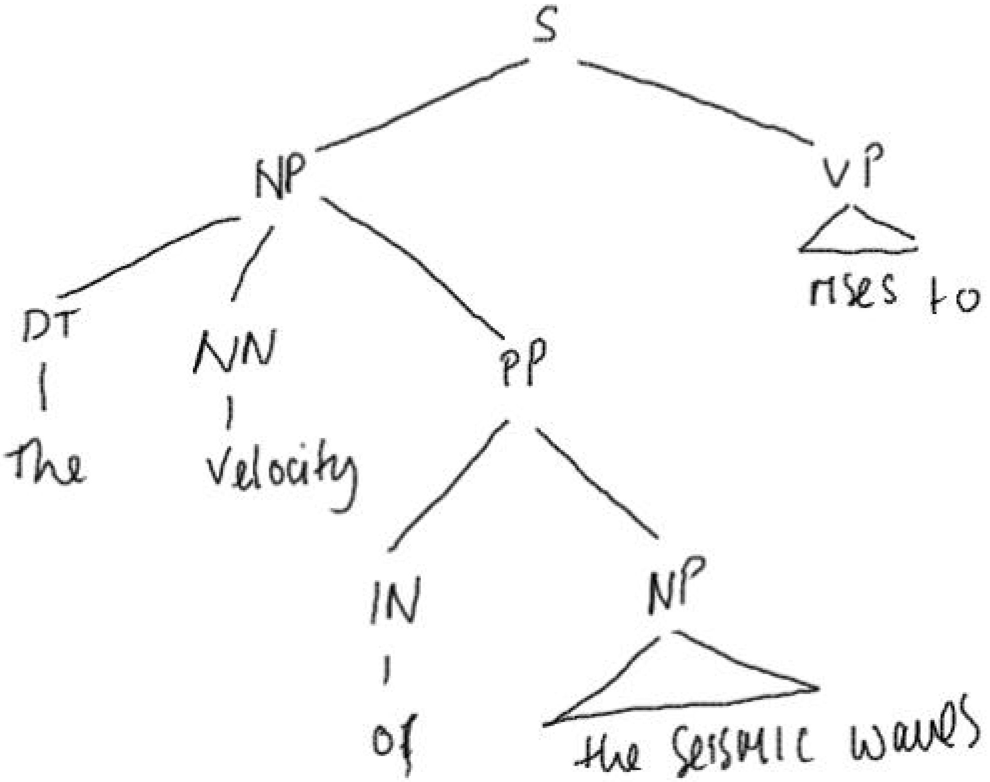
Parsing.

- We think of strings of language as having structure at long scales:

" The velocity of the seismic waves rises to.

why is this singular?

- hard to capture with HMM



How can we extract this structure?

- complex q'n with many partial answers
- One answer: Probabilistic Context Free Gramm

PCFG consists of

- a set of terminals $\{w^k\}, k = 1 \cdots N$
- a set of nonterminals $\{N^i\}$ $i = 1 \cdots n$
- a start symbol N^1
- a set of rules $N^i \rightarrow \zeta^j$

  ζ^j is a sequence of terminals and non-terminals

- a set of probabilities on rules

$$P(N^i \rightarrow \zeta^j \mid N^i) = P(N^i \rightarrow \zeta^j)$$

note $$\sum_j P(N^i \rightarrow \zeta^j) = 1$$

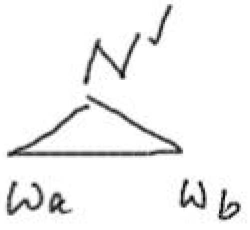
Notation:

sentence is $w_1 \cdots\cdots w_m$

span w_{ab} is $w_a \cdots w_b$

if by a set of rewrites we can go from N^j to w_{ab}

then N^j dominates w_{ab} and w_{ab} is yield of N^j



N^j_{ab} means N^j dominates $a \cdots b$

$$P(w_{1m}) = \sum_t P(w_{1m}, t)$$

all trees that yield w_{1m}

Conditions:

need

Place-invariance:

$$\forall_k \quad P(N^j_{k(k+c)} \rightarrow \zeta) \text{ is the same}$$

Context free:

$$P(N^j_{kl} \rightarrow \zeta \mid \text{anything outside } k-l) = P(N^j_{kl} \rightarrow \zeta)$$

