_i) + \text{Const}$$

where $X_i \in \{1 \cdots r_i\}$

$-\log P$ is sometimes called Energy or Cost

MAP:

$$\min_{X_1 \cdots} -\log P \quad \text{(P1)} \equiv \text{(D1)} \equiv \text{(D2)}$$

algs so far:

- Turn (D2) into L.P.
  - tractable if $r = 1$ AND properties of A (via TUM).

- if $r > 1$:
  - α-expansion
  - α-β swap

  → if conditions apply to $A (\equiv M_{:,j})$

- (P1) - if graph is tree, Max product belief prop is exact.
  (So this applies to (D1), (D2) appropriate

- - if NOT loopy max product, and hope.