$\cancel{E[X] > \sqrt{E[X^2]}}$ NO

$$
\begin{aligned}
\text{VARIANCE} &= E\left[\left(X - E[X]\right)^2\right] \\
&= E\left[X^2 - 2E[X]X + E[X]^2\right] \\
&= E[X^2] - 2E[X]E[X] + E[X]^2 \\
&= \underline{E[X^2] - E[X]^2}
\end{aligned}
$$

$$E[X] \leq \sqrt{\underline{E[X^2]}}$$

not variance

$$\operatorname{cov}(X,Y) = E[(X-E[X])(Y-E[Y])]$$

$$= E[XY] - 2E[X]E[Y] + E[X]E[Y]$$

$$= E[XY] - E[X]E[Y]$$

1] <u>If X, Y are indep</u>,

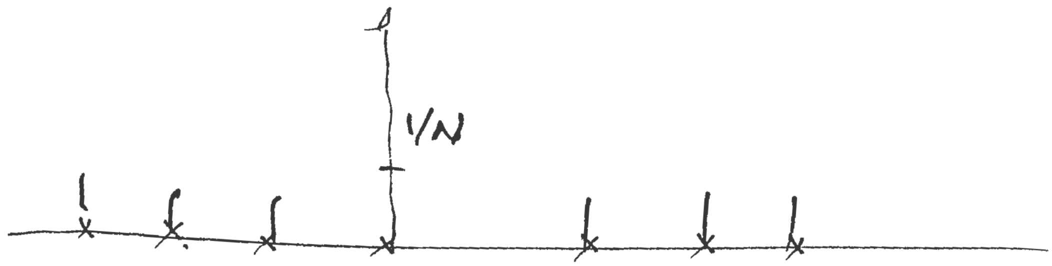
$$E[X,Y] = E[X]E[Y]$$

$$E[XY] = \sum_{x,y \in D_x \times D_y} xy\, P(X=x, Y=y)$$

$$= \sum_{x \in D} \sum_{y \in D} xy\, P(X=x)P(Y=y)$$

$$= \sum_x \sum_y (xP(X=x))(yP(Y=y))$$

$$= \underbrace{\left[\sum_x xP(X=x)\right]}_{E[X]} \underbrace{\left[\sum_y yP(Y=y)\right]}_{E[Y]}$$



N

Empirical dist

$E[X]$

$E[(X - E[X])^2]$

$E[(X - E[X])(Y - E[Y])]$

$$\frac{1}{N} \sum_i (x_i - \text{mean}(x))(y_i - \text{mean}(y))$$

$$= r \cdot \text{std}(x) \cdot \text{std}(y)$$

Markov

$$P(\{|X| \geq a\}) \leq \frac{E[|X|]}{a}$$

$a > 0$

Chebyshev

$$P(\{|X - E[X]| \geq a\}) \leq \frac{\mathrm{Var}[X]}{a^2}$$

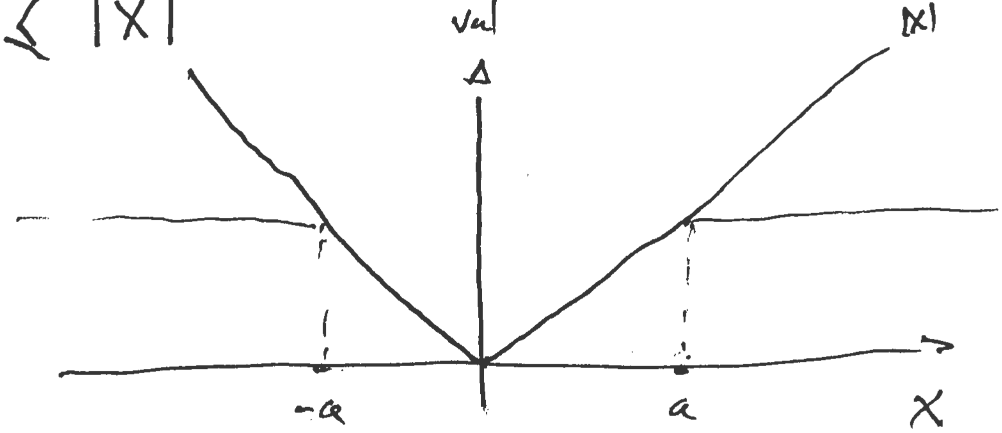
$$\mathbb{1}_{[\varepsilon]}(X) = \begin{cases} 1 & \text{if } X \in \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{1}_{[\{|X| \leq a\}]}(X) = \begin{cases} 1 & \text{if } -a < X < a \\ 0 & \text{otherwise} \end{cases}$$

$$E[\mathbb{1}_{[\Sigma]}(X)] = P(\Sigma)$$

$$P(\{|X| \geq a\}) \leq \frac{E[|X|]}{a}$$

$$a\,\mathbb{1}_{[\{|X| \geq a\}]}(X) \leq |X|$$



$$E\left[a\,\mathbb{1}_{[\{|X| \geq a\}]}(X)\right] \leq E[|X|]$$

$$a\,P(\{|X| \geq a\}) \leq E[|X|]$$