Camera calibration: Calibration from vanishing points

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Calibration using vanishing points

- Can sometimes calibrate from vanishing points

Source: A. Efros, A. Criminisi

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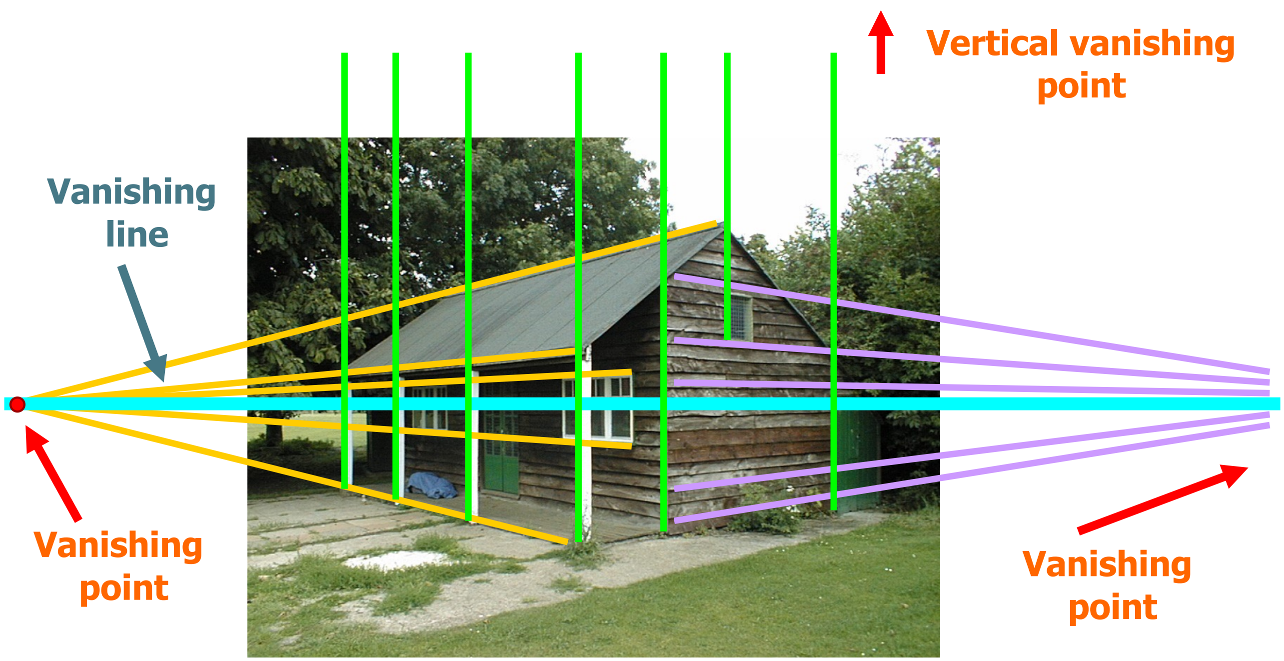
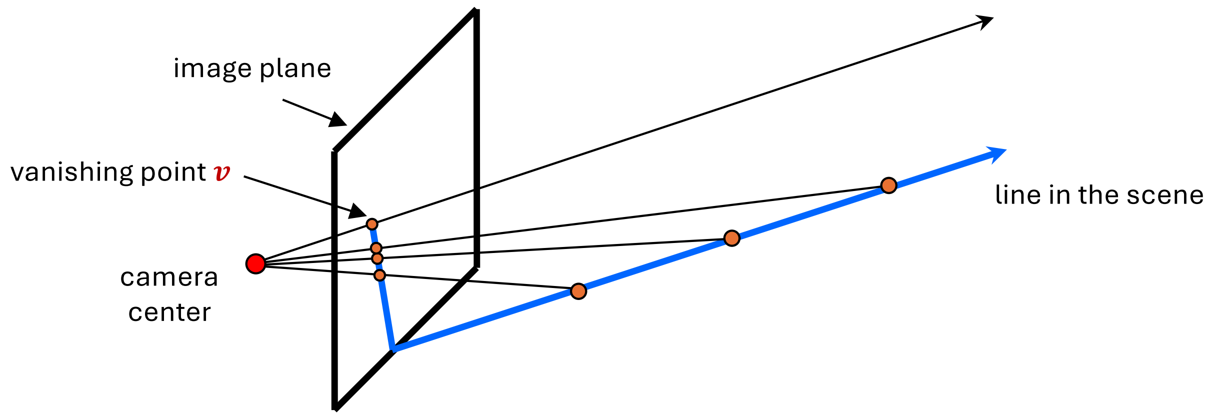
- Can sometimes calibrate from vanishing points