Ancestor free:

$$P(N^j_{kl} \rightarrow \zeta \mid \text{any } \underset{N^j_{kl}}{\text{ancestors}} \text{ outsid}$$

$$= P(N^j_{kl} \rightarrow \zeta)$$

Under these conditions, computing $P(w,t)$ is straightforward

Properties

- Predictive power tends to be greater for language than HMM's w/ same # of parameters
- PCFG's favour smaller trees over larger trees.
- Probability can be wasted on infinite trees

Example:

$$S \to \text{rhubarb} \qquad P = 1/3$$

$$S \to SS \qquad P = 2/3$$

w	$P(w) = \sum_t P(w,t)$
rhubarb	$1/3$
rhubarb rhubarb	$2/3 \times 1/3 \times 1/3 = 2/27$
rhubarb rhubarb rhubarb	$(2/3)^2 \left(\frac{1}{3}\right)^3 \times 2 = 8/243$

$$\text{So} \quad \sum_{w} P(w) = \sum_{w,t} P(w,t)$$

$$= 1/3 + 2/27 + 8/243 + \ldots$$

$$= 1/2 \qquad !$$

(Half of the probability has gotten stuck in infinite trees) issues: eval: $P(w)$?

inference: $\underset{t}{\operatorname{argmax}}\ P(w,t)$

learning: rule probs.

- consider only Chomsky Normal Form grammars

rules are:

$$\left.\begin{array}{l} N^i \rightarrow N^j N^k \\ N^i \rightarrow w^j \end{array}\right\} \text{only forms available}$$

parameters are:

$P(N^i \rightarrow N^j N^k)$ n^3 table for n nontermina

$P(N^i \rightarrow w^j)$ nV table for n nontermina
V terminal(

and

$$\sum_{j,k} P(N^i \rightarrow N^j K^k) + \sum_j P(N^i \rightarrow w^j) = 1$$

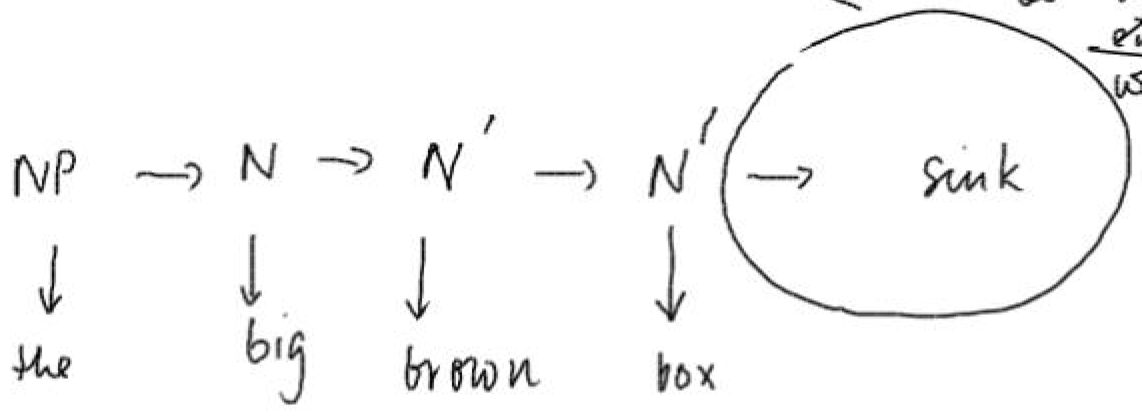
Example:

Probalistic regular grammar

(which is rather like an HMM)

