

Edge detection



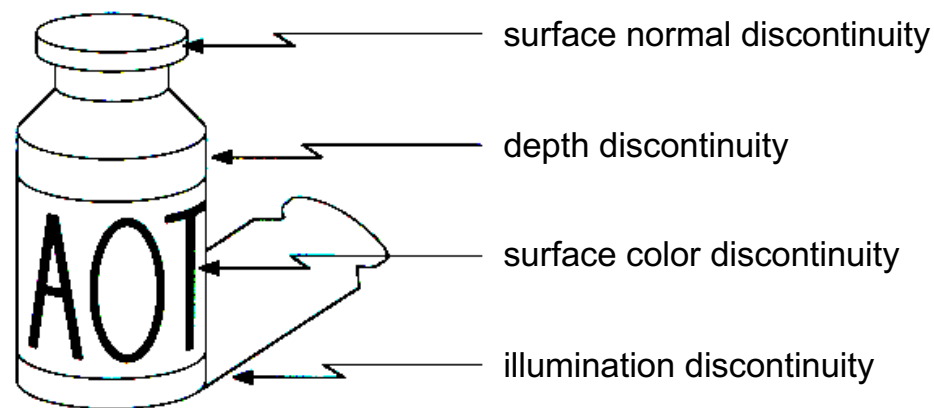
[Winter in Kraków photographed by Marcin Ryczek](#)

Overview

- Motivating edge detection
- Image gradients
- Derivative of Gaussian filters
- Canny edge detector
- Role of edge detection in image understanding
- Orientations

Edge detection

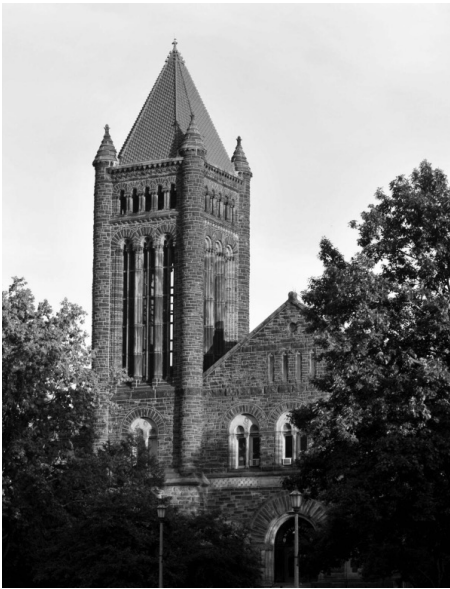
- **Goal:** Identify sudden changes (discontinuities) in an image
- Intuitively, edges carry most of the semantic and shape information from the image



Sources: D. Lowe and S. Seitz

Edge detection

Input photo

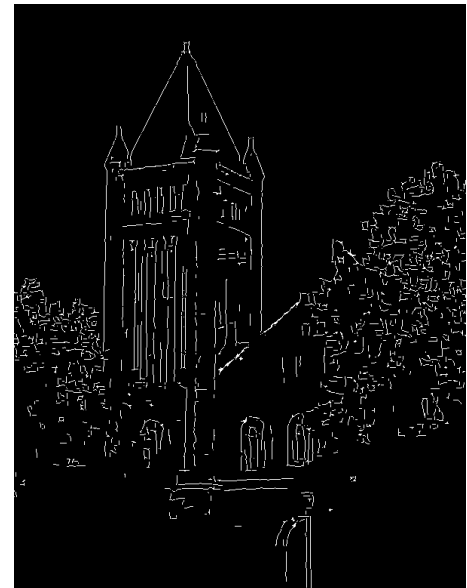


Ideal: artist's line drawing



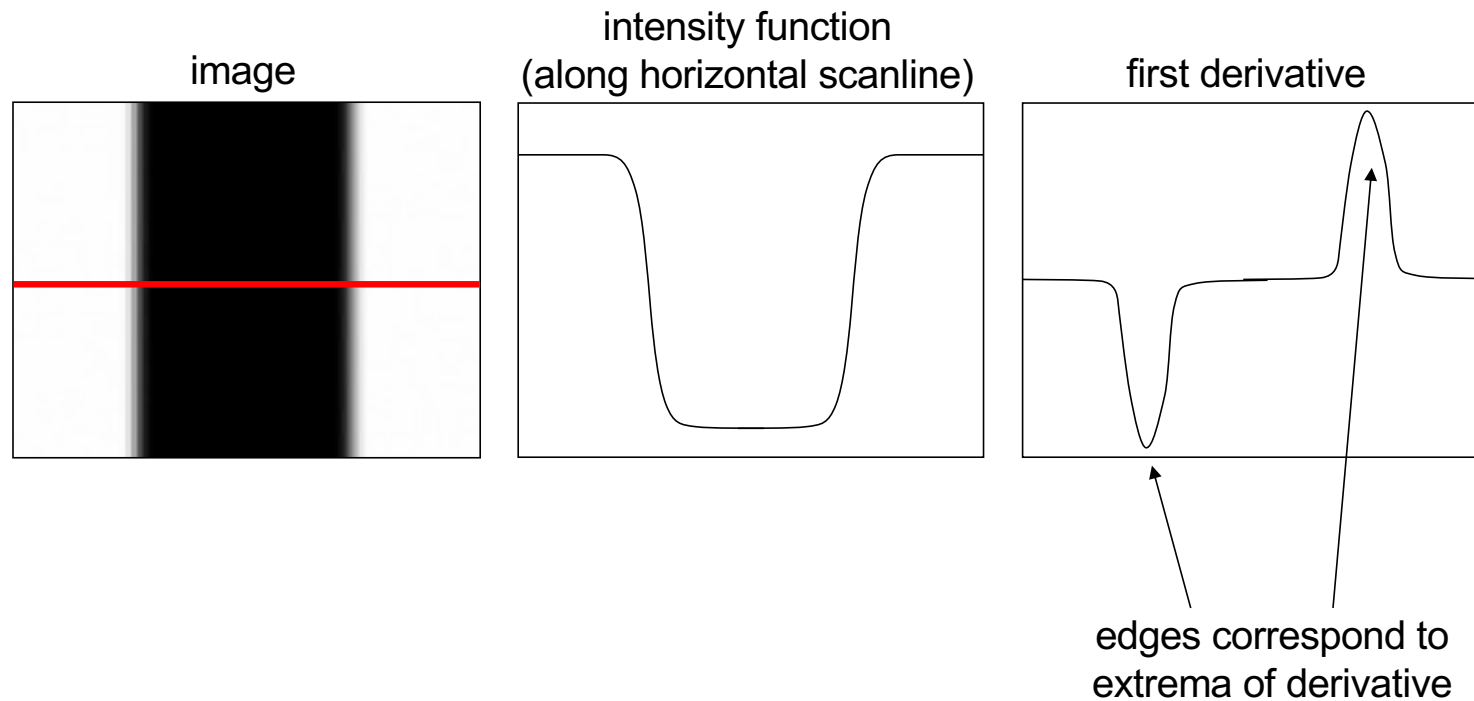
[Image source](#)

Reality



Edge detection

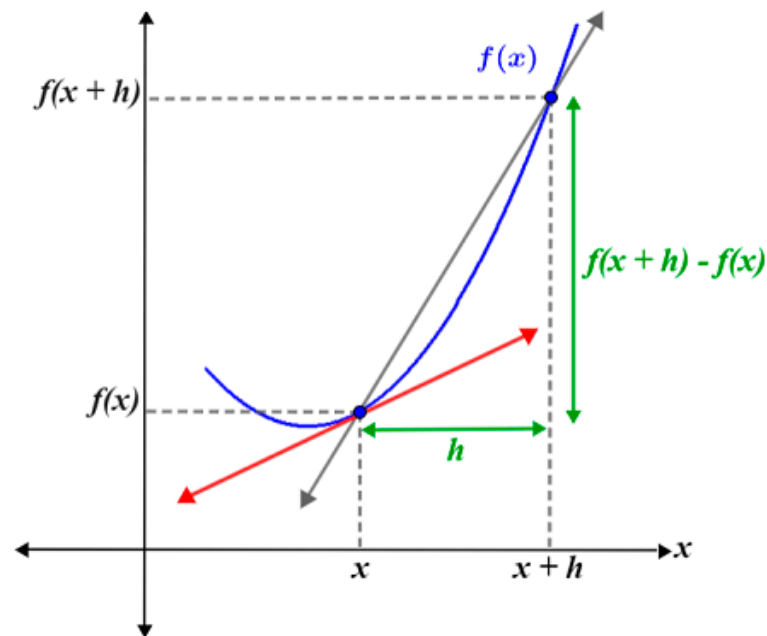
- An edge is a place of rapid change in the image intensity function



Partial derivatives of an image

- For 2D function $f(x, y)$, the partial derivative w.r.t. x is

$$\frac{\partial f(x, y)}{\partial x} = \lim_{h \rightarrow 0} \frac{f(x + h, y) - f(x, y)}{h}$$



Partial derivatives of an image

- For 2D function $f(x, y)$, the partial derivative w.r.t. x is

$$\frac{\partial f(x, y)}{\partial x} = \lim_{h \rightarrow 0} \frac{f(x + h, y) - f(x, y)}{h}$$

- For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x, y)}{\partial x} = \frac{f(x + 1, y) - f(x, y)}{1}$$

- To implement the above as convolution, what would be the associated filter?