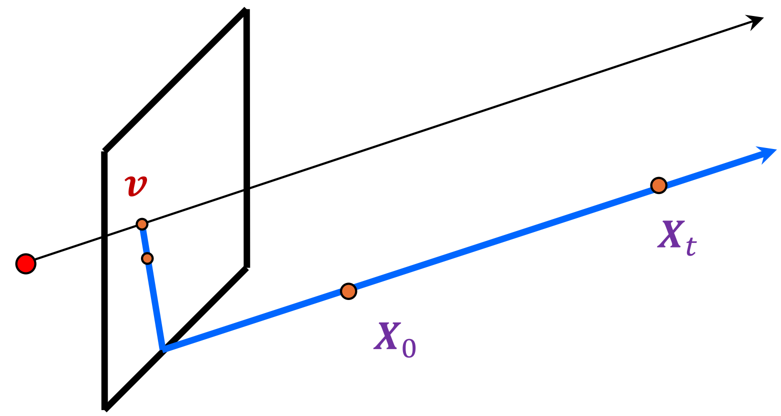
Source: A. Efros, A. Criminisi

Review: Vanishing points



- All lines having the same direction share the same vanishing point

Computing vanishing points



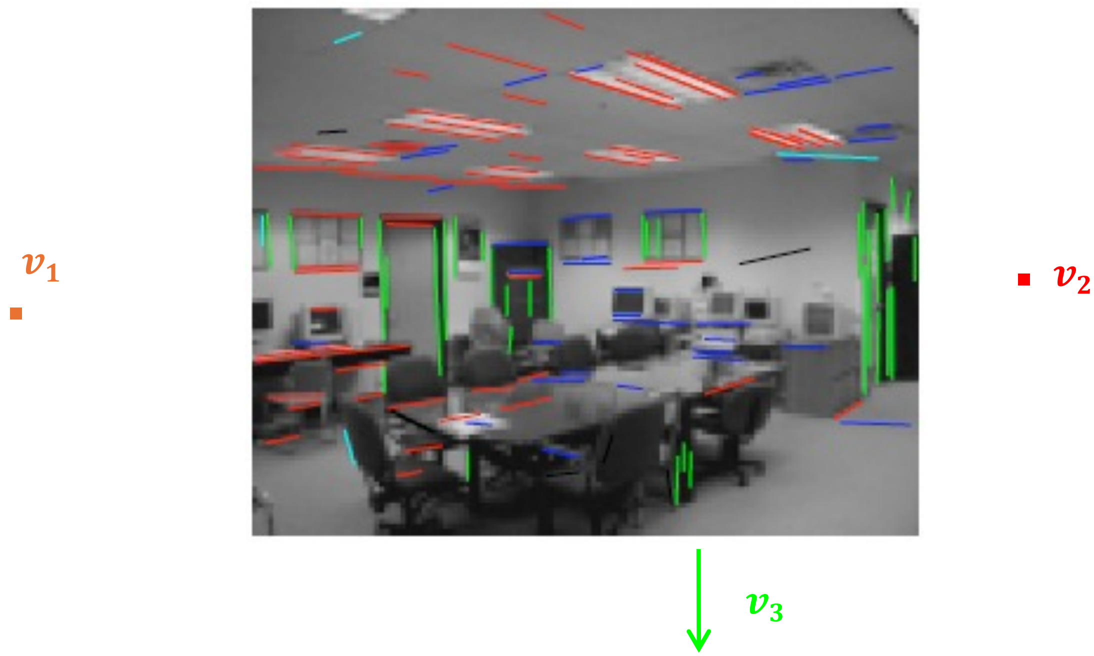
- Line is

$$\boldsymbol{X}_t = \begin{pmatrix} X_0 + tD_1 \\ Y_0 + tD_2 \\ Z_0 + tD_3 \\ 1 \end{pmatrix} \cong \begin{pmatrix} X_0/t + D_1 \\ Y_0/t + D_2 \\ Z_0/t + D_3 \\ 1/t \end{pmatrix} \qquad \boldsymbol{X}_\infty = \begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ 0 \end{pmatrix}$$

