

1) Five fair dice are rolled. Find the probability that:

- a) all five numbers are different
- b) at least two dice show the same number
- c) at least one die shows a 6

2) Each firing of a missile at a target has a .21 probability of striking the target. The outcome of each firing is independent of the outcome of any others.

- a) What is the probability of destroying the target with three shots?
- b) I keep firing until I destroy the target. What is the expected number of shots I have to fire?

3) Roll two dice. Let A ="the sum is even" and B ="the sum is divisible by 3" (i.e. $B=\{3, 6, 9, 12\}$). Are A and B independent? explain

4) Roll two dice. Let A ="The first die is odd" and B ="the second die is odd" and C ="the sum is odd". Show these events are pairwise independent, but not independent

5) You want to invent a game where the player bets \$1, and rolls two dice. If the sum is 7, the player wins \$ k , and otherwise loses their bet.

- a) What value of \$ k makes the game fair?
- b) If $k=1$, what is the expected value of the game to the player?

6) I have two normal random variables. X has zero mean, and unit variance. Y has zero mean and unit variance.

- a) what is $\text{mean}(2X)$?
- b) what is the variance of $(2X-4Y)$?
- c) what is the standard deviation of $(X-Y)$

7) I flip a coin 3 times. I do not know p , the probability it comes up heads. I estimate p using maximum likelihood. I observe $h=0$ heads and $t=3$ tails.

- a) What value does maximum likelihood report for p ?
- b) Why is it not safe to assume that $p=0$?
- c) Describe (briefly) another estimation procedure that might help

8) In 1969, Dr Spock came on trial. He would have liked women on his jury. The jury was drawn from a panel of 350 people chosen by a clerk. There were 102 women in the panel. At the next stage, the judge chose 100 potential jurors from the 350 on the panel. This pool included 9 women.

- (a) 350 people are chosen at random from a very large population, which is 50% female. What is the probability that the sample contains 102 women?
- (b) 100 people are chosen at random, without replacement, from a group of 102 women and 248 men. What is the probability that this group contains 9 women?

9) The correlation between height and weight for men aged 18-74 in the US is about 0.4. For each of the statements below, say whether this follows from the data, and explain your answer.

(a) Taller men tend to be heavier

(b) The correlation between weight and height for men aged 18-74 in the US is about 0.4

(c) Heavier men tend to be taller

(d) If you put on 10 pounds, you are likely to grow taller

10) Urns I, II, and III each contain four pieces of paper, with the numbers 1... 4 on them. I shake each urn, and draw one piece of paper from each.

(a) What is the probability that all four numbers are the same?

(b) What is the probability that all four numbers are different?