

Light and shading



P. Claesz, [Still Life with a Skull and a Writing Quill](#), 1628

Some phenomena



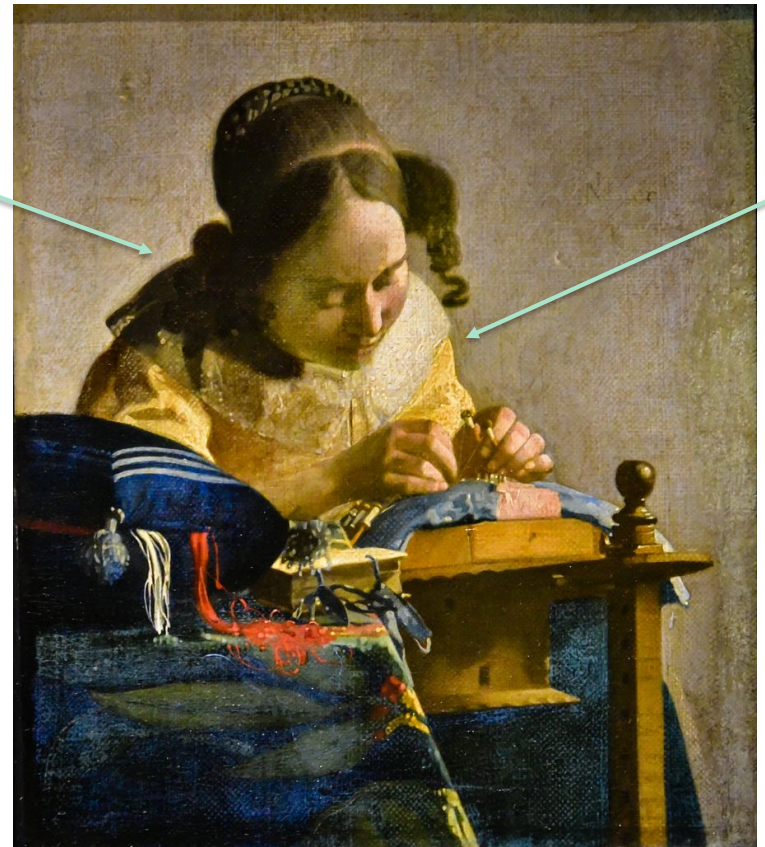
P. Claesz, [Still Life with a Skull and a Writing Quill](#), 1628

Artist physics can't be trusted!



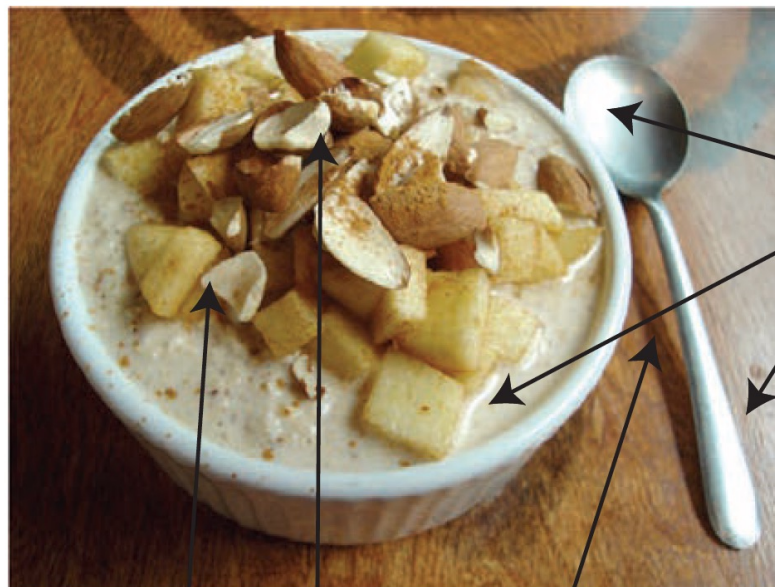
Johannes
Vermeer

B



C

A



Specularities

Cast shadow

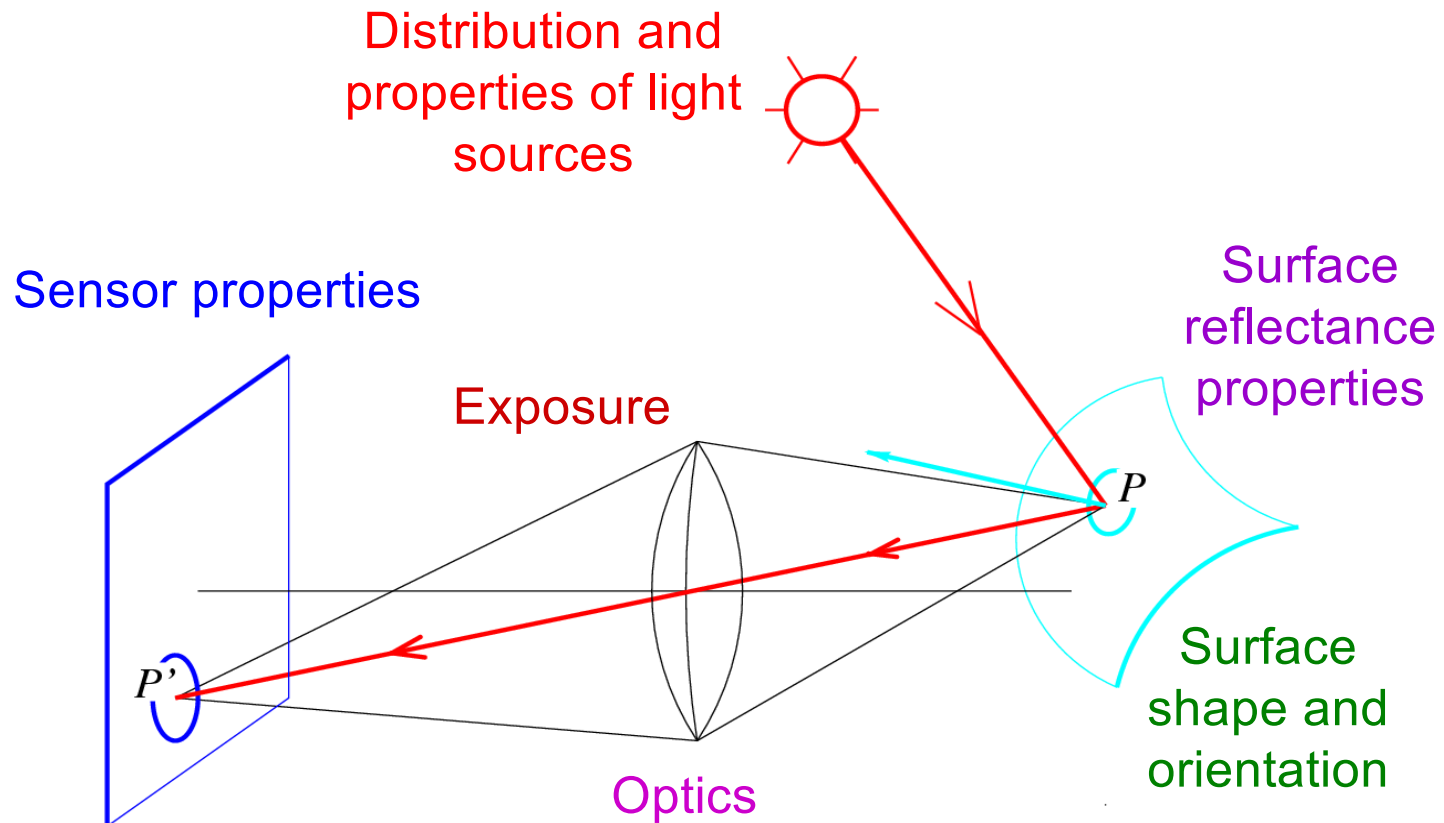
Diffuse reflection, bright

Diffuse reflection, dark

Figure 1.4 This photograph, published on flickr by mlinksva, illustrates a variety of illumination effects. There are specularities on the metal spoon and on the milk. The bright diffuse surface is bright because it faces the light direction. The dark diffuse surface is dark because it is tangential to the illumination direction. The shadows appear at surface points that cannot see the light source.

Image formation

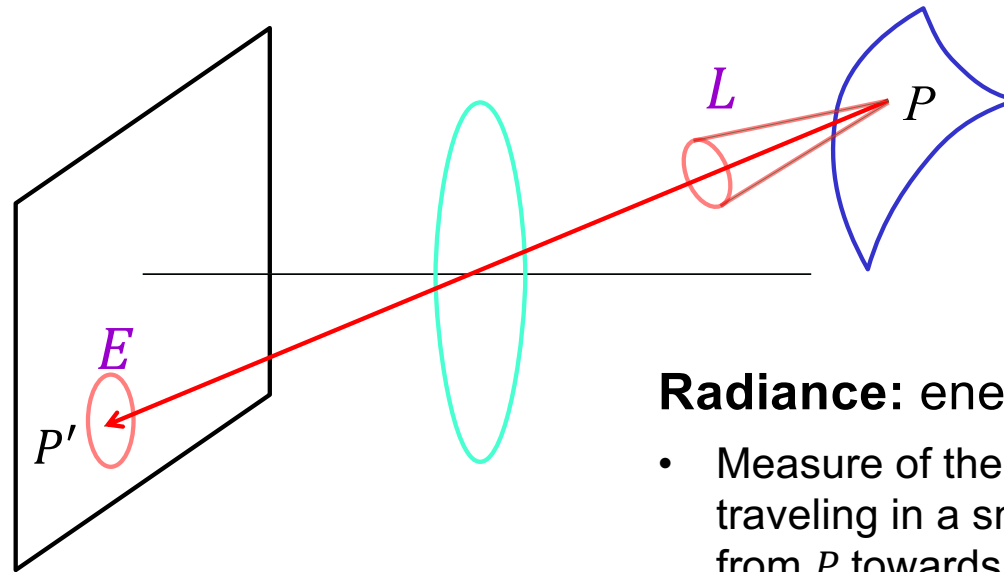
- What determines the *brightness* of an image pixel?



Outline

- Small taste of radiometry
- In-camera transformation of light
- Reflectance properties of surfaces
- Diffuse and specular reflection
- Shape from shading
- Estimating direction of light sources

Radiometry of image formation



Irradiance: energy arriving at a surface

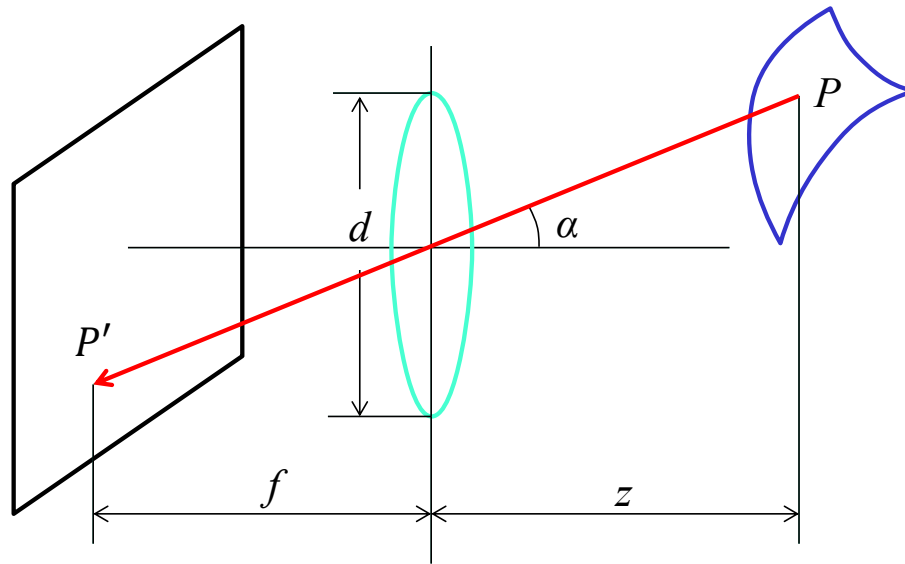
- Incident power per unit area (not foreshortened)
- Units: Watts per square meter

Radiance: energy carried by a ray

- Measure of the density of photons traveling in a small cone of directions from P towards P'
- Power per unit area perpendicular to the direction of travel, per unit solid angle
- Units: Watts per square meter per steradian

What is the relationship between E and L ?