$N^i \to w^j N^k$

$N^i \to w^j$



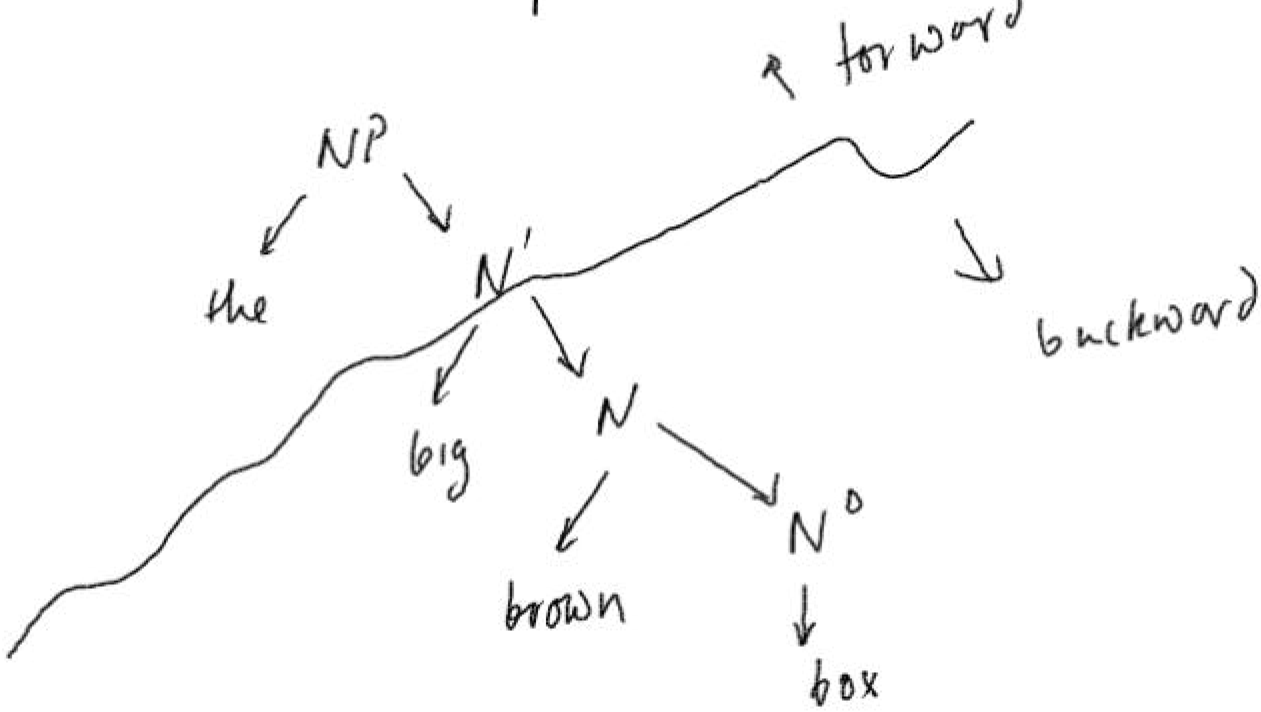
if we add this, we have ~~someth~~ ~~that~~ can we ~~impl~~ ~~ed~~ ~~with a~~ HM