Overview

- Motivating edge detection
- Image gradients
- Derivative of Gaussian filters

“Fun” facts

Convolution is commutative

$$f * g = g * f$$

(easy proof: use the convolution theorem, multiplication commutes!)

Differentiation can be represented with convolution

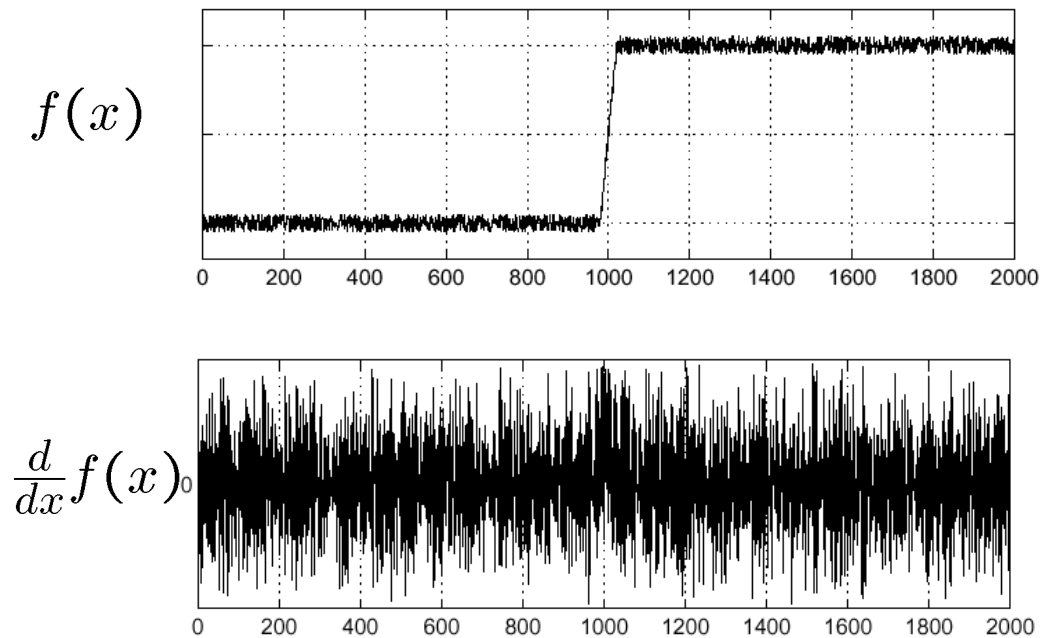
there is some k so that

$$\frac{df}{dx} = k * f = f * k$$

(Won't prove this, but we've sort of seen it already)

Finding noisy edges

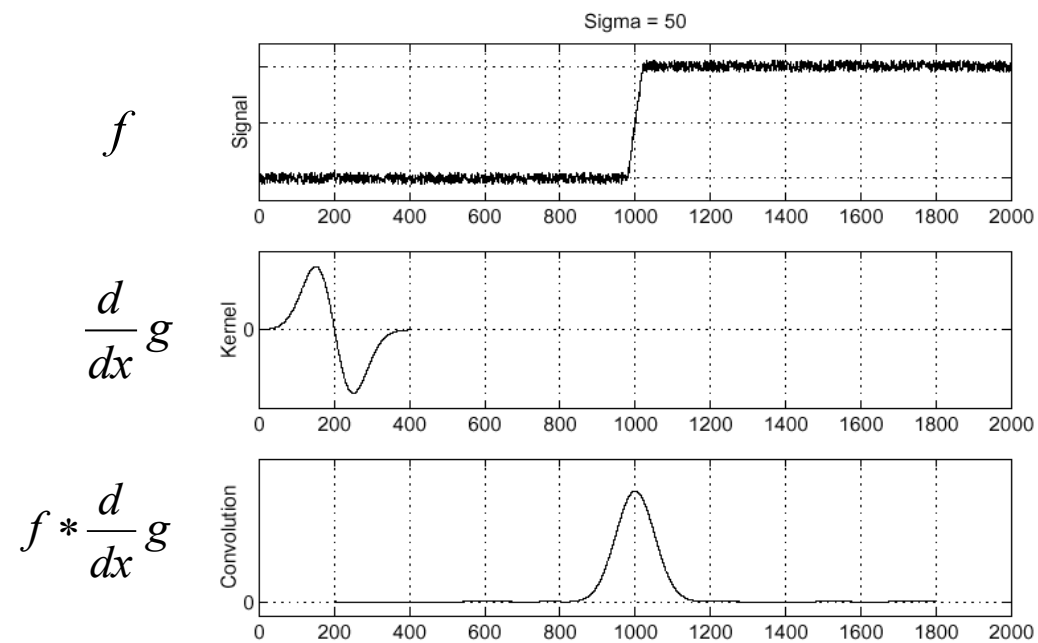
- Consider a single row or column of the image:



- Where is the edge?

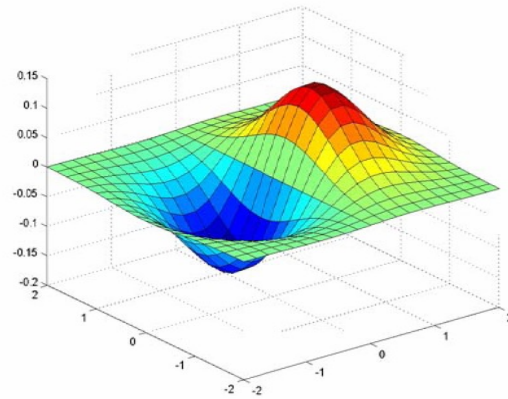
Source: S. Seitz

Filtering with derivative of Gaussian

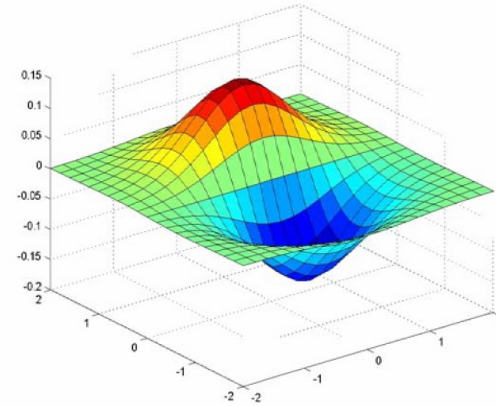
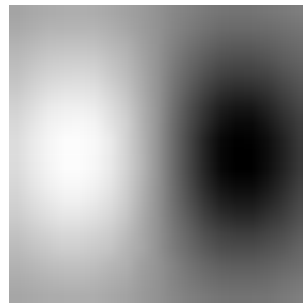


Source: S. Seitz

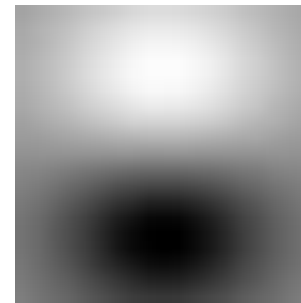
2D Derivative of Gaussian (d.o.g. or dog) filters



x -direction

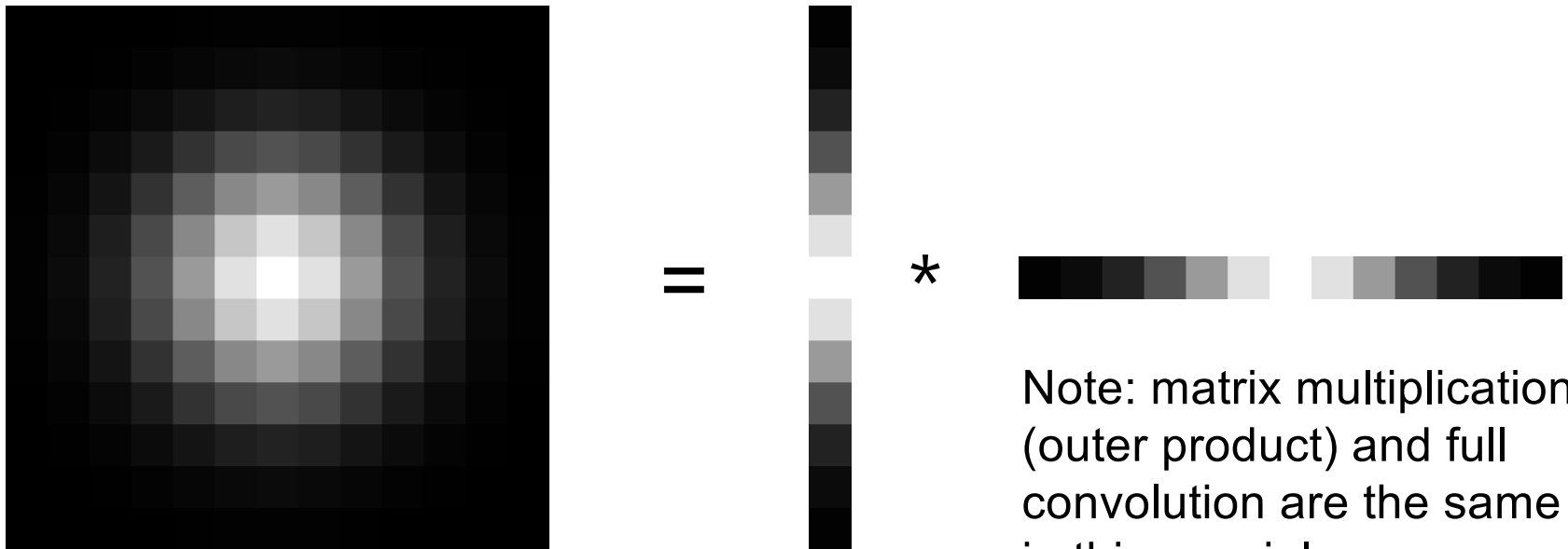


y -direction



Separability of the Gaussian filter

$$\frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{y^2}{2\sigma^2}\right)$$



Note: matrix multiplication (outer product) and full convolution are the same in this special case

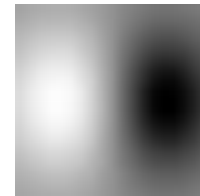
2D Derivative of Gaussian filters

- (Unnormalized) 2D Gaussian:

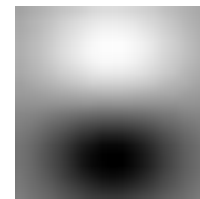
$$g(x, y) \propto \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) = \exp\left(-\frac{x^2}{2\sigma^2}\right) \exp\left(-\frac{y^2}{2\sigma^2}\right)$$

- (Unnormalized) Gaussian derivatives:

$$\frac{\partial g}{\partial x} \propto -x \exp\left(-\frac{x^2}{2\sigma^2}\right) \exp\left(-\frac{y^2}{2\sigma^2}\right)$$



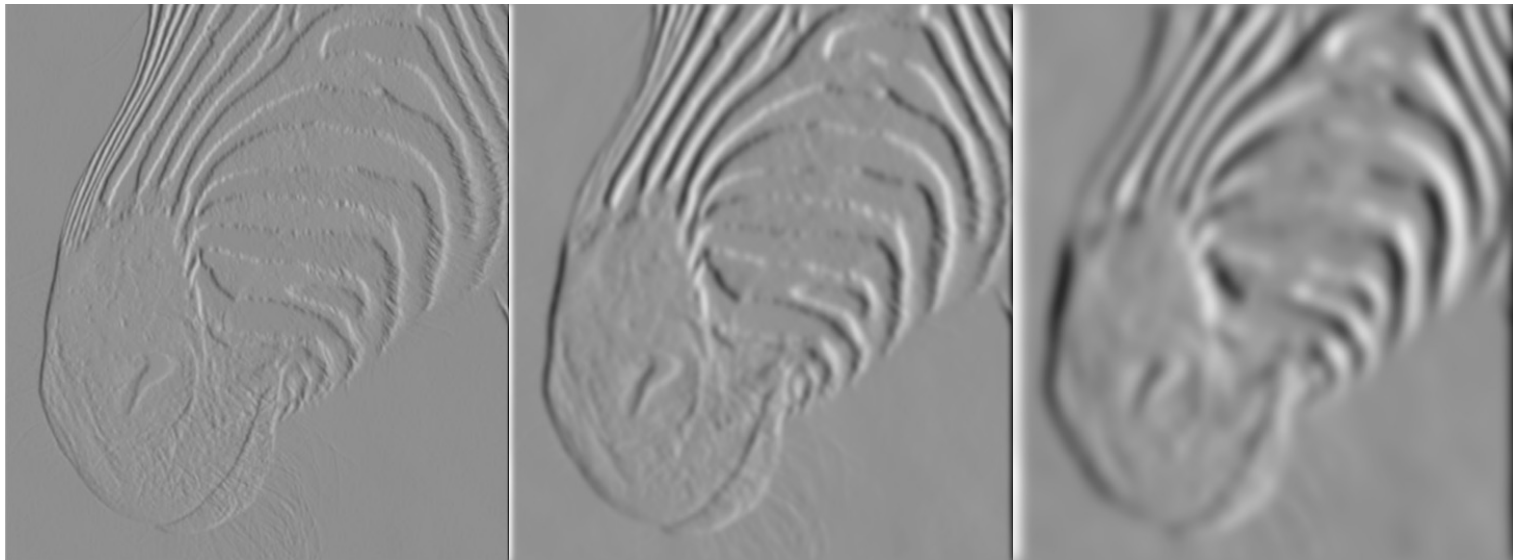
$$\frac{\partial g}{\partial y} \propto -y \exp\left(-\frac{y^2}{2\sigma^2}\right) \exp\left(-\frac{x^2}{2\sigma^2}\right)$$



- These are products of a 1D Gaussian in one direction and 1D derivative of Gaussian in the other direction!

Derivative of Gaussian: Scale

- Using Gaussian derivatives with different values of σ finds structures at different scales or frequencies



$\sigma = 1$

$\sigma = 3$

$\sigma = 7$

Source: D. Forsyth

Summing up: Types of filters

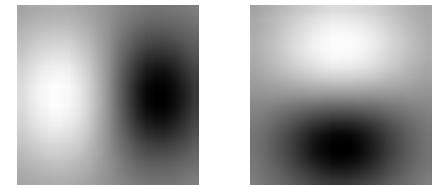
- Smoothing filters

- Gaussian: remove “high-frequency” components; “low-pass” filter
- Can the values of a smoothing filter be negative?
- What should the values sum to?
 - **One**: constant regions are not affected by the filter



- Derivative filters

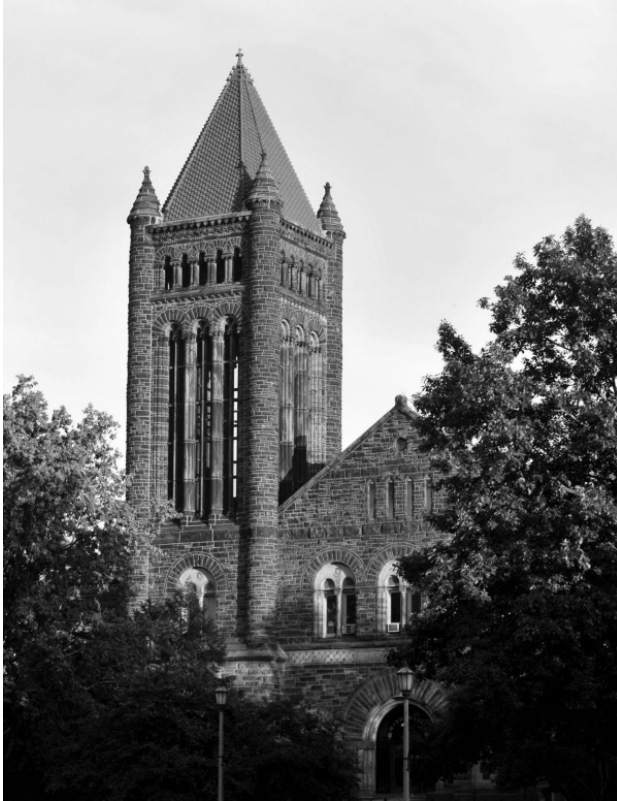
- Derivatives of Gaussian: compute smoothed differences; “band-pass” filters
- Can the values of a derivative filter be negative?
- What should the values sum to?
 - **Zero**: no response in constant regions



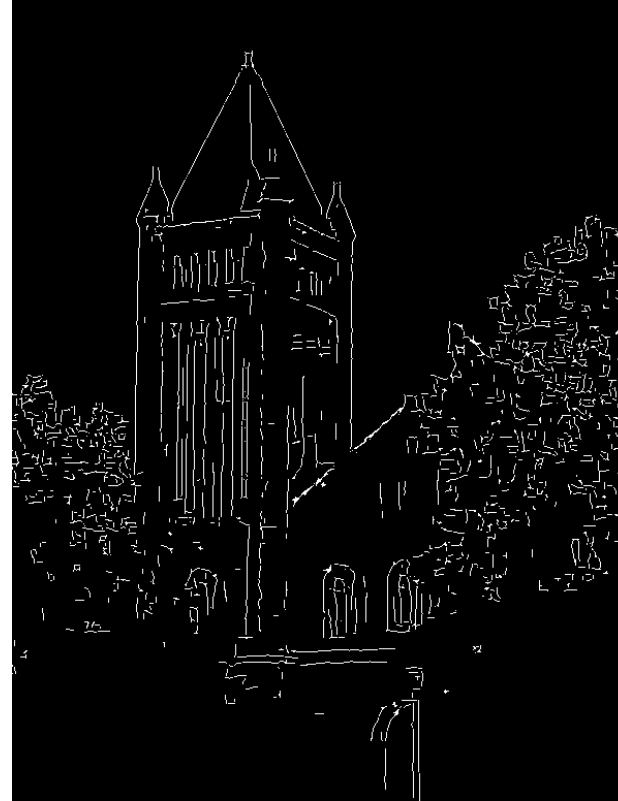
Edge detection: Overview

- Motivating edge detection
- Image gradients
- Derivative of Gaussian filters
- Canny edge detector

Building an edge detector



original image

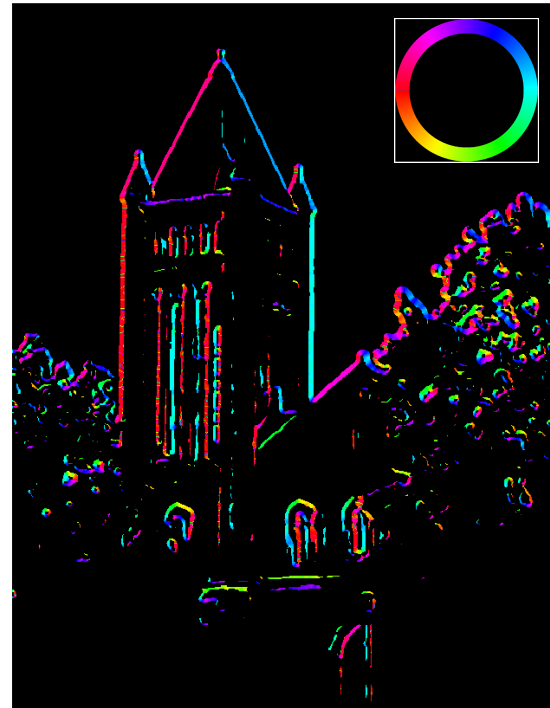
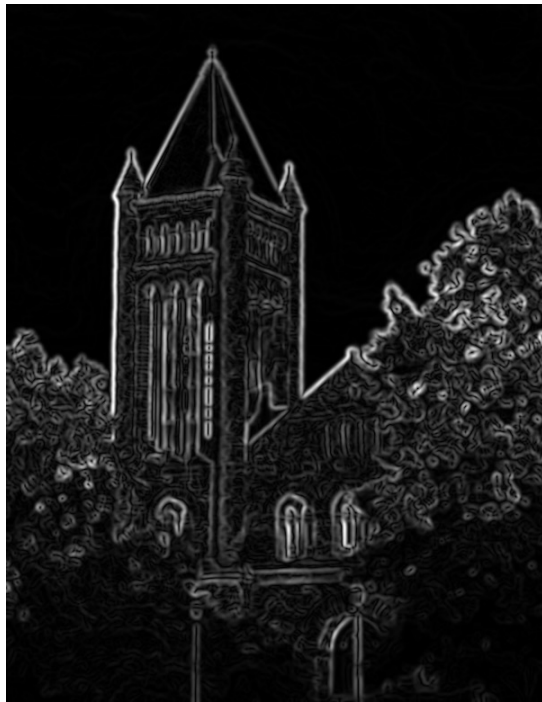


final output

J. Canny, [A Computational Approach To Edge Detection](#), IEEE Trans. PAMI, 8:679-714, 1986

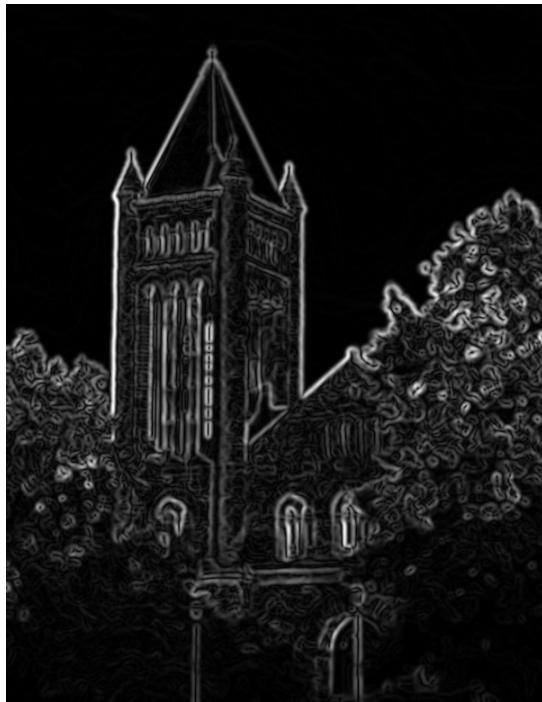
Building an edge detector

1. Compute x and y derivative images
2. Find magnitude and orientation of the gradient



Building an edge detector

1. Compute x and y derivative images
2. Find magnitude and orientation of the gradient



Let's threshold the gradient magnitude

Source: S. Gupta

Building an edge detector

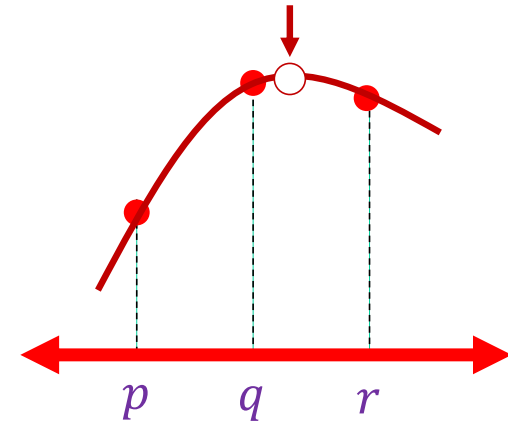
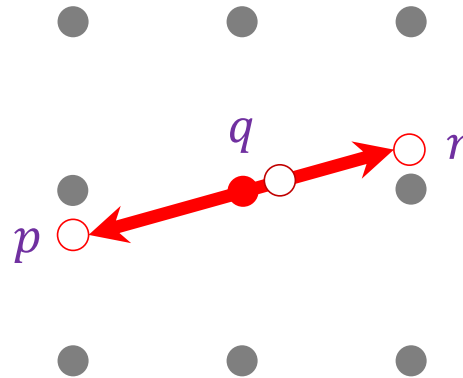
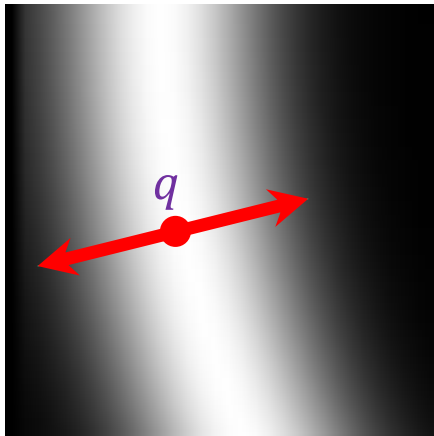
1. Compute x and y derivative images
2. Find magnitude and orientation of the gradient



We get thick trails, not neat edge curves

Let's threshold the gradient magnitude

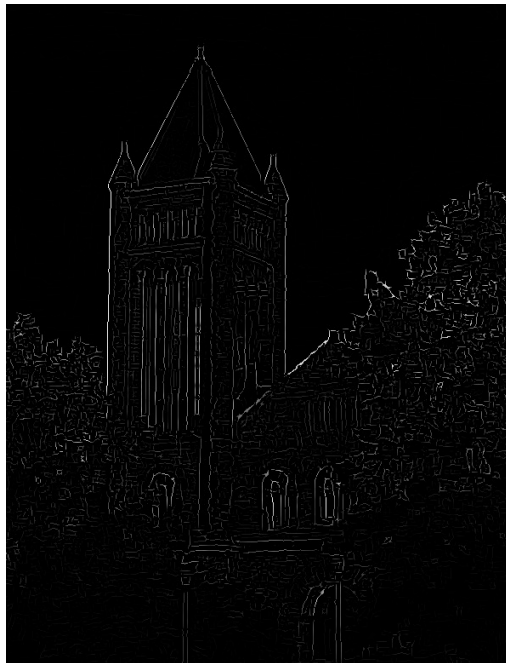
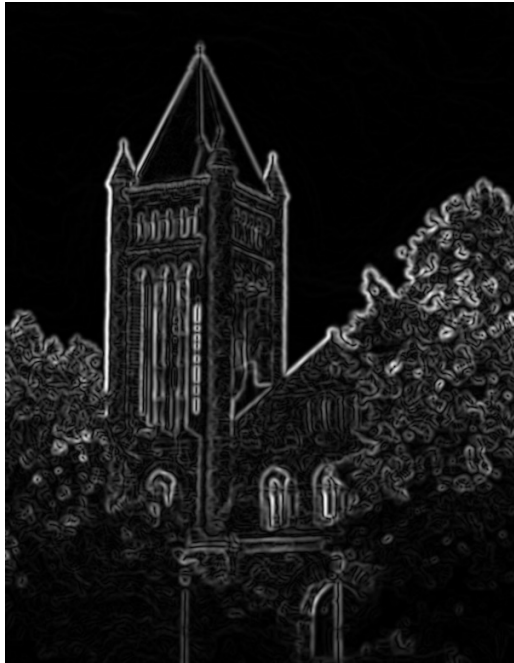
Non-maximum suppression



1D image "slice" normal to the edge

- For each location q above threshold, check that the gradient magnitude is higher than at adjacent points p and r along the direction of the gradient
 - Need to interpolate to get the gradient magnitude values at p and r
 - Can even use nonlinear interpolation to get sub-pixel edge localization!

Non-maximum suppression



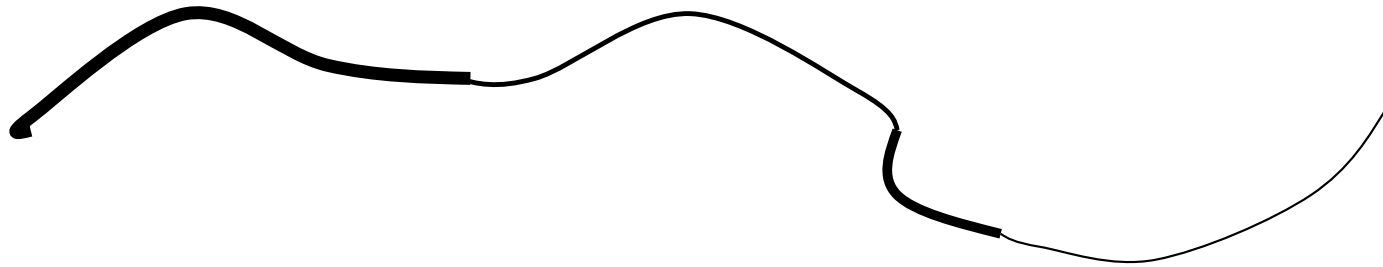
NMS

NMS > threshold

Another problem: pixels
along this edge didn't
survive the thresholding

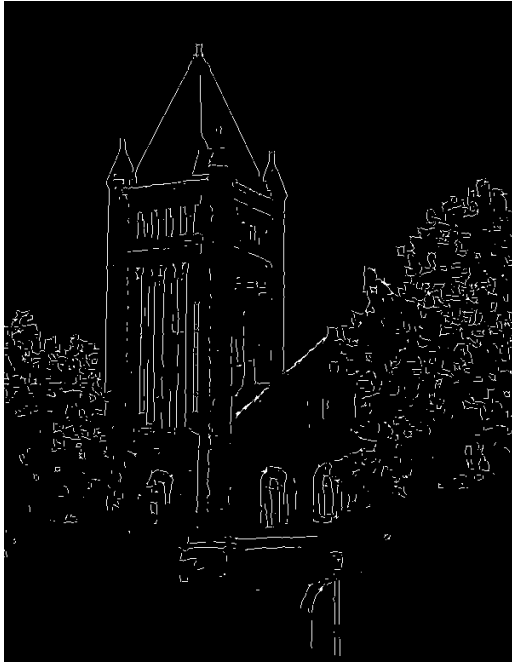
Hysteresis thresholding

- Use a high threshold to start edge curves, and a low threshold to continue them

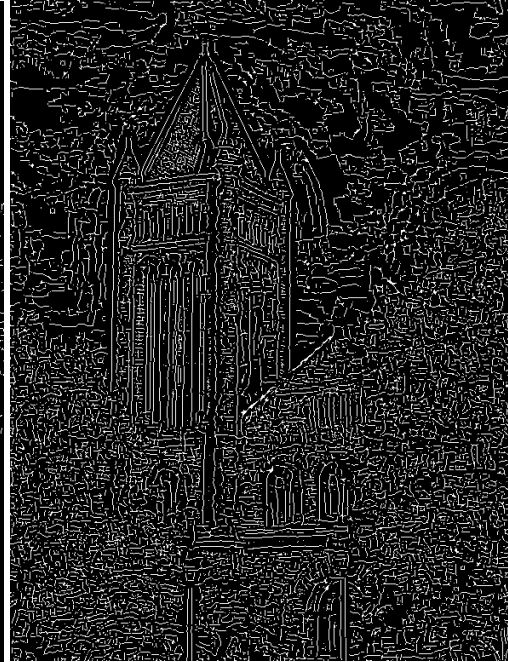


Source: Steve Seitz

Hysteresis thresholding



high threshold
(strong edges)



low threshold
(weak edges)



hysteresis threshold

Recap: Canny edge detector

1. Compute x and y derivative images
2. Find magnitude and orientation of the gradient
3. Non-maximum suppression:
 - Thin wide “ridges” down to single pixel width
4. Linking and thresholding (hysteresis):
 - Define two thresholds: low and high
 - Use the high threshold to start edge curves and the low threshold to continue them

J. Canny, [A Computational Approach To Edge Detection](#), IEEE Trans. PAMI, 8:679-714, 1986.

Overview

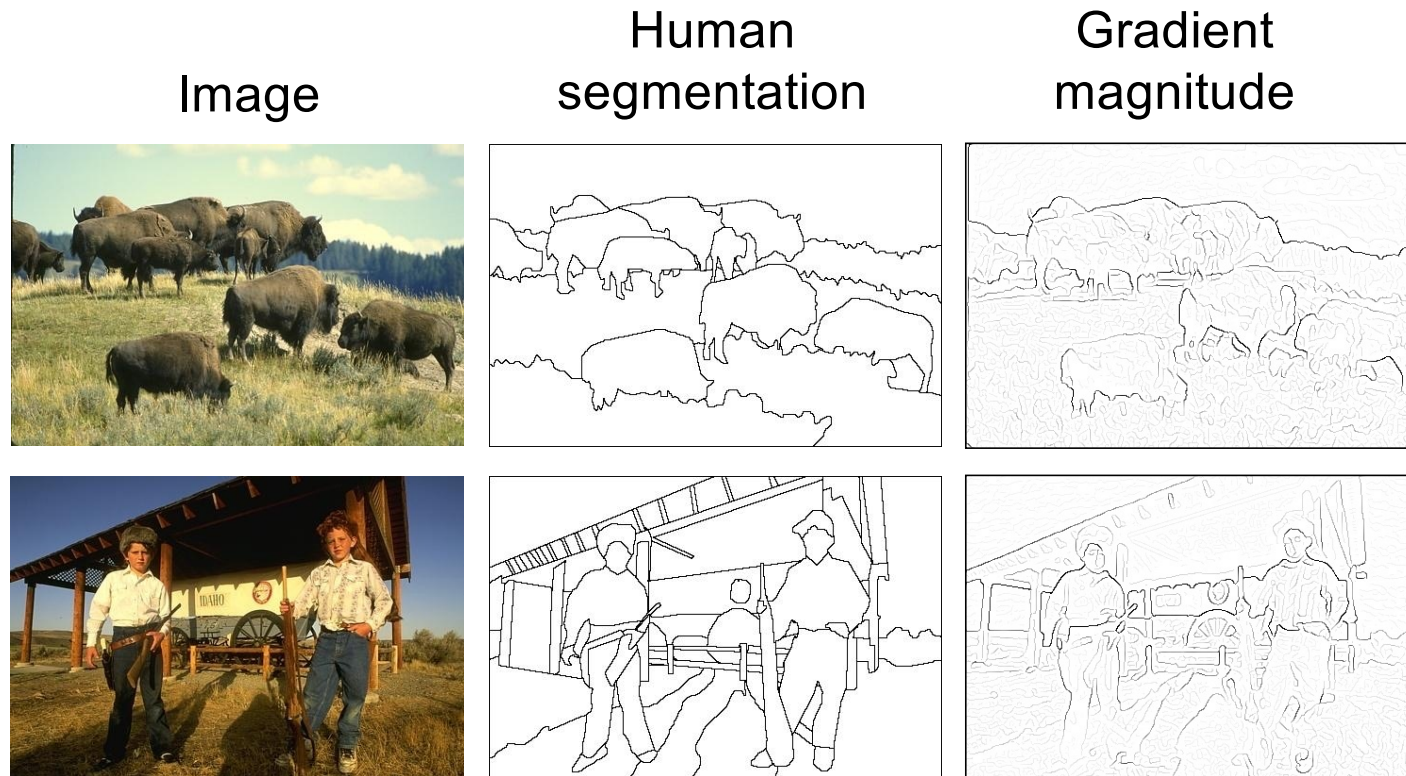
- Motivating edge detection
- Image gradients
- Derivative of Gaussian filters
- Canny edge detector
- What is the role of edge detection in image understanding?

Are edges an “input” or an “output”?



Figure from Marr (1982), attributed to R. C. James

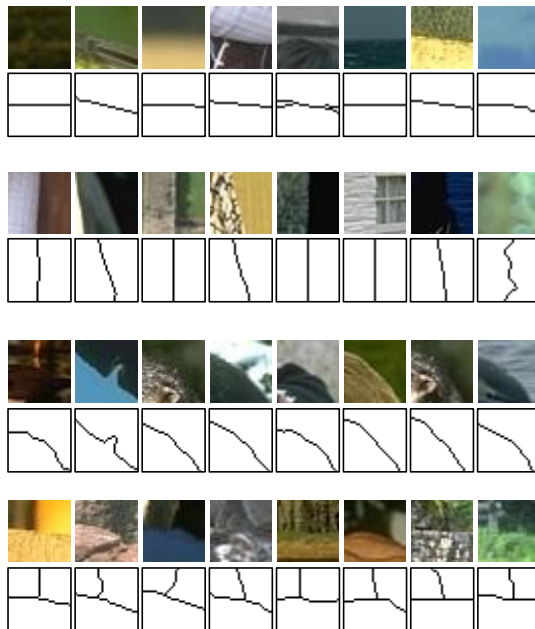
Image gradients vs. meaningful contours



D. Martin, C. Fowlkes, D. Tal, and J. Malik. [A Database of Human Segmented Natural Images and its Application to Evaluating Segmentation Algorithms and Measuring Ecological Statistics](#). ICCV 2001