Fundamental radiometric relation

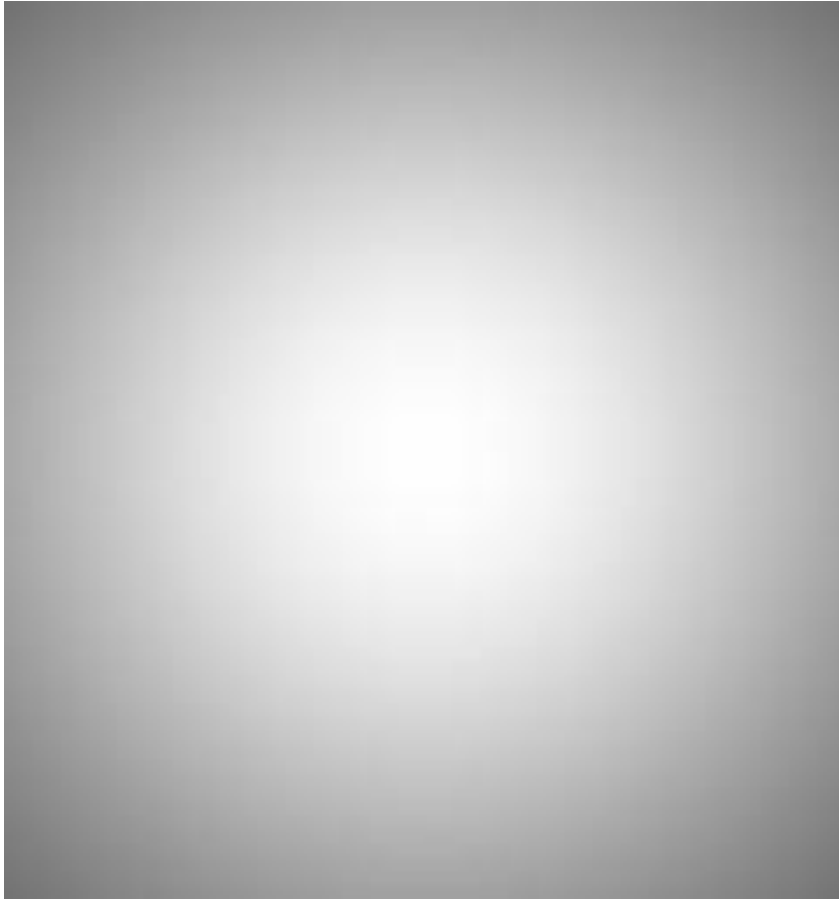


$$E = \left[\frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha \right] L$$

- Image irradiance (E) is linearly related to scene radiance (L)
- Irradiance is *directly* proportional to the area of the lens ($\frac{\pi d^2}{4}$) and *inversely* proportional to the squared distance between the lens and the image plane (f)
- The irradiance decreases as the angle between the viewing ray and the optical axis (α) increases

For derivation, see, e.g., Szeliski 2.2.3

Fundamental radiometric relation

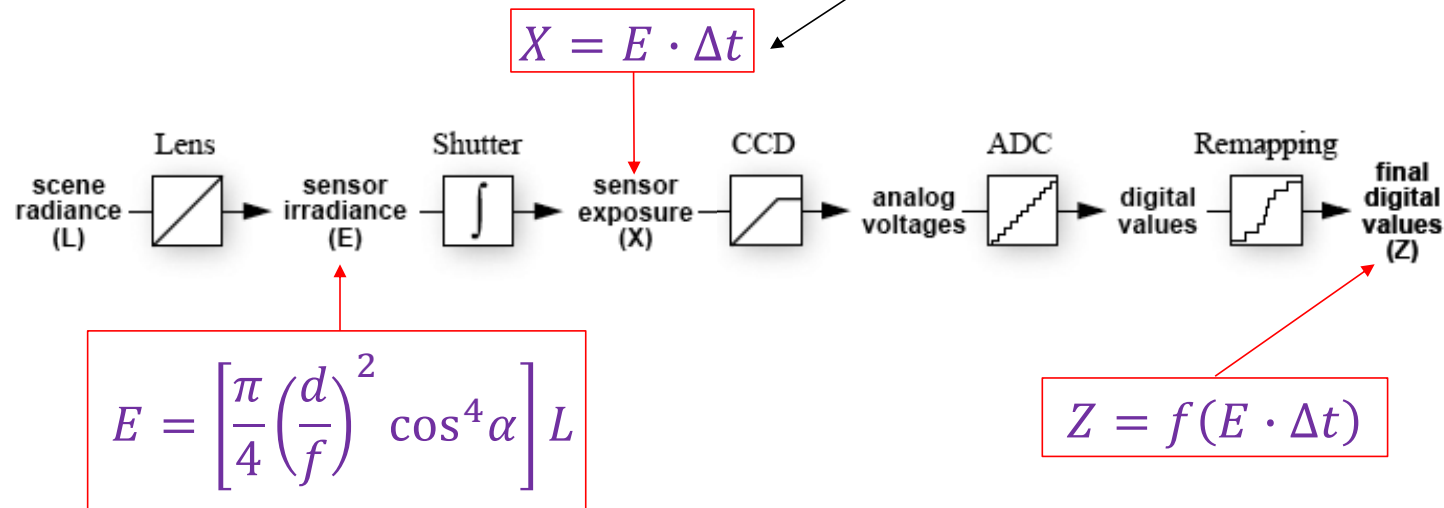


$$E = \left[\frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha \right] L$$

S. B. Kang and R. Weiss. [Can we calibrate a camera using an image of a flat, textureless Lambertian surface?](#)
ECCV 2000

From light rays to pixel values

A more complicated model is
Sometimes appropriate here



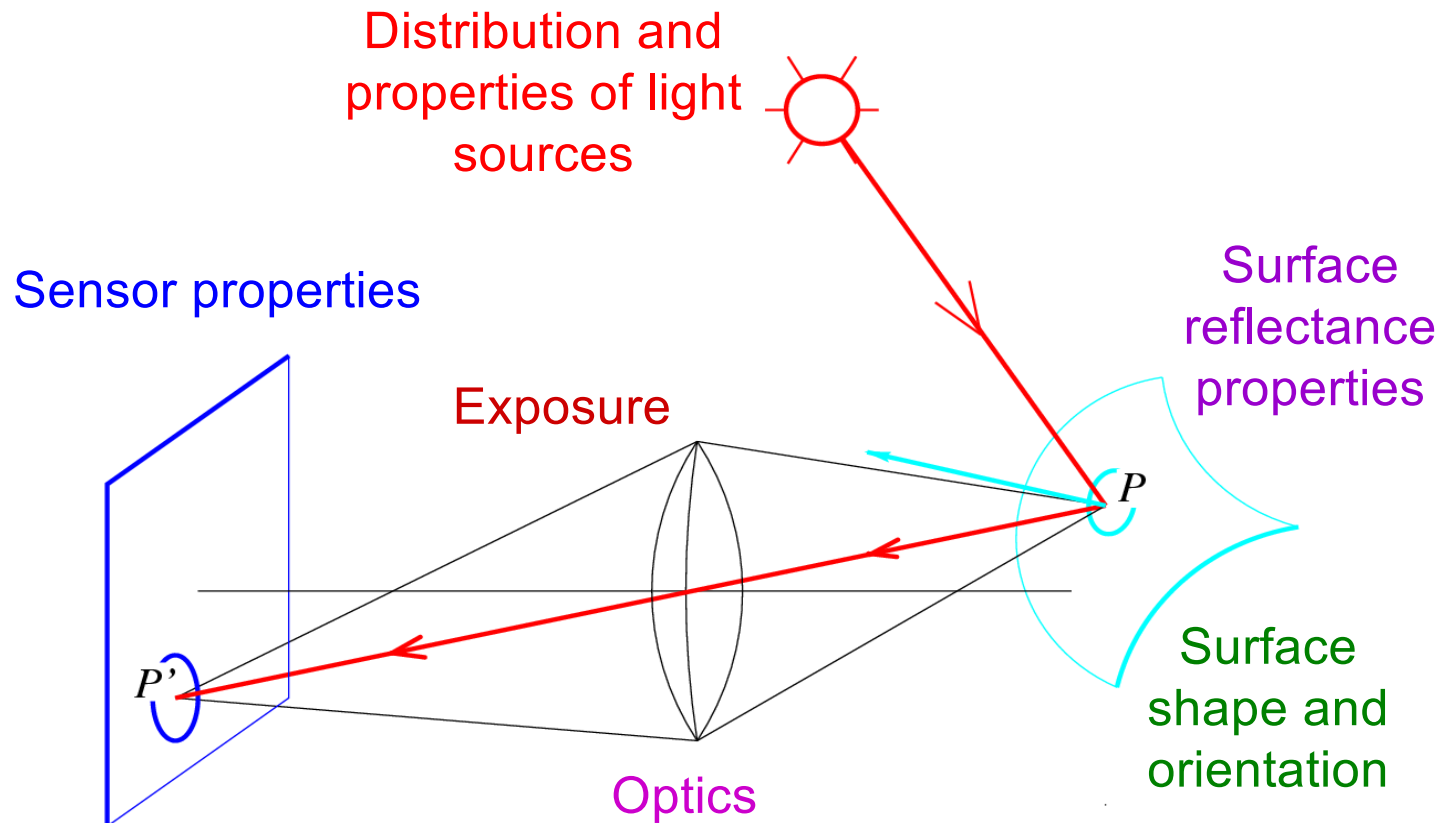
- **Camera response function:** the mapping f from irradiance to pixel values
 - Needed for applications like estimation of scene reflectance properties, creating high dynamic range (HDR) images
 - For further reading: M. Brown, [Understanding the In-Camera Image Processing Pipeline for Computer Vision](#), CVPR 2016 Tutorial

Outline

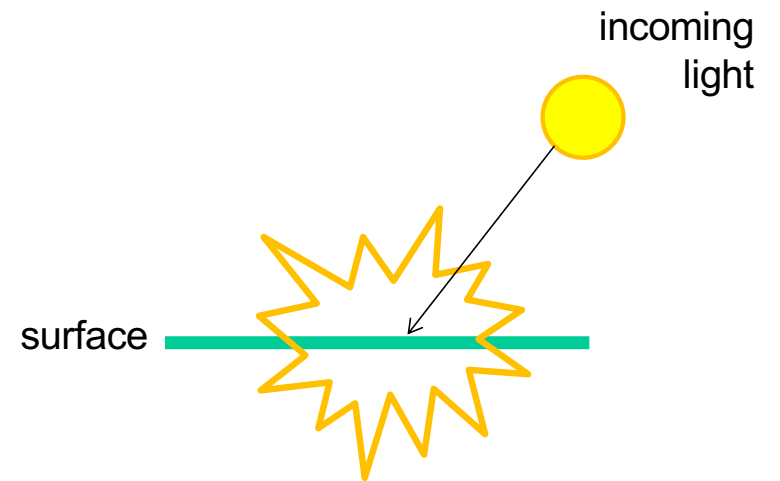
- Small taste of radiometry
- In-camera transformation of light
- Reflectance properties of surfaces

Recall: Image formation

- What determines the brightness of an image pixel?

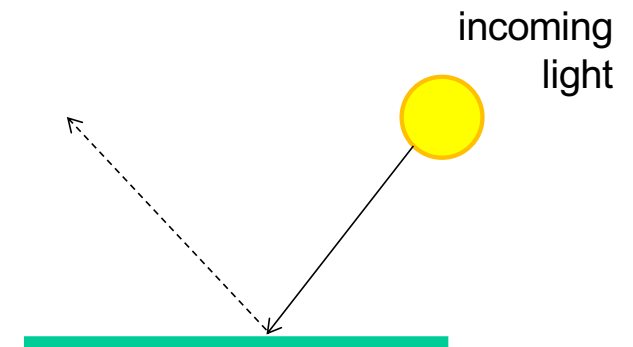


What can happen to light when it hits a surface?