Now in an HMM, we worked with

$$P(w_{1t-1}, X_t = i)$$

$$P(w_{tT} \mid X_t = i)$$

but PRG parse



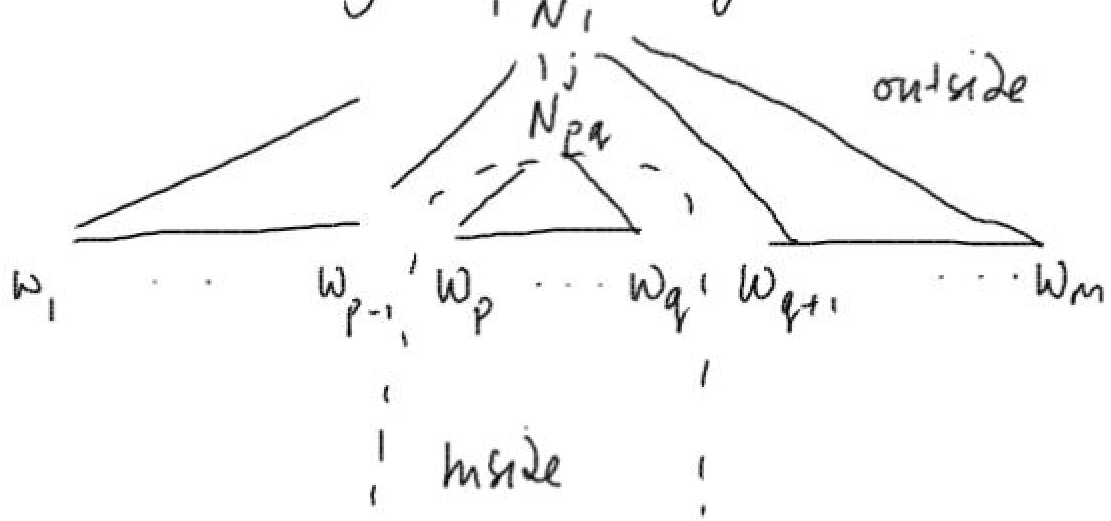
All this suggests an approach to PCFG's

Outside prob $\alpha_j(p,q)$ ← forward

$= P(w_{1(p-1)}, N^{j}_{pq}, w_{(q+1)m})$ ← backward

Inside prob $\beta_j(p,q) = P(w_{pq} | N^{j}_{pq})$

this is cutting up string



Notice

$$\alpha_j(p,q) \cdot \beta_j(p,q)$$

$$= P(w_{1 \cdots m}, \text{ tree with } N^j \text{ yielding } w_{pq})$$

$$\sum_j \alpha_j(p,q) \beta_j(p,q)$$

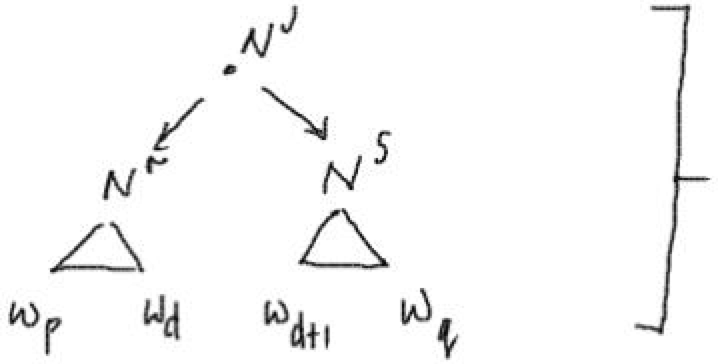
$$= P(w_{1 \cdots m}, \text{ tree with some } N \text{ yielding } w_{pq})$$

How to calculate probabilities:

Inside probs:

$$\beta_j(K,K) = P(N^j \to \omega_K)$$

now consider $\beta_j(p,q)$



This is a ~~pictur~~ ~~of~~ all possible cases.

so

$$\beta_j(p,q) = \sum_{r,s} \left[\sum_{d=p+1}^{q-1} \Big\{ P(N^j \to N^r N^s) \cdot \beta_r(p,d) \cdot \beta_s(d+1,$$

Outside probs:

$$\alpha_1(1, m) = 1$$

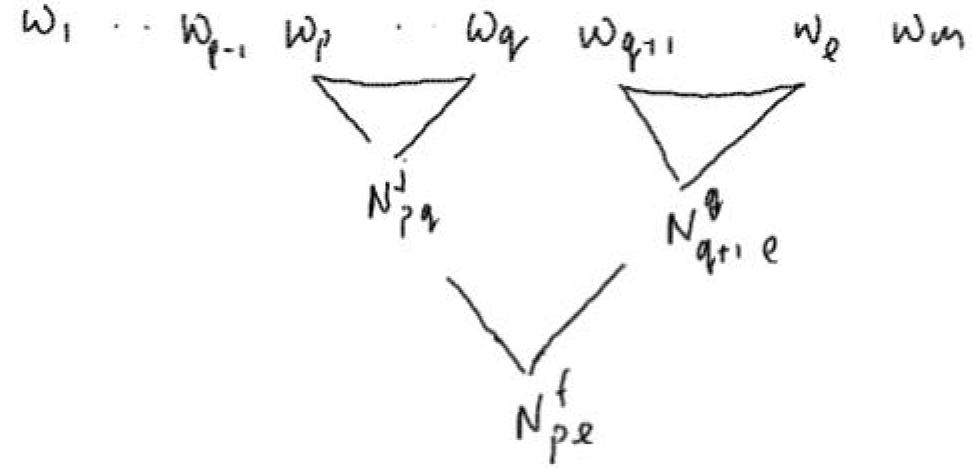
$= P($tree yielding $w_1 \cdots w_m$, roote
at N^1

$$\alpha_j(1, m) = 0$$

Interior strings

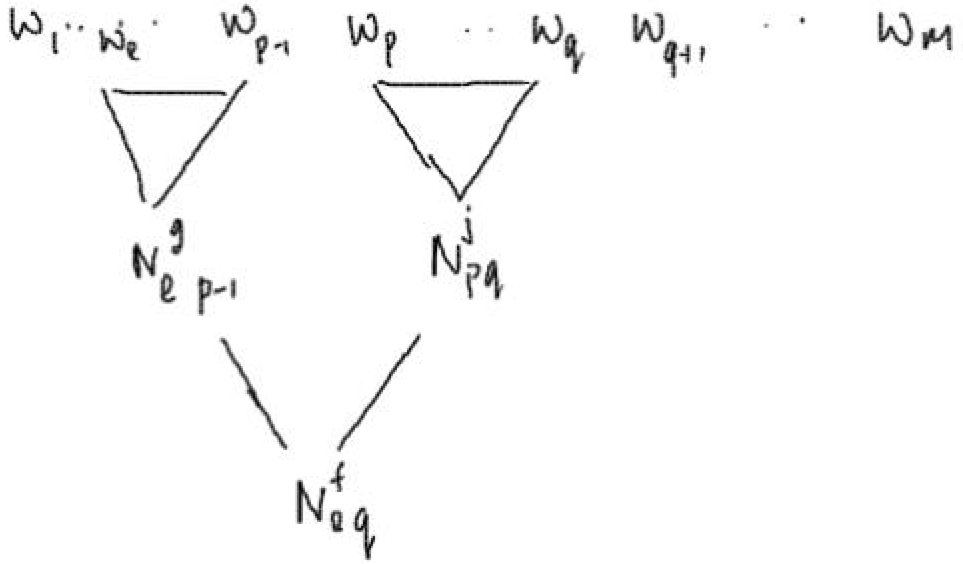
$$w_1 \cdots \cdots w_{p-1}\; w_p \qquad w_q\; w_{q+1} \cdots \cdots w_m$$

we could have



So

or we could have



be careful of $N^j \to N^j N^j$

$$\alpha_j(p,q) = \Bigg[\sum_{f,g\neq j}\ \sum_{e=q+1}^{m} \Big(\alpha_f(p,e)\, P(N^f \to N^j N^g) \times \beta_g(q+1,e)\Bigg]$$

$$+ \Bigg[\sum_{fg}\ \sum_{e=1}^{p-1} \alpha_f(e,q)\, P(N^f \to N^j N^g) \times \beta_g(e,p-1)\Bigg]$$

So we have an algorithm for computing $P(\omega)$

- compute β's bottom up
- (then compute α's top down)

$$P(\omega) = \alpha_1(1 \cdots m) . \beta_1(1 \cdots m)$$

$$= \sum_j \alpha_j(k,k) . \beta_j(k,k)$$

<u>Inference</u>:

- recall HMM
- key is to keep track of the highest probability path from state j at time t, forward
- call this an accumulator