Data-driven edge detection

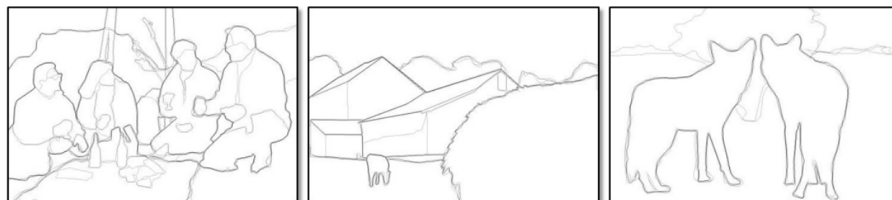
Training data



Input images



Ground truth

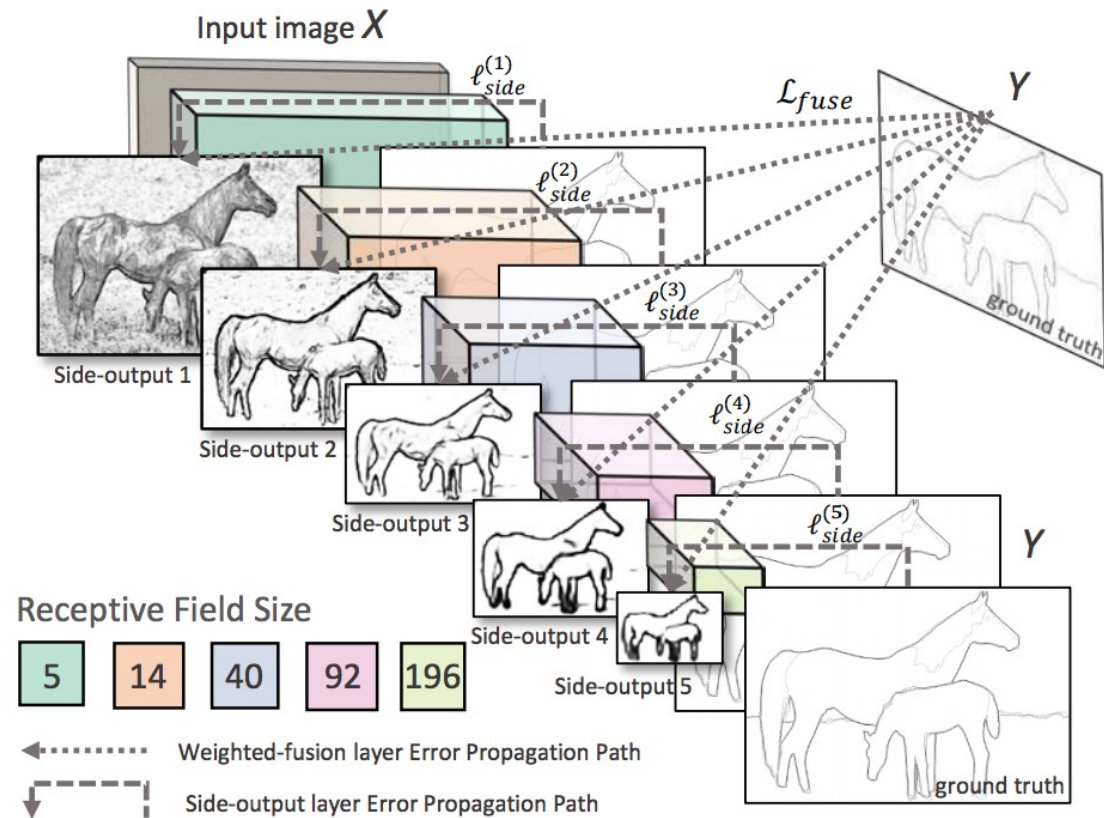


Output



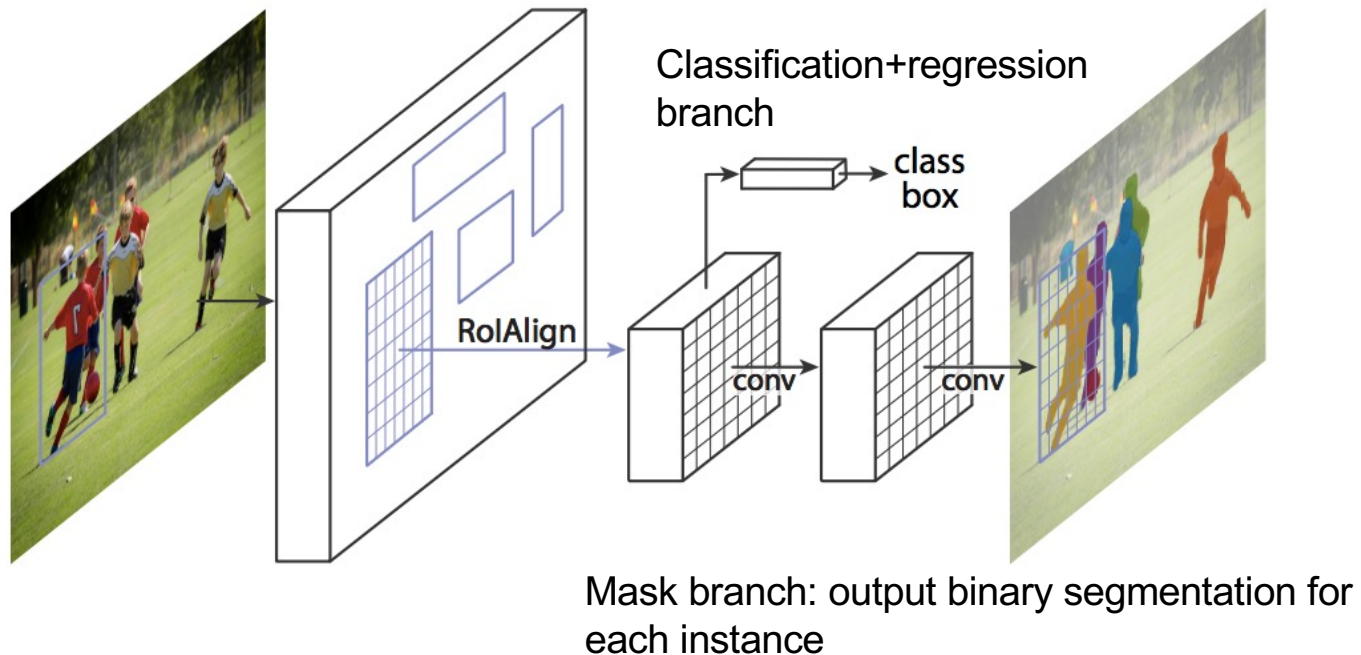
P. Dollar and L. Zitnick, [Structured forests for fast edge detection](#), ICCV 2013

Data-driven edge detection



S. Xie and Z. Tu, [Holistically-nested edge detection](#), ICCV 2015

Most successful approach in practice: Top-down segmentation



K. He, G. Gkioxari, P. Dollar, and R. Girshick, [Mask R-CNN](#), ICCV 2017 (Best Paper Award)

Overview

- Motivating edge detection
- Image gradients
- Derivative of Gaussian filters
- Canny edge detector
- Role of edge detection in image understanding
- **Orientations**

Problem:

Scaling the image scales gradient magnitude

$$f \rightarrow kf \text{ implies } \sqrt{\frac{\partial f^2}{\partial x} + \frac{\partial f^2}{\partial y}} \rightarrow k \sqrt{\frac{\partial f^2}{\partial x} + \frac{\partial f^2}{\partial y}}$$

Which causes problems with thresholds, etc

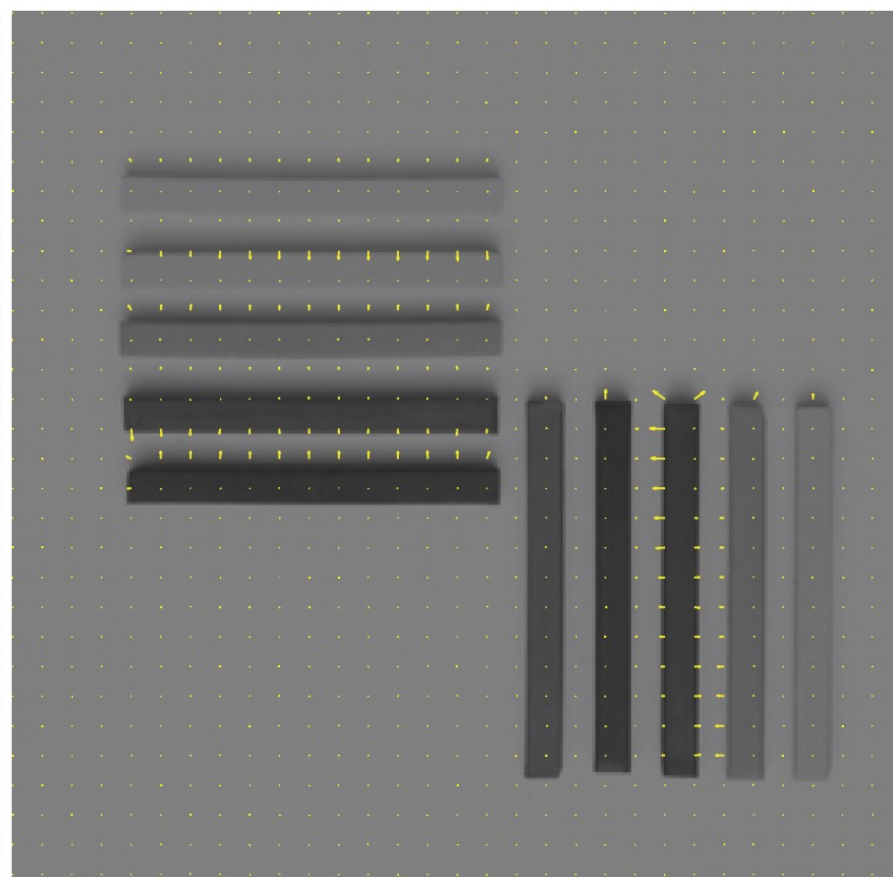
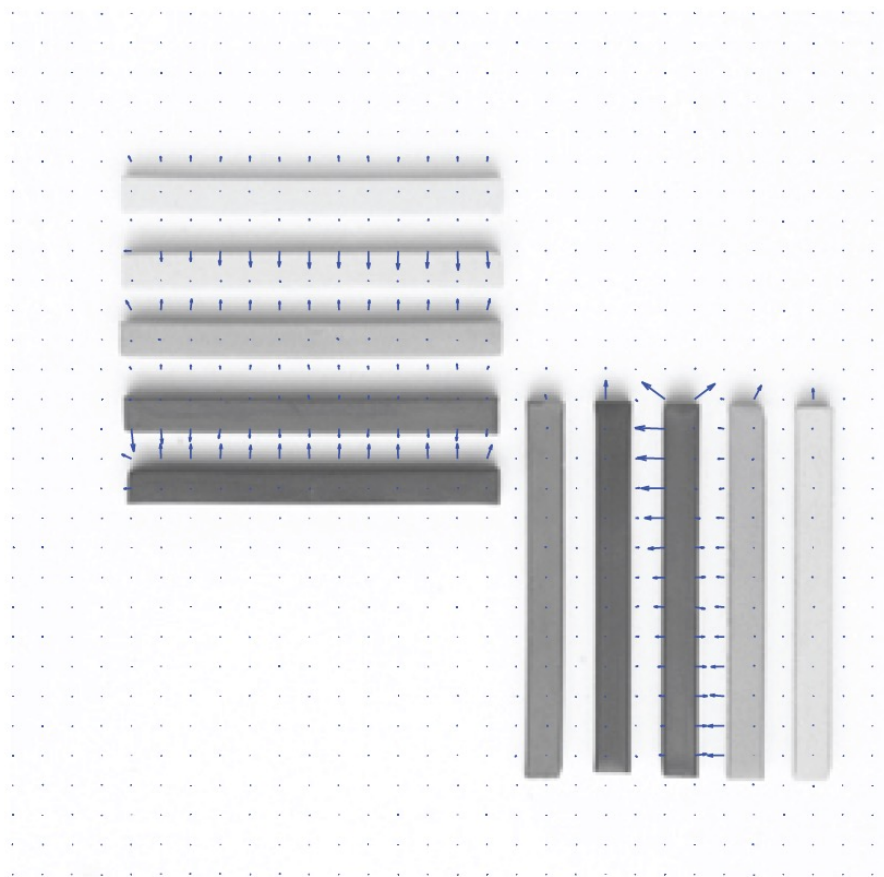
- image gets darker, some edges disappear
- image gets lighter, some edges appear