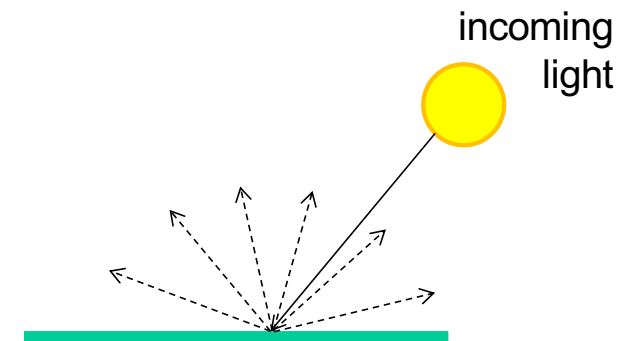


Basic models of reflection

- **Specular reflection:** light is reflected about the surface normal



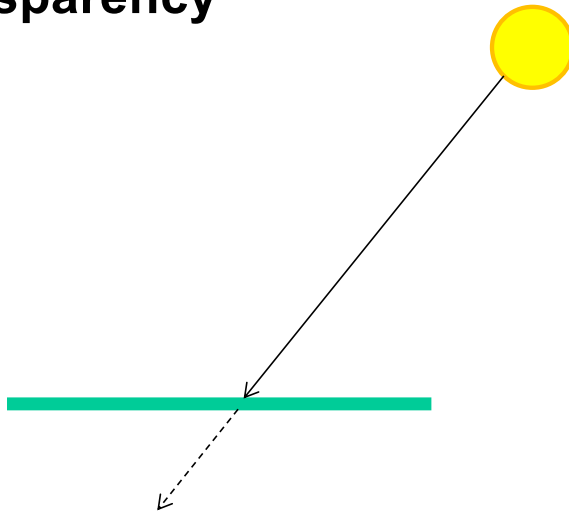
- **Diffuse reflection:** light scatters equally in all directions



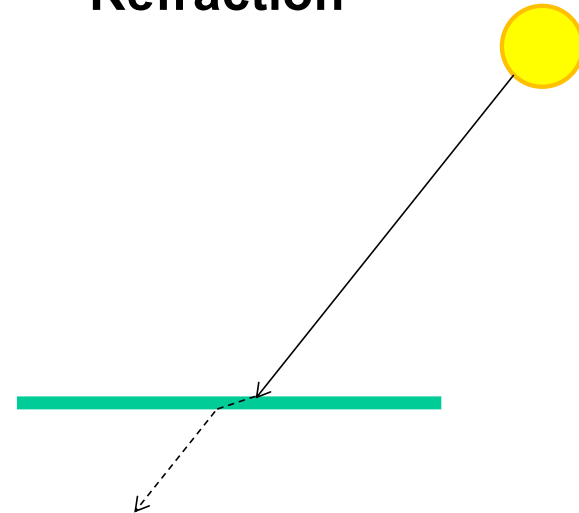
Other possible effects



- **Transparency**

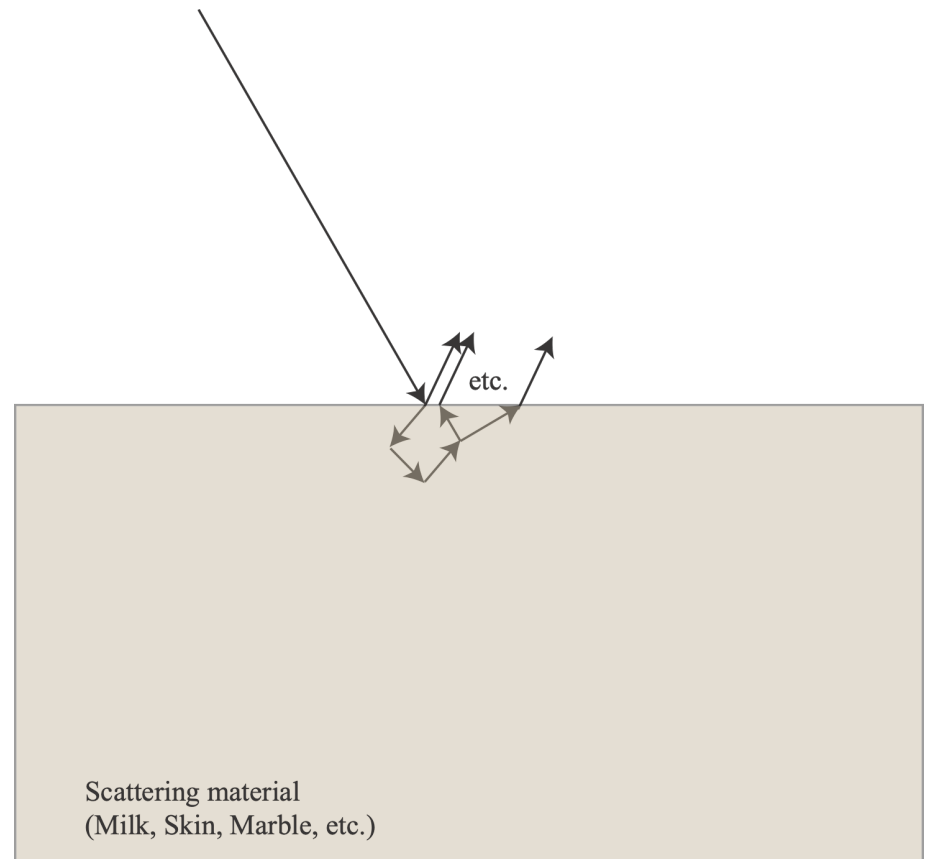


- **Refraction**



Other possible effects

- **Subsurface scattering**



Slide from D. Hoiem

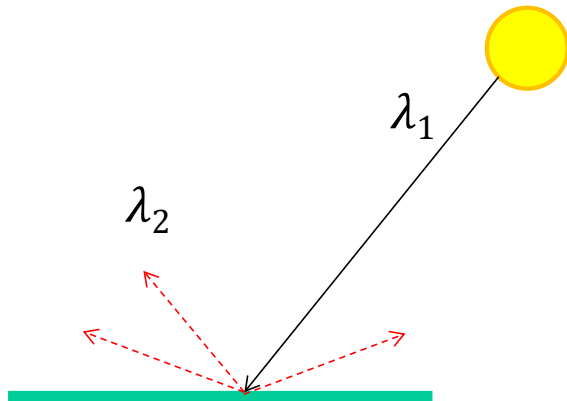
[Image source](#)



subsurface scattering in skin (not rendered!)

Other possible effects

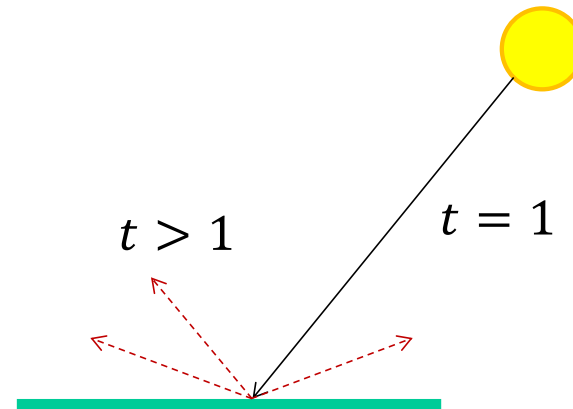
- Fluorescence



Slide from D. Hoiem

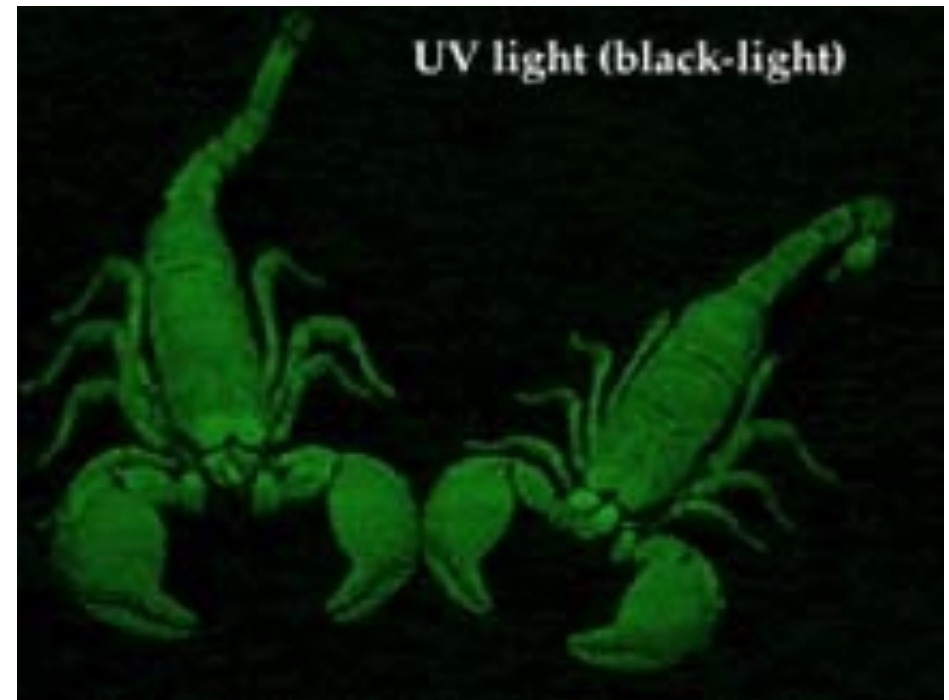
[Image source](#)