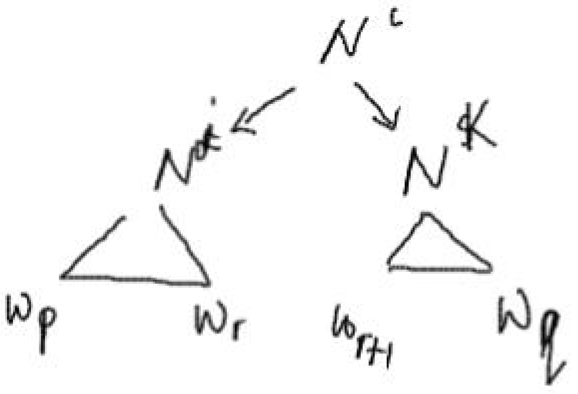
In this case, we want best parse tree spanning a substring rooted with a non-terminal

write

$\delta_i(p,q)$ = prob. of highest inside pro
parse tree spanning
p, q, using N^i at top
(i.e. N^i_{pq})

1) $\delta_i(p,p) = P(N^i \to w_p)$

2) we have



so we must have

$$\delta_i(p,q) = \max_{\substack{j,k,\\ p\le r<q}} \left\{ \begin{array}{l} P(N^i \to N^j N^k) \times \\ \delta_j(p,r) \\ \delta_k(r+1,q) \end{array} \right.$$

how good the best tree is

also, keep

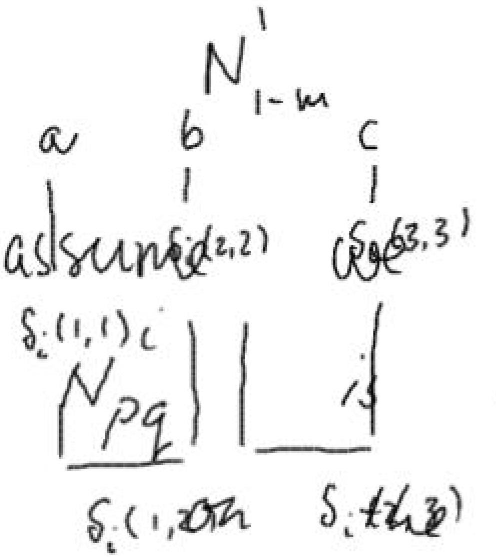
$$\Psi_i(p,q) = \arg\max$$

which is ~~the tree~~ (j, k, r)

3) $\delta_1(1, m)$ is prob of most prob parse

Example

We know root is



now assume we know N^i_{pq} in , $\psi_i(p,q) = (j,k,$

then on the left, we must have

$N^j_{pr}\ \delta_i(1,3)$ which is either $\delta_i(1,1)\,\delta_j(2,3)$ or

and on the right $\delta_i(1,2)\,\delta_j(3,3)$

$$N^k_{(r+1)\,q}$$

Training:

- assume we know terminals, non-terminals, start, all rules
- must now determine rule probs from data
- Do this with EM

Single sentence

$$\hat{P}(N^j \to \zeta) = \frac{\#(N^j \to \zeta)}{\sum_{\gamma} \#(N^j \to \gamma)}$$

Now

$$\alpha_j(p,q)\,\beta_j(p,q) = P(N^1 \Rightarrow w_{1m},$$

$$N^j \Rightarrow w_{pq} \mid \text{Gramma}$$

$$= P(N^1 \Rightarrow w_{1m} \mid G) \times$$

$$P(N^j \Rightarrow w_{pq} \mid N^1 \Rightarrow w_{1m}, G$$

$$P(N^1 \Rightarrow w_{1m} \mid G) = \alpha_1(1,m)\,\beta_1(1,m) = \pi$$

So:

number ~~fraction~~ of times N^j is used

$$= \sum_{q=1}^{m} \cancel{\alpha} \sum_{q=p}^{m} P(N^j \Rightarrow w_{pq} \mid N^1 \Rightarrow w_{1m}, G)$$

$$= \sum_{q=1}^{m} \sum_{p=1}^{m} \frac{\alpha_j(p,q)\,\beta_j(p,q)}{\pi}$$

Two kinds of rule

nonterms → nonterms I
term II

I: need:

$$P(N^j \to N^r N^s \Rightarrow w_{pq} \mid N^1 \Rightarrow w_{1m}, G)$$

$$= \frac{\sum_{d=p}^{q-1} \alpha_j(p,q)\, P(N^j \to N^r N^s)\, \beta_r(p,d)\, \beta_s(d+1}{\pi}$$

then

number of times $N^j \to N^r N^s$ given N^j

$$= \sum_{p=1}^{m-1} \sum_{q=p+1}^{m} \Big[P(N^j \to N^r N^s \Rightarrow w_{pq} \mid N^1 \Rightarrow w_1$$

which yields a re-estimation formula

II $\text{count}(N^j \rightarrow w^k \mid N^1 \Rightarrow w_{1m}, G)$

$$= \cancel{\frac{\sum_{h=1}^{m} \alpha_j(h,h)\, P(N^j \rightarrow w_h, w_h = w}{\pi}}$$

$$= \frac{\sum_{h=1}^{m} \alpha_j(h,h)\, \delta(w_h = w^k) \, . \, \beta_j(h,h)}{\pi}$$

which gives the re-est formula.

More than one sentence is only slightly more complex — one must count over all sents.

Important facts:

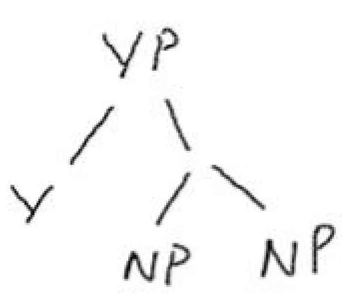
- slow:
  - each iteration for 1 sentence costs $O(m^3 n^3)$

    sent length ↑ ↑ number of non-term
- local maxima are a major problem; eg 300 trials give 300 different local maxima (Charniak)

- Some evidence that method ~~prom~~ needs more non-terminals than are strictly needed

Some options:

- Lexicalization:
  - Current parser does not know what verb is involved in, say



We should have rules that know what the word is

- known as lexicalization
- but how much should be known?
- some tension here between lexicalization (one or more rules per word?) and estimation (many instances of a ~~word~~ rule)

- try to break words into classes?

Partially unsupervised learning:

- Hard for us to build an activity treebank (don't know rules)
- But we could <u>segment</u>
-

- Some evidence grammar learning works better on a segmented corpus (Pereira + Schabes; Schabes et al)

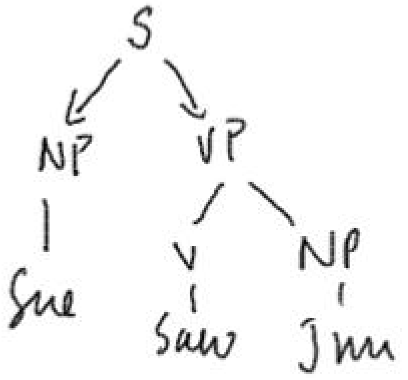
Learning Structure:

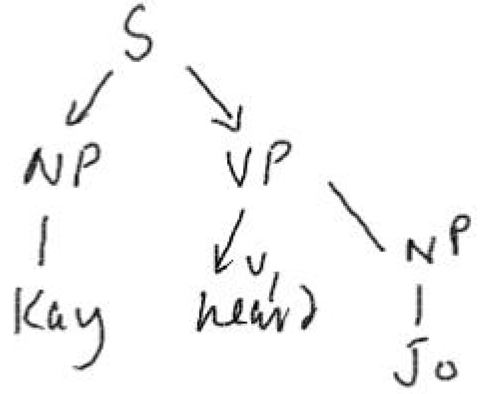
- Some evidence MDL methods apply

Data oriented parsing:

- Parse by cut + paste of parse trees

We have





we can (a) make new sents by cut + paste

— This is the essence of parsing
expose what cut + pastes are
syntactically acceptable

+(b) parse sents (with a very nasty search)