- $\boldsymbol{X}_\infty$ is a *point at infinity,* $\boldsymbol{v}$ is its projection: $\boldsymbol{v} \cong \boldsymbol{P}\boldsymbol{X}_\infty$

Calibration from vanishing points

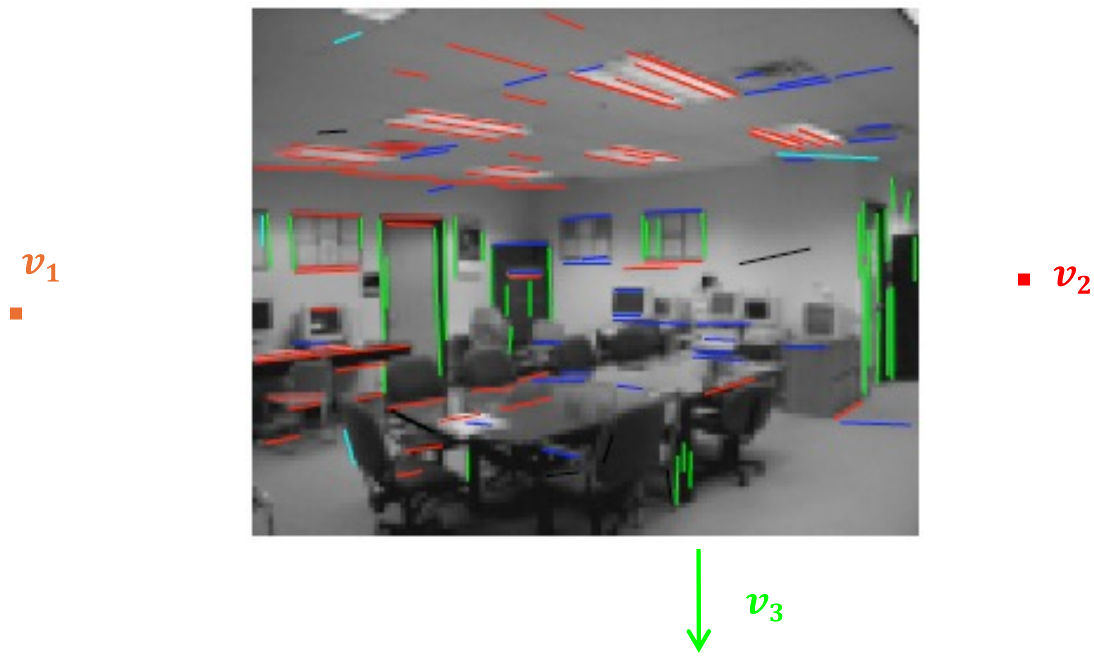
- Consider a scene with three orthogonal vanishing directions:



- Here: $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ are *finite* vanishing points and $\boldsymbol{v}_3$ is an *infinite* vanishing point

Calibration from vanishing points

- Consider a scene with three orthogonal vanishing directions:



- We can align the world coordinate system with these directions

Projection of the world coordinate system

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \boldsymbol{p}_1$$

$$\boldsymbol{p}_1 \quad \boldsymbol{p}_2 \quad \boldsymbol{p}_3 \quad \boldsymbol{p}_4$$

Projection of the world coordinate system

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \boldsymbol{p}_2$$

$$\boldsymbol{p}_1 \quad \boldsymbol{p}_2 \quad \boldsymbol{p}_3 \quad \boldsymbol{p}_4$$

Projection of the world coordinate system

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \boldsymbol{p}_3$$

$$\underbrace{\boldsymbol{p}_1 \quad \boldsymbol{p}_2 \quad \boldsymbol{p}_3}_{} \quad \boldsymbol{p}_4$$

Vanishing points in x, y, z directions (i.e., $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, $\boldsymbol{v}_3$)