- Phosphorescence



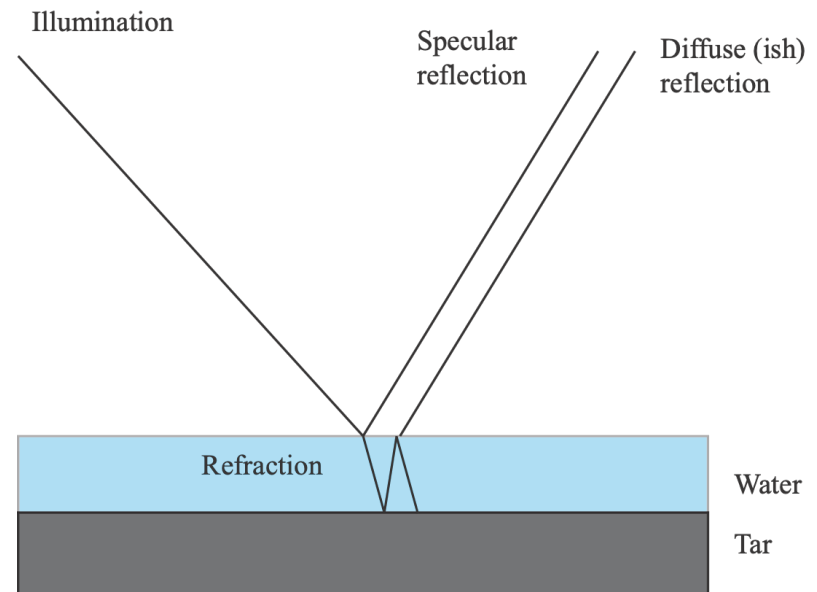
Fluorescence in nature

Many examples, mostly obscure:
scorpions, deep sea fish, teeth, nylon, chitons



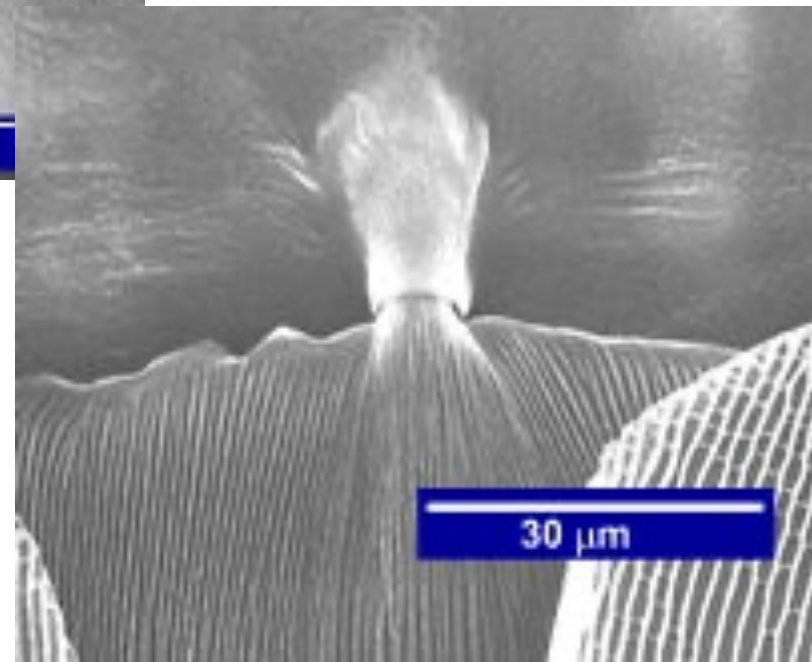
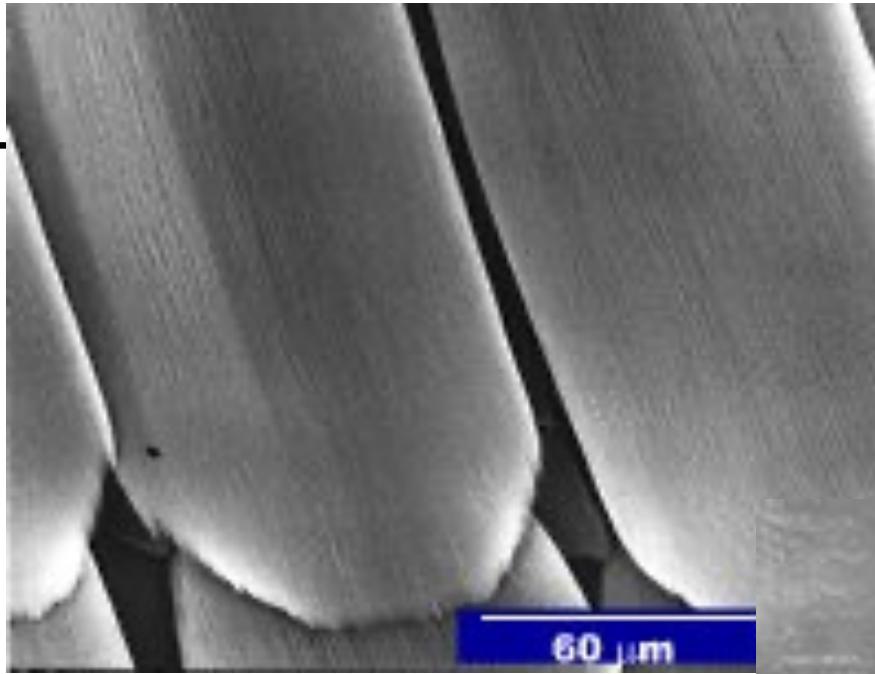
Films on surfaces

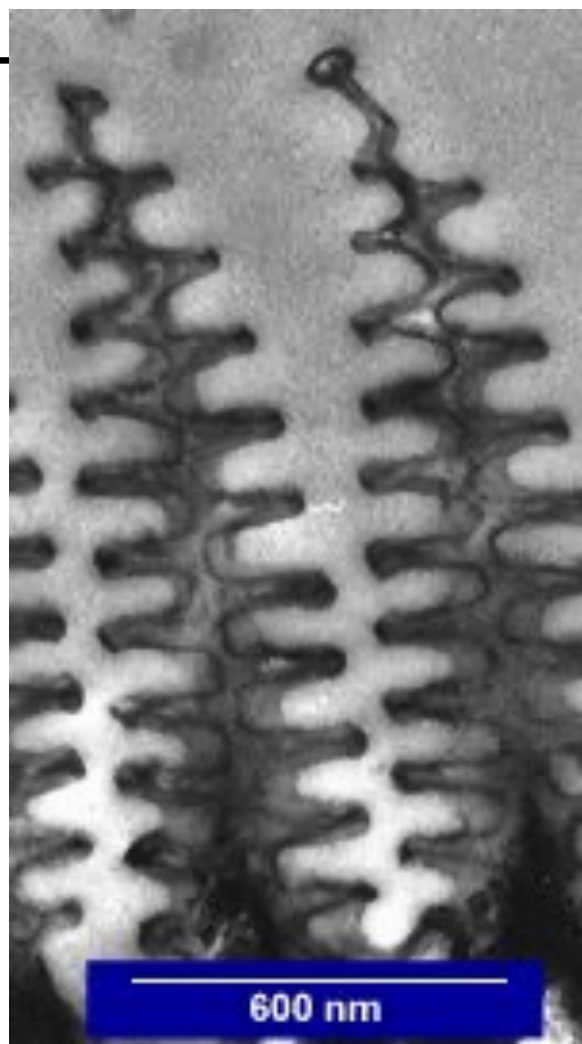
- eg water
- Assume:
 - film is thin
- You see:
 - specular reflection+diffuse term



Interference effects

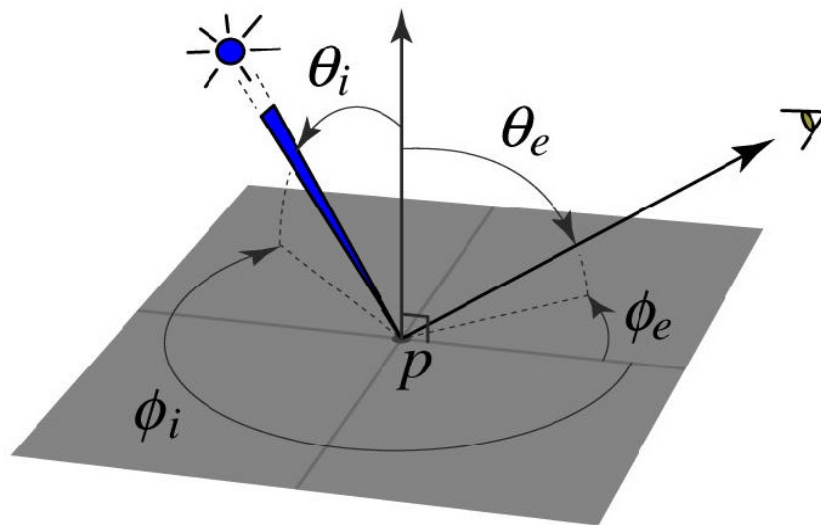
- Sometimes seen on films
 - if the film is the right number of wavelengths thick
 - waves will interfere destructively (resp constructively)
 - can give rise to intense colors
 - oil films on water often do this





Bidirectional reflectance distribution function (BRDF)

- How bright a surface appears when viewed from one direction when light falls on it from another
- Definition: ratio of the radiance in the emitted direction to irradiance in the incident direction



Function of (at least) four parameters: incident and outgoing θ, ϕ

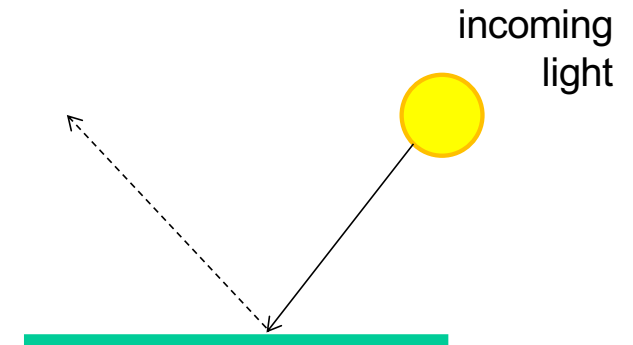
Bidirectional reflectance distribution function (BRDF)

- Table of what goes out vs what went in
- Definition:
 - ratio of the radiance in the emitted direction to irradiance in the incident direction
- Can be measured (goniometry), but measurement is expensive
- Can be incredibly complicated and is often wildly unstable!

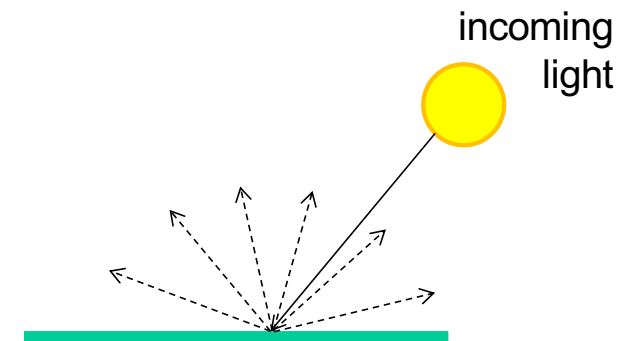


Basic models of reflection in detail

- **Specular reflection:** light is reflected about the surface normal

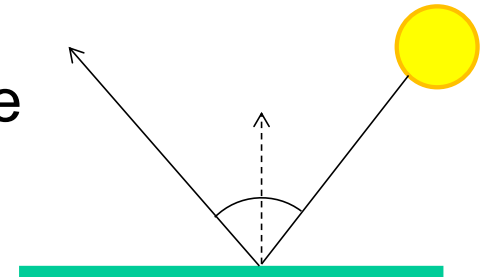


- **Diffuse reflection:** light scatters equally in all directions



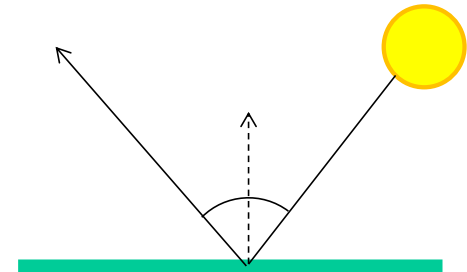
Specular reflection

- Radiation arriving along a source direction leaves along the **specular direction** (source direction reflected about normal)
- Classic case: Mirror
- Diagnosis
 - When you look at a specular surface from different directions, appearance changes
 - True specular surfaces are “really like” mirrors
 - Form a clear image
- Q:
 - Why do mirrors reverse left and right, but not up and down?



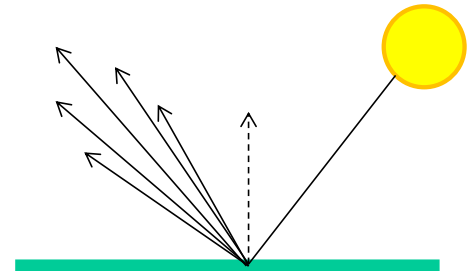
Specularities

- On real surfaces, energy usually goes into a “lobe” of directions
 - So image is blurred
 - More usually, you see only the source



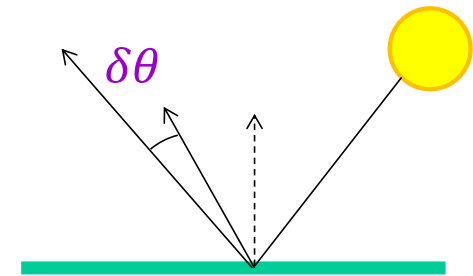
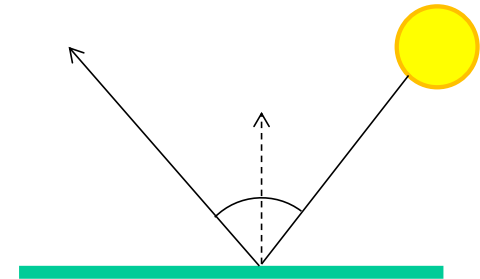
Specularities: narrow bright patches

- On metals: color of the metal
- Others: color of the light source

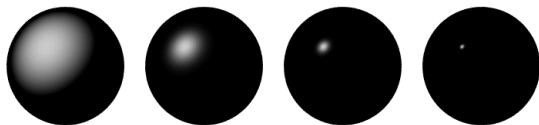


Specular reflection

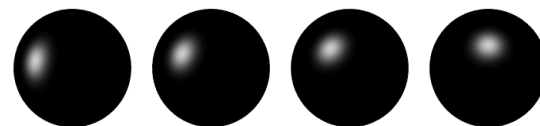
- **Phong model:** reflected energy falls off with $\cos^n(\delta\theta)$