Hysteresis helps, but doesn't cure

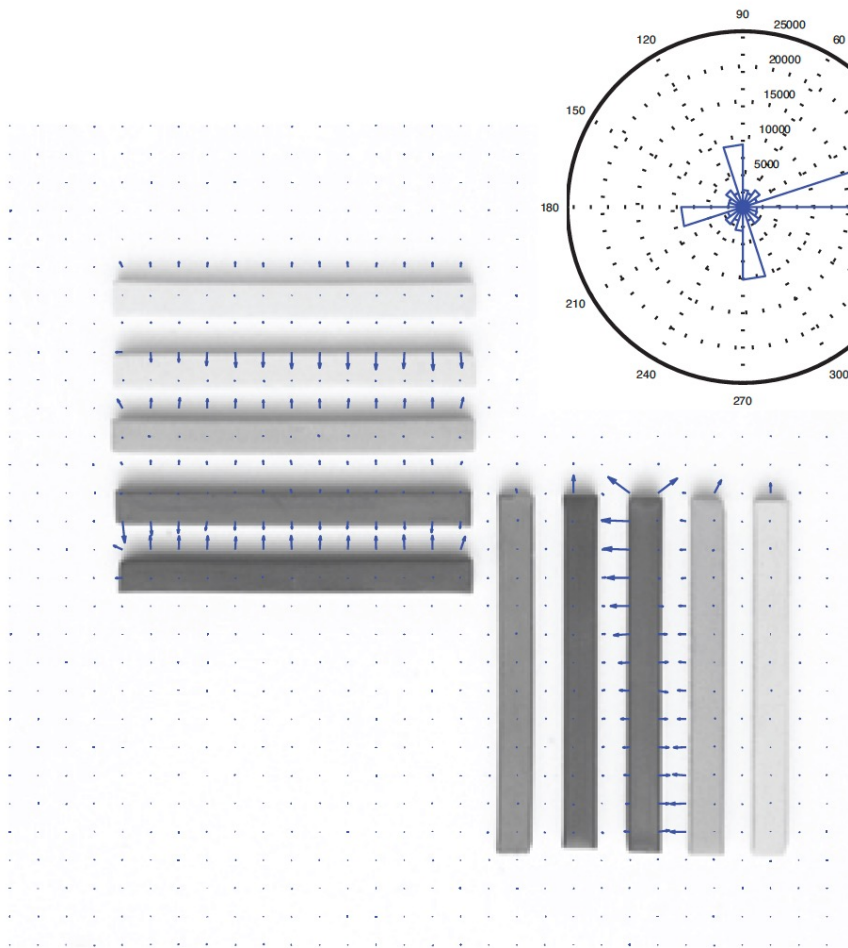
Gradient orientations don't change with illumination

$$f \rightarrow kf \text{ implies } \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) \rightarrow k \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

Q: build a representation out of orientations?

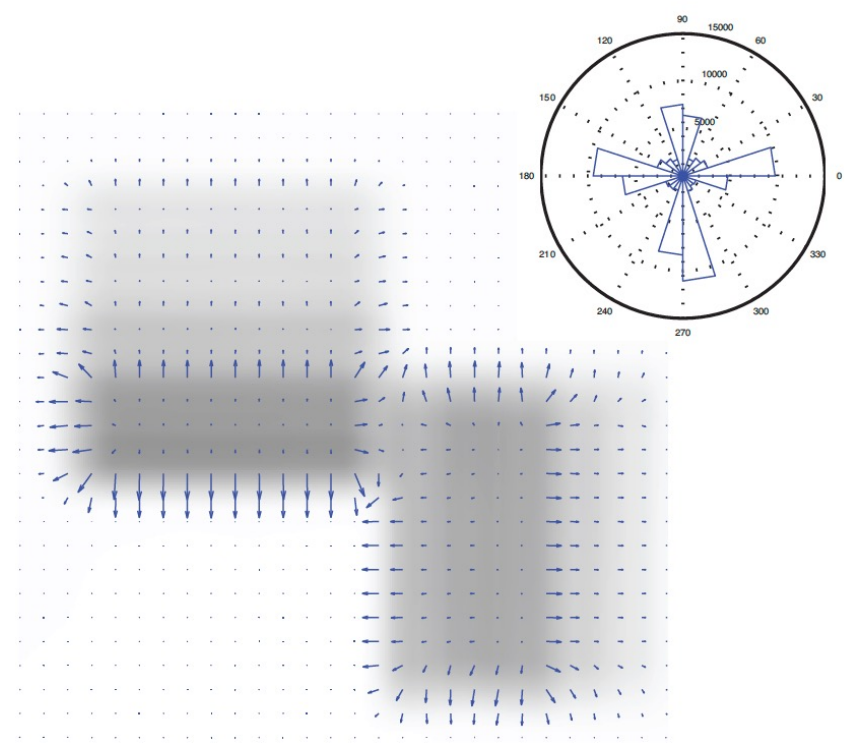
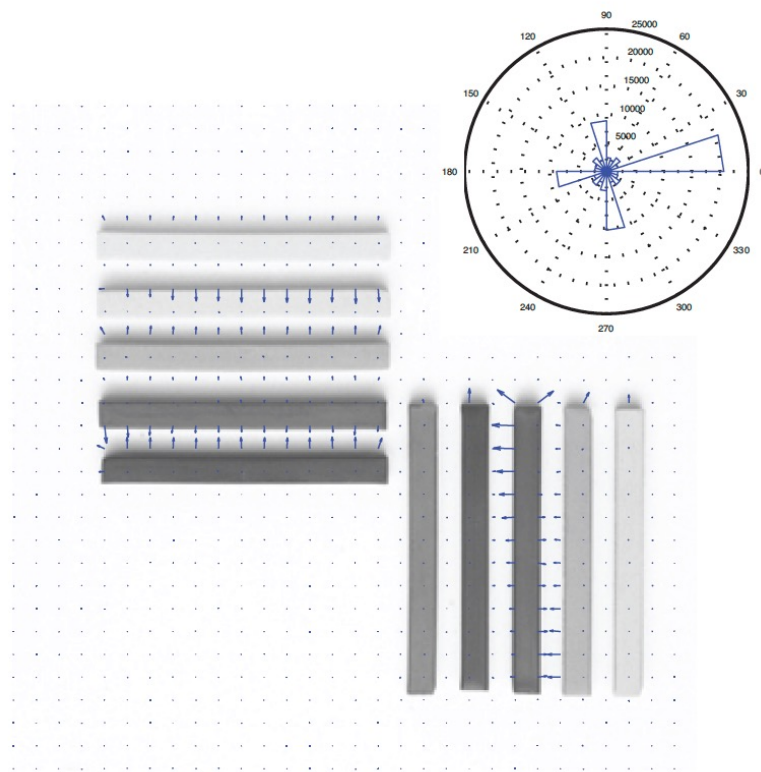


Orientation histograms

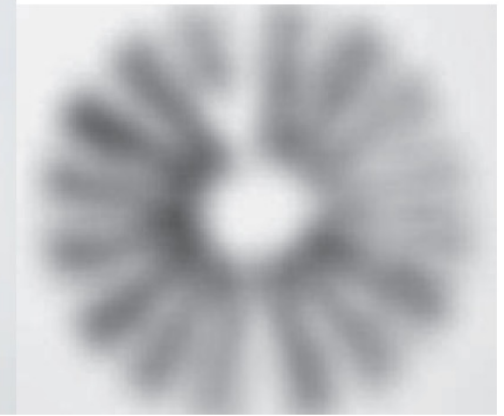
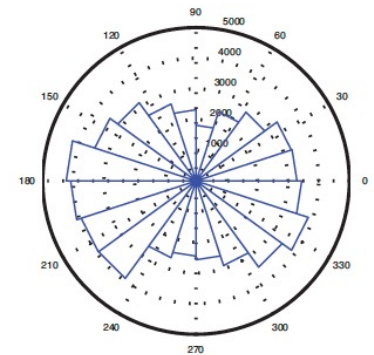
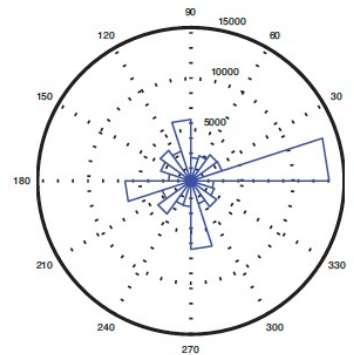
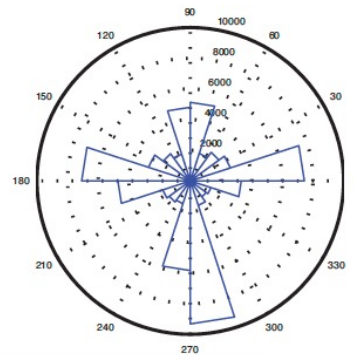
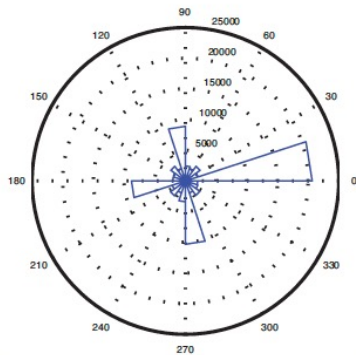


→ Rose plot, which shows number of vectors at each range of angles

Orientation histogram depends on scale



Different patterns have different orientation histograms



Notice something important...