Projection of the world coordinate system

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \boldsymbol{p}_4$$

$\boldsymbol{p}_1 \quad \boldsymbol{p}_2 \quad \boldsymbol{p}_3 \quad \boldsymbol{p}_4$

Vanishing points in x, y, z directions (i.e., $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$)

Projection of world origin

- Problem: this only gives us the four columns up to independent scale factors, additional constraints needed to solve for them

Calibration from vanishing points

- Align the world coordinate system with three orthogonal vanishing directions in the scene:

$$\boldsymbol{v}_i \cong \boldsymbol{K}[\boldsymbol{R}|\boldsymbol{t}]\begin{pmatrix}\boldsymbol{e}_i \\ 0\end{pmatrix}$$

$$\boldsymbol{v}_i \cong \boldsymbol{K}\boldsymbol{R}\boldsymbol{e}_i$$

$$\boldsymbol{e}_i \cong \boldsymbol{R}^T\boldsymbol{K}^{-1}\boldsymbol{v}_i$$

$$\boldsymbol{e}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \boldsymbol{e}_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \boldsymbol{e}_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

Calibration from vanishing points

- Align the world coordinate system with three orthogonal vanishing directions in the scene:

$$\boldsymbol{v}_i \cong \boldsymbol{K}\boldsymbol{R}\boldsymbol{e}_i$$

$$\boldsymbol{e}_i \cong \boldsymbol{R}^T\boldsymbol{K}^{-1}\boldsymbol{v}_i$$

- Orthogonality constraint: $\boldsymbol{e}_i^T\boldsymbol{e}_j = 0$

$$\boldsymbol{v}_i^T\boldsymbol{K}^{-T}\boldsymbol{R}\boldsymbol{R}^T\boldsymbol{K}^{-1}\boldsymbol{v}_j = 0$$

$$\boldsymbol{v}_i^T\boldsymbol{K}^{-T}\boldsymbol{K}^{-1}\boldsymbol{v}_j = 0$$

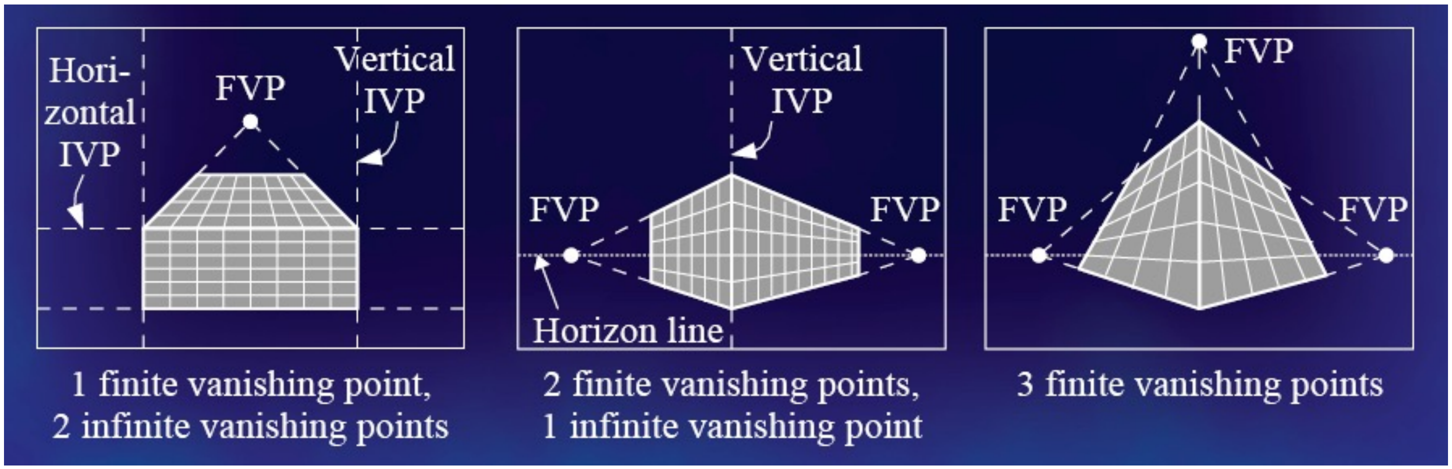
Constraint on K,
- Free with three vanishing points