Changing the exponent

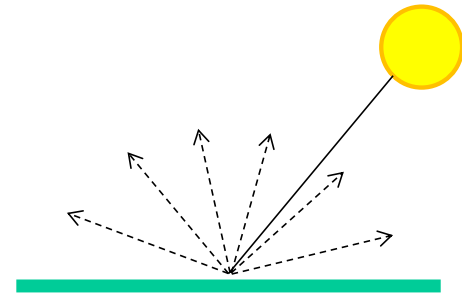


Moving the light source



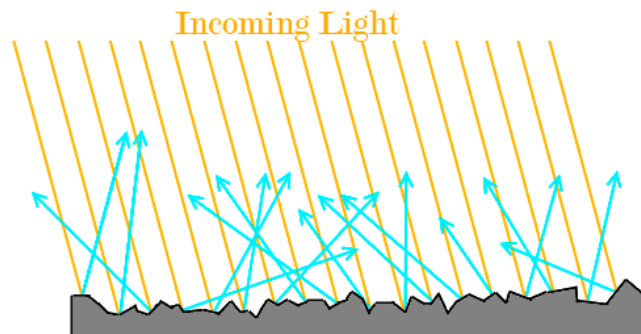
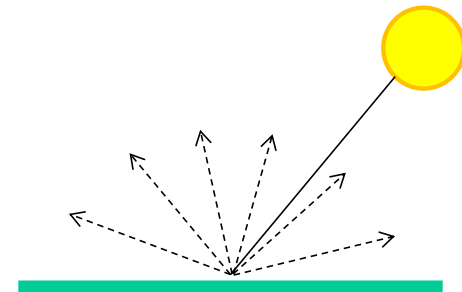
Diffuse reflection

- Light scatters equally in all directions
 - E.g., brick, matte plastic, rough wood



Diffuse reflection

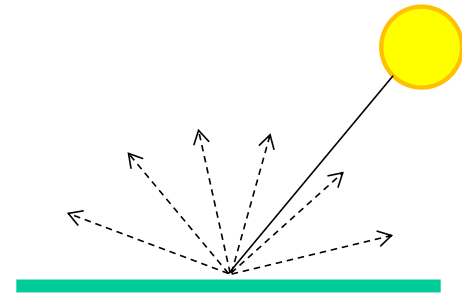
- Light scatters equally in all directions
 - E.g., brick, matte plastic, rough wood
- One cause: *microfacets* that scatter incoming light randomly



[Image source](#)

Diffuse reflection

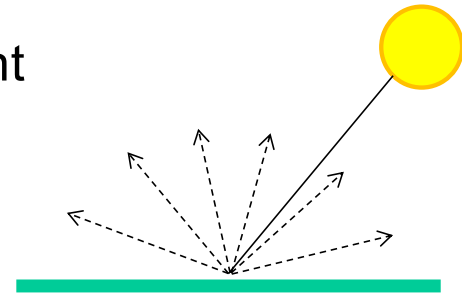
- Light scatters equally in all directions
 - E.g., brick, matte plastic, rough wood



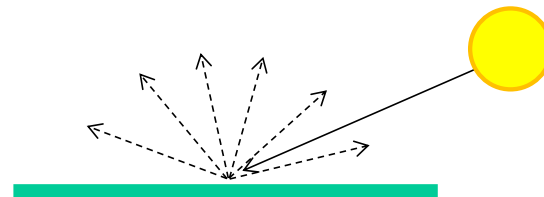
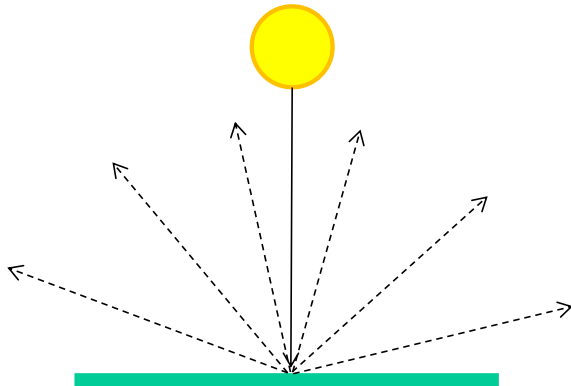
- Diagnosis:
 - Surface has the same brightness when looked at from different directions
 - (under fixed illumination)
- Extremely common
 - Very often surfaces are “largely” diffuse

Diffuse reflection

- Light scatters equally in all directions
 - For a fixed incidence angle, BRDF is constant

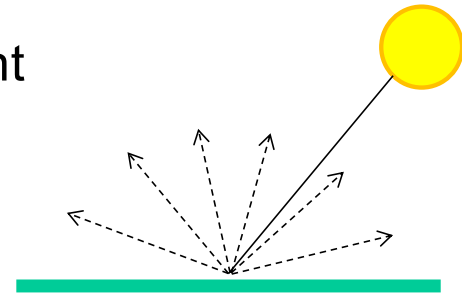


- What if we change the incidence angle?

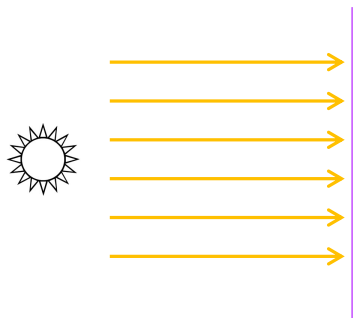


Diffuse reflection

- Light scatters equally in all directions
 - For a fixed incidence angle, BRDF is constant



- What if we change the incidence angle?

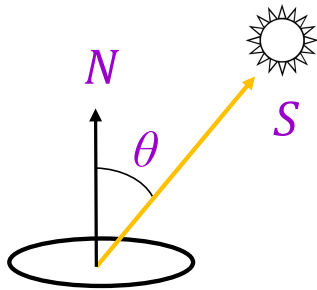


brighter

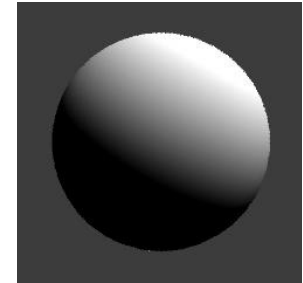


darker

Diffuse reflection: Lambert's law



$$\begin{aligned} I &= \rho (S \cdot N) \\ &= \rho \|S\| \cos \theta \end{aligned}$$



I : reflected intensity (technically: *radiosity*, or total power leaving the surface per unit area)

ρ : albedo (fraction of incident irradiance reflected by the surface)

S : direction of light source (magnitude proportional to intensity of the source)

N : unit surface normal

Diffuse vs. specular: Significance for vision applications

Same lighting, as close as possible camera settings, but different **camera position**



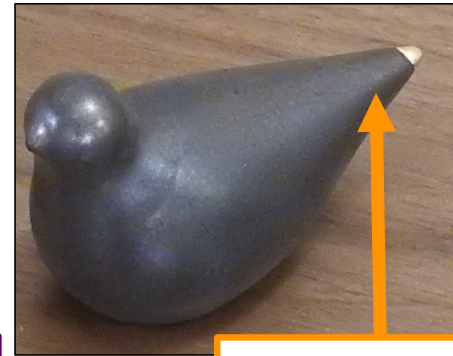
Diffuse



Same
appearance



Specular



Totally different
appearance



Outline

- Small taste of radiometry
- In-camera transformation of light
- Reflectance properties of surfaces
- Diffuse and specular reflection
- Shape from shading

Photometric stereo, or shape from shading

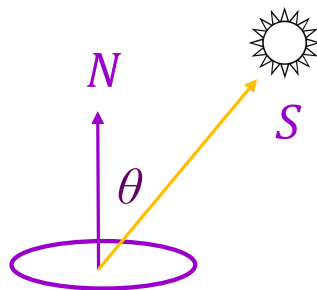
- Can we reconstruct the shape of an object based on shading cues?



Luca della Robbia,
Cantoria, 1438

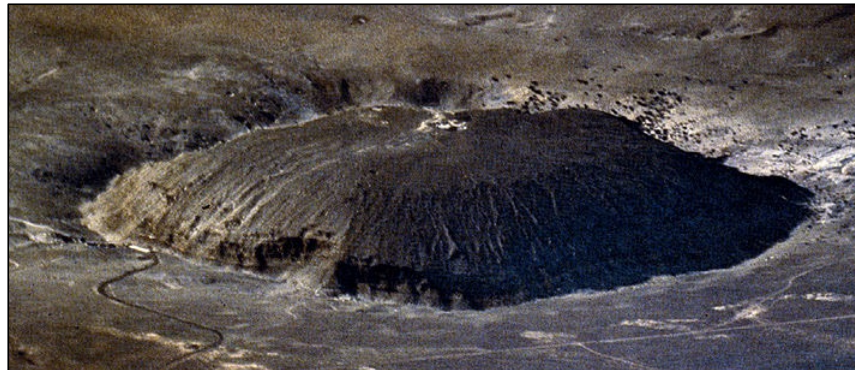
Photometric stereo, or shape from shading

- Can we reconstruct the shape of an object based on shading cues?
- Assuming a Lambertian object, given the image intensity (I), can we recover the light source direction (S) and the surface normal (N)?
- Can we do this from a single image?



$$\begin{aligned} I &= \rho (S \cdot N) \\ &= \rho \|S\| \cos \theta \end{aligned}$$

Shape from shading ambiguity



Source: [J. Johnson and D. Fouhey](#)

[Image source](#)

Shape from shading ambiguity

- Humans assume light from above (and the blueness also tells you distance)

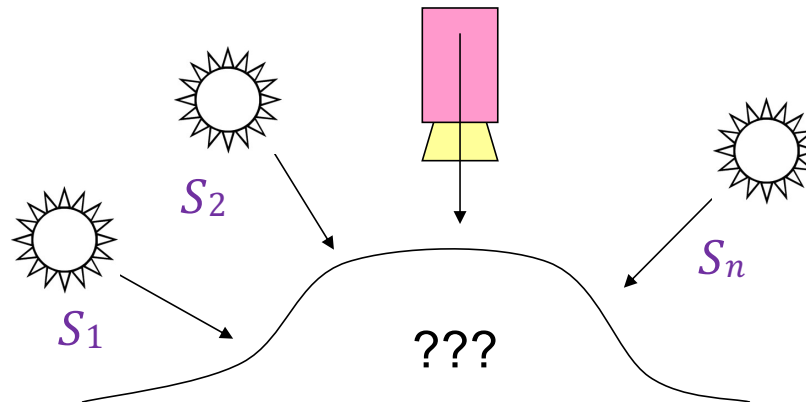


Source: [J. Johnson and D. Fouhey](#)

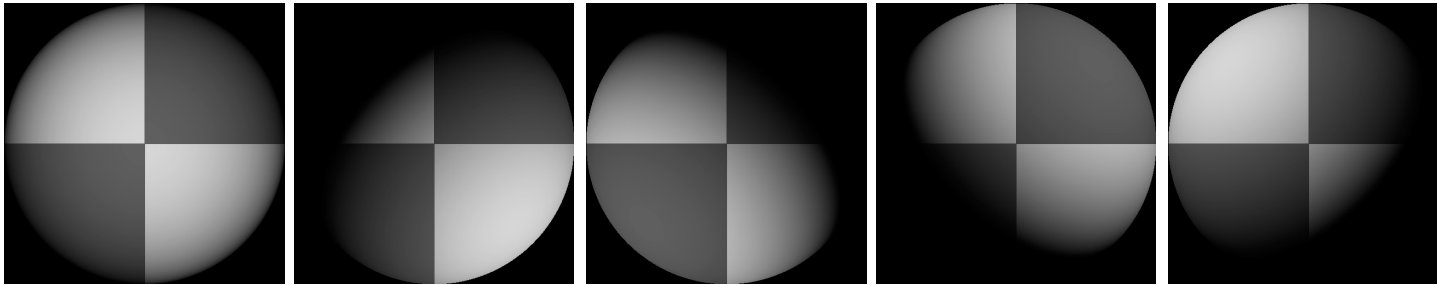
[Image source](#)

Photometric stereo

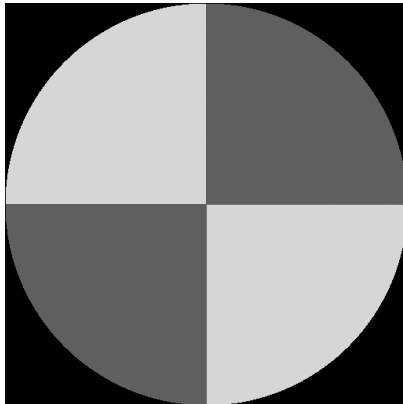
- Assume:
 - A Lambertian object
 - A *local shading model* (each point on a surface receives light only from sources visible at that point)
 - A set of *known* light source directions
 - A set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
 - Orthographic projection
- Goal: reconstruct object shape and albedo



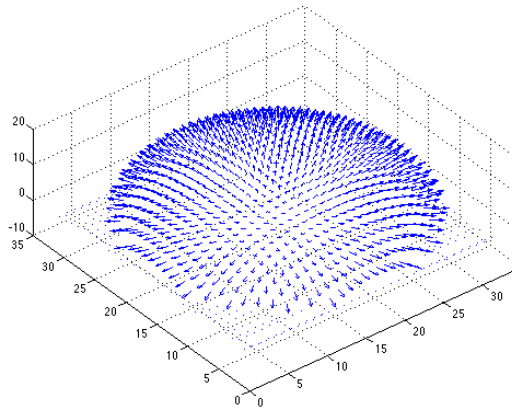
Example 1



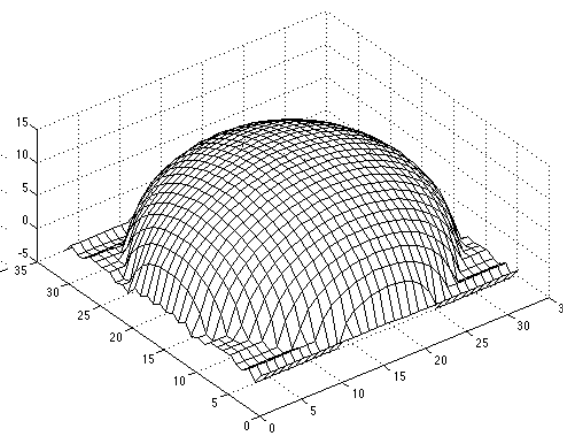
Recovered
albedo



Recovered normal
field



Recovered surface
model



Example 2

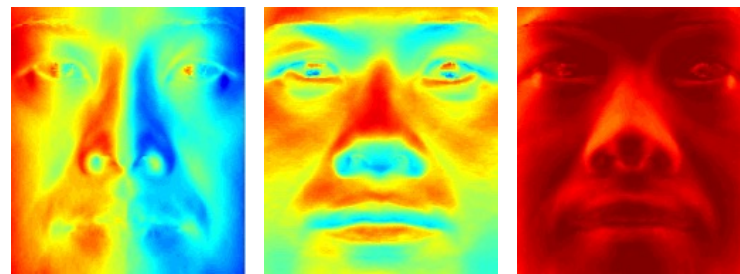
Input



Recovered
albedo



Recovered normal field



x

y

z

Recovered
surface model

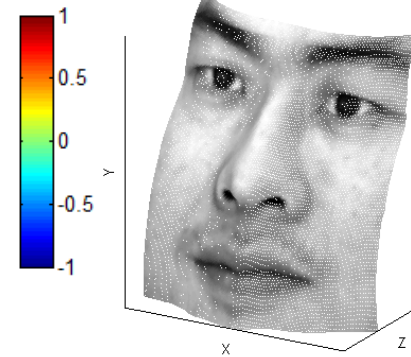


Image model

- **Known:** source vectors S_j and pixel values $I_j(x, y)$
- **Unknown:** surface normal $N(x, y)$ and albedo $\rho(x, y)$

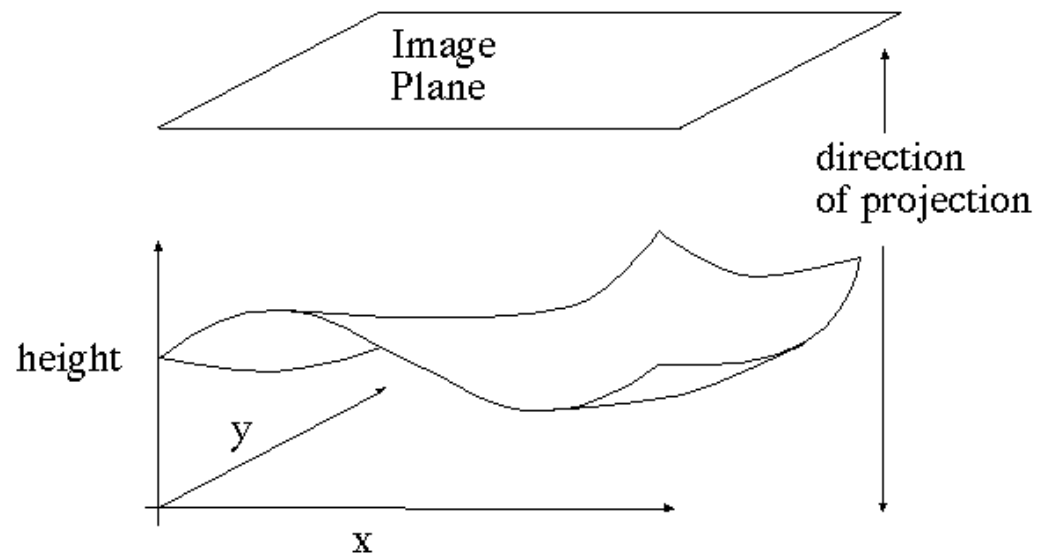


Image model

- **Known:** source vectors S_j and pixel values $I_j(x, y)$
- **Unknown:** surface normal $N(x, y)$ and albedo $\rho(x, y)$
- Assume that the response function of the camera is a linear scaling by a factor of k
- Lambert's law:

$$\begin{aligned} I_j(x, y) &= k \rho(x, y) (N(x, y) \cdot S_j) \\ &= (\rho(x, y) N(x, y)) \cdot (k S_j) \\ &= g(x, y) \cdot V_j \end{aligned}$$

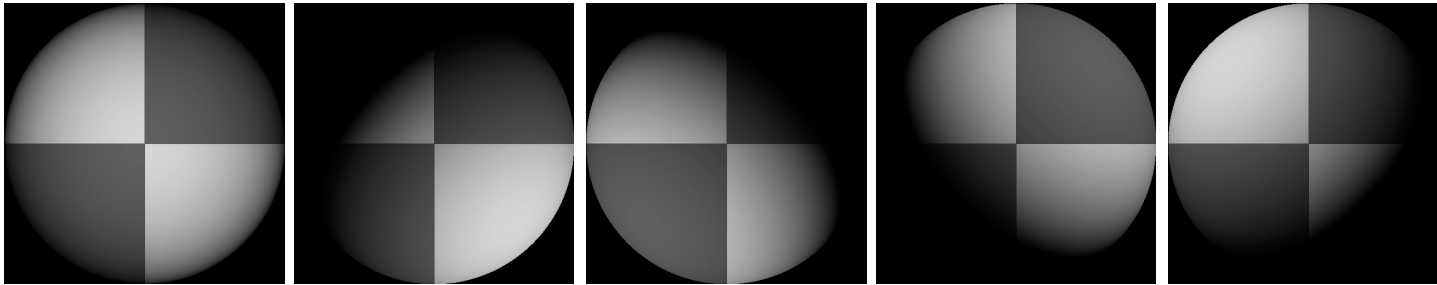
Least squares problem

- For each pixel, set up a linear system:

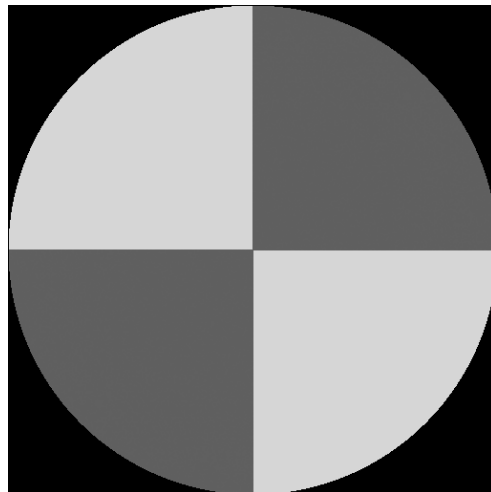
$$\begin{array}{ccc} \begin{bmatrix} V_1^T \\ V_2^T \\ \vdots \\ V_n^T \end{bmatrix} & g(x, y) = & \begin{bmatrix} I_1(x, y) \\ I_2(x, y) \\ \vdots \\ I_n(x, y) \end{bmatrix} \\ \begin{array}{c} n \times 3 \\ \text{known} \end{array} & \begin{array}{c} \downarrow \\ 3 \times 1 \\ \text{unknown} \end{array} & \begin{array}{c} n \times 1 \\ \text{known} \end{array} \end{array}$$

- Obtain least-squares solution for $g(x, y)$, which we defined as $\rho(x, y)N(x, y)$
- Since $N(x, y)$ is the *unit* normal, $\rho(x, y)$ is given by the magnitude of $g(x, y)$
- Finally, $N(x, y) = \frac{1}{\rho(x, y)} g(x, y)$

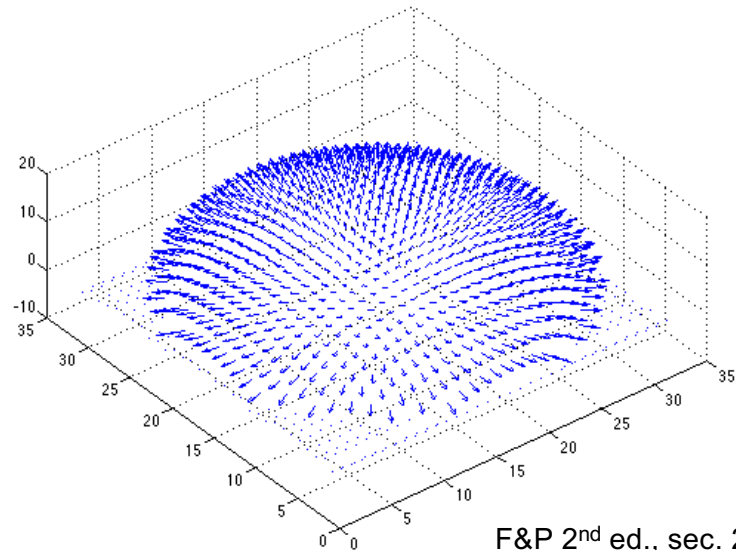
Synthetic example



Recovered albedo



Recovered normal field



F&P 2nd ed., sec. 2.2.4

Recovering a surface from normals

- Recall: the surface is written as
- Write the estimated vector g as

$$(x, y, f(x, y))$$

$$g(x, y) = \begin{bmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{bmatrix}$$

- This means the unit normal has the following form:
- Then we obtain values for the partial derivatives of the surface:

$$N(x, y) = \frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \begin{bmatrix} f_x \\ f_y \\ 1 \end{bmatrix}$$

$$f_x(x, y) = \frac{g_1(x, y)}{g_3(x, y)}$$

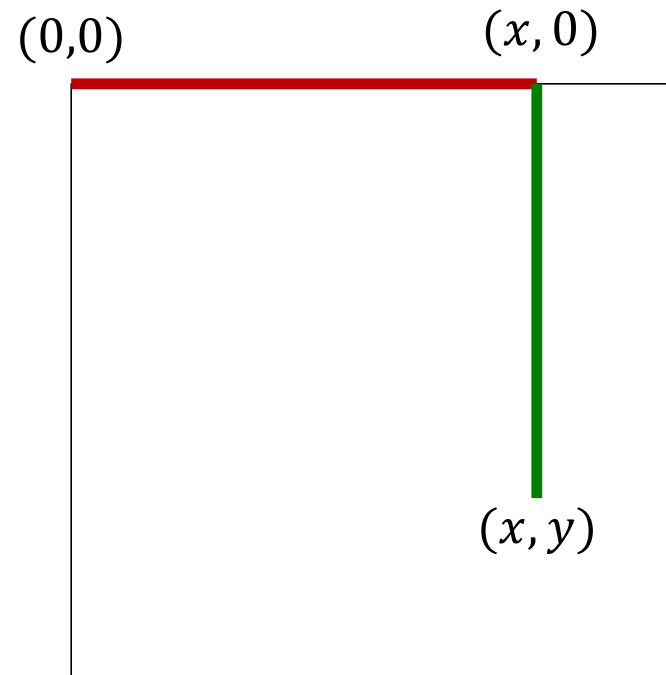
$$f_y(x, y) = \frac{g_2(x, y)}{g_3(x, y)}$$

Recovering a surface from normals

- We can now recover the surface height at any point by integration along some path, e.g.

$$f(x, y) = \int_0^x f_x(s, 0) ds + \int_0^y f_y(x, t) dt + C$$

- For robustness, it is better to take integrals over many different paths and average the results



Recovering a surface from normals

- We can now recover the surface height at any point by integration along some path, e.g.

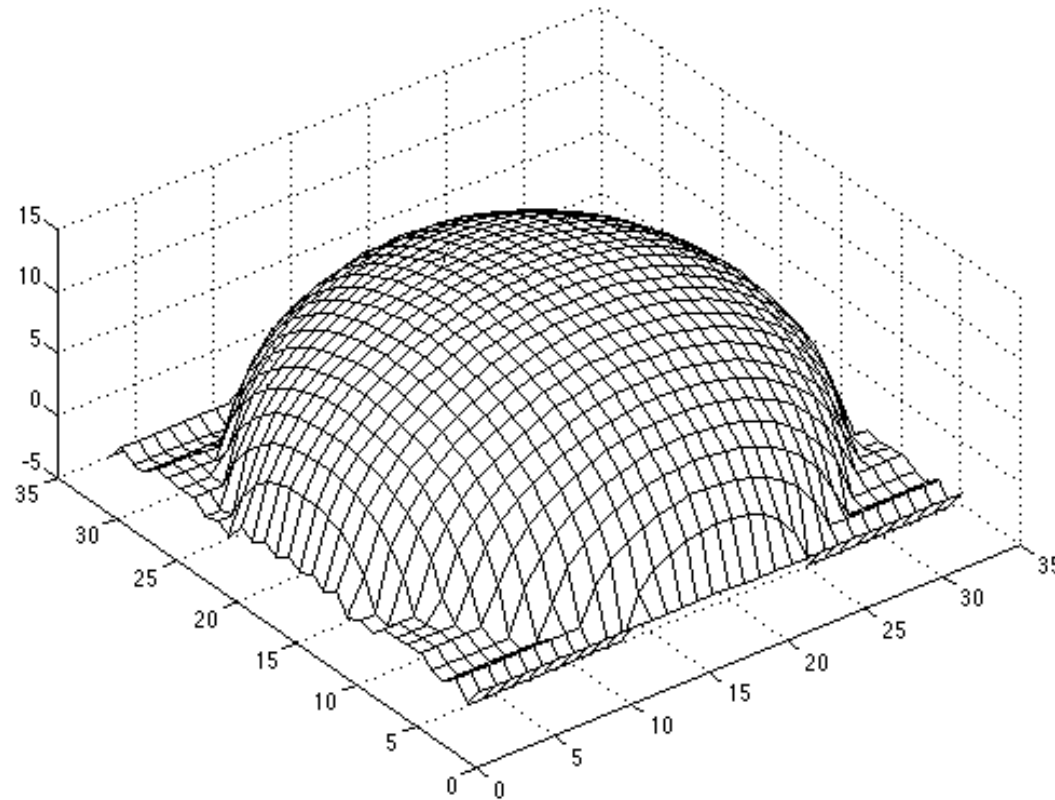
$$f(x, y) = \int_0^x f_x(s, 0) ds + \int_0^y f_y(x, t) dt + C$$

- Note: *integrability* must be satisfied: for the surface f to exist, the mixed second partial derivatives must be equal (or at least similar in practice):

$$\frac{\partial}{\partial y} \left(\frac{g_1(x, y)}{g_3(x, y)} \right) = \frac{\partial}{\partial x} \left(\frac{g_2(x, y)}{g_3(x, y)} \right)$$

- For robustness, it is better to take integrals over many different paths and average the results

Surface recovered by integration



Limitations of model

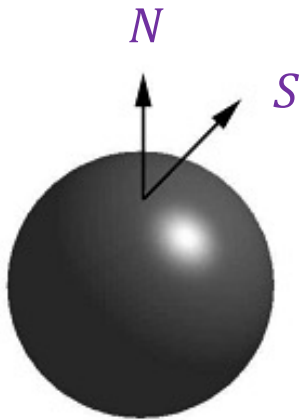
- Orthographic camera model
- Simplistic reflectance and lighting model
- No shadows
- No interreflections
- No missing data
- Integration is tricky

Outline

- Small taste of radiometry
- In-camera transformation of light
- Reflectance properties of surfaces
- Diffuse and specular reflection
- Shape from shading
- Estimating direction of light sources

Finding the direction of the light source

$$I(x, y) = N(x, y) \cdot S(x, y)$$

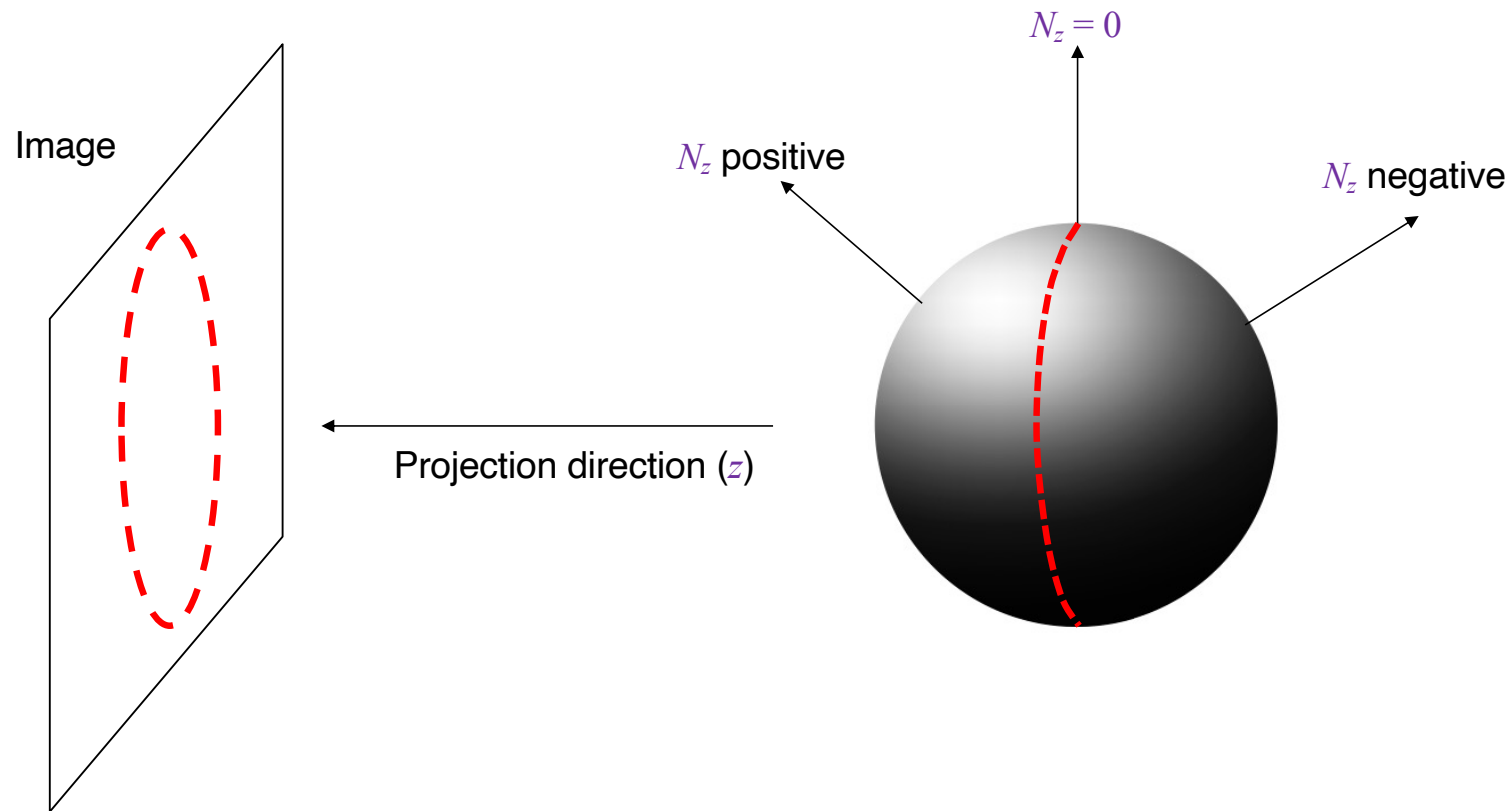


- Full 3D case:

$$\begin{bmatrix} N_x(x_1, y_1) & N_y(x_1, y_1) & N_z(x_1, y_1) \\ N_x(x_2, y_2) & N_y(x_2, y_2) & N_z(x_2, y_2) \\ \vdots & \vdots & \vdots \\ N_x(x_n, y_n) & N_y(x_n, y_n) & N_z(x_n, y_n) \end{bmatrix} \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \begin{bmatrix} I(x_1, y_1) \\ I(x_2, y_2) \\ \vdots \\ I(x_n, y_n) \end{bmatrix}$$

Finding the direction of the light source

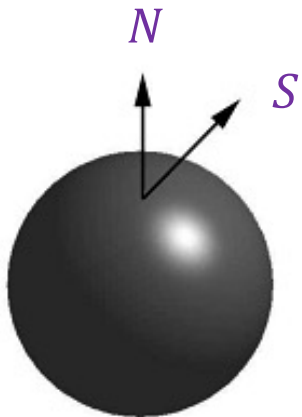
Consider points on the *occluding contour*:



P. Nillius and J.-O. Eklundh. [Automatic estimation of the projected light source direction](#). CVPR 2001

Finding the direction of the light source

$$I(x, y) = N(x, y) \cdot S(x, y)$$



- Full 3D case:

$$\begin{bmatrix} N_x(x_1, y_1) & N_y(x_1, y_1) & N_z(x_1, y_1) \\ N_x(x_2, y_2) & N_y(x_2, y_2) & N_z(x_2, y_2) \\ \vdots & \vdots & \vdots \\ N_x(x_n, y_n) & N_y(x_n, y_n) & N_z(x_n, y_n) \end{bmatrix} \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \begin{bmatrix} I(x_1, y_1) \\ I(x_2, y_2) \\ \vdots \\ I(x_n, y_n) \end{bmatrix}$$

- For points on the occluding contour ($N_z = 0$):

$$\begin{bmatrix} N_x(x_1, y_1) & N_y(x_1, y_1) \\ N_x(x_2, y_2) & N_y(x_2, y_2) \\ \vdots & \vdots \\ N_x(x_n, y_n) & N_y(x_n, y_n) \end{bmatrix} \begin{bmatrix} S_x \\ S_y \end{bmatrix} = \begin{bmatrix} I(x_1, y_1) \\ I(x_2, y_2) \\ \vdots \\ I(x_n, y_n) \end{bmatrix}$$

Finding the direction of the light source



P. Nillius and J.-O. Eklundh. [Automatic estimation of the projected light source direction](#). CVPR 2001

Application: Detecting composite photos

Fake photo



Real photo



M. K. Johnson and H. Farid. [Exposing Digital Forgeries by Detecting Inconsistencies in Lighting](#).
ACM Multimedia and Security Workshop, 2005

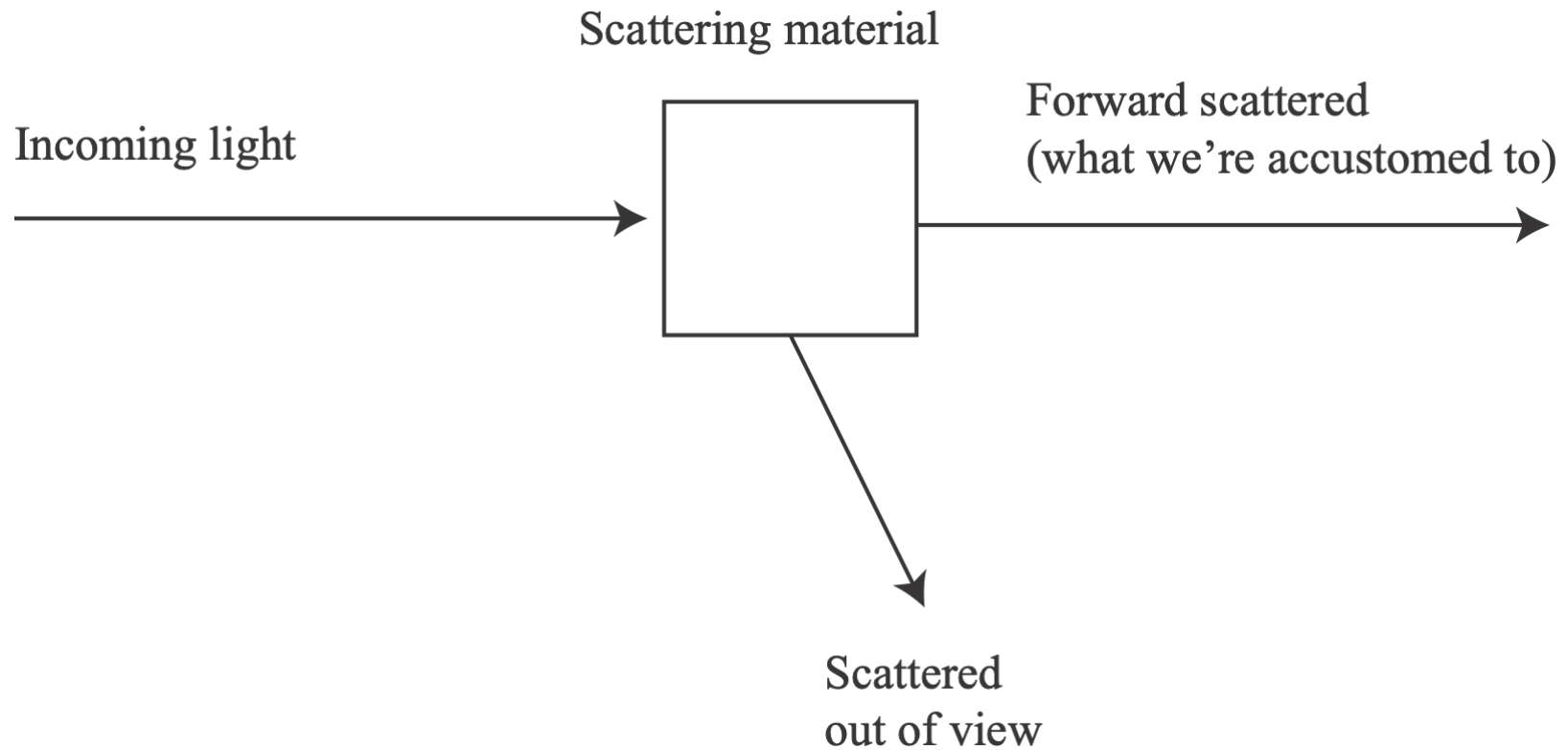
Bits and Pieces, Obstacles and Problems

- Why does blueness reveal depth?
- What are the effects of interreflection?
- Does shading in a single image reveal shape?

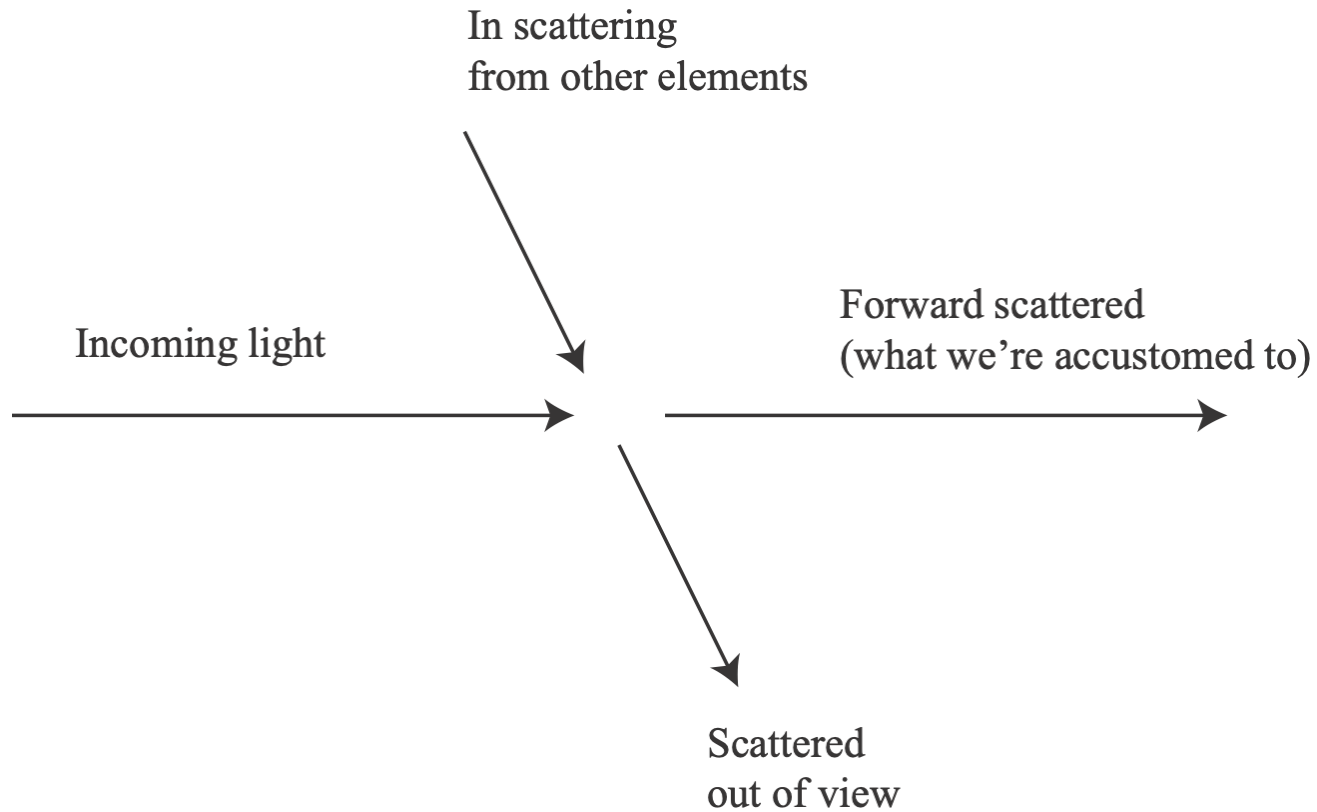
Participating media

- for example,
 - smoke,
 - wet air (mist, fog)
 - rain
 - dusty air
 - air at long scales
- Light leaves/enters a ray travelling through space
 - leaves because it is scattered out
 - enters because it is scattered in
- New visual effects

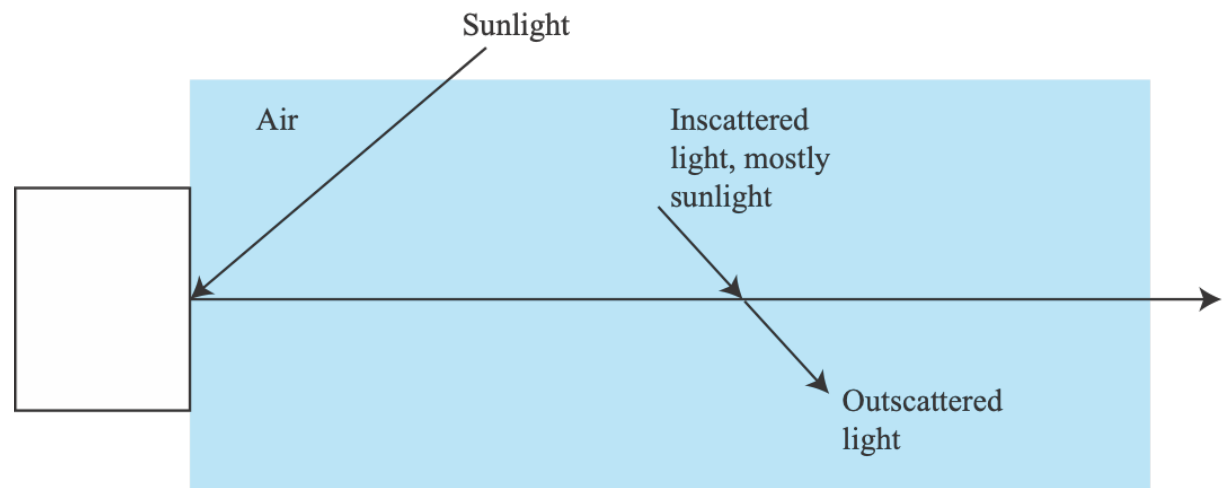
Light hits a small box of material



A ray passing through scattering material



Airlight as a scattering effect



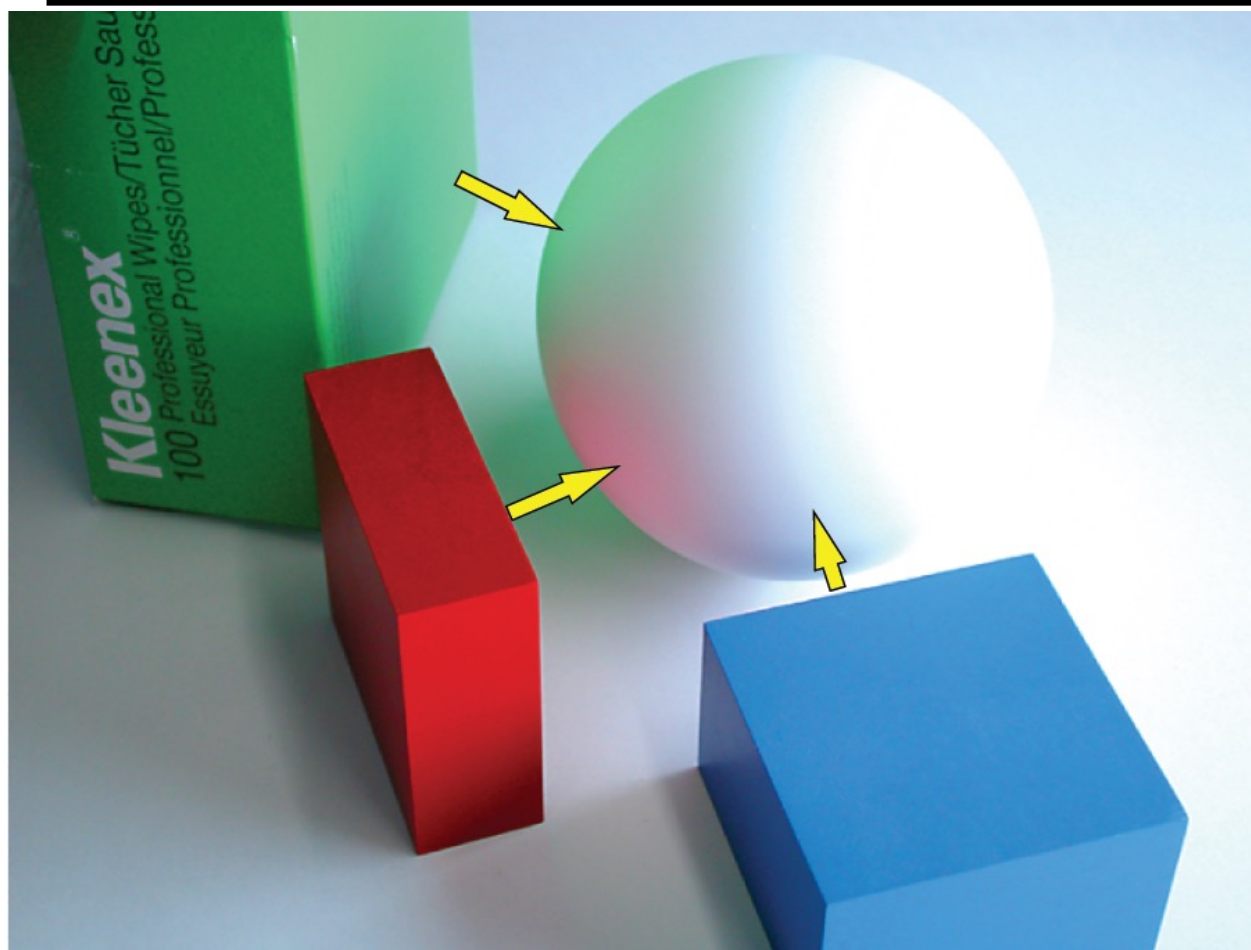


From Lynch and Livingstone, Color and Light in Nature



From Lynch and Livingstone, Color and Light in Nature

Interreflections



Odd fact: this does not seem to be a major problem for Photometric stereo

Q: why?

From Koenderink slides on image texture and the flow of light

Shape from shading

- Given a single shaded image of an object, recover:
 - Shape
 - Albedo
- People seem to be able to do this
- In Computer Vision:
 - Open since the early 70's
 - Mostly, still doesn't work
 - Mostly, attention has moved elsewhere

Shading is an amazing single view cue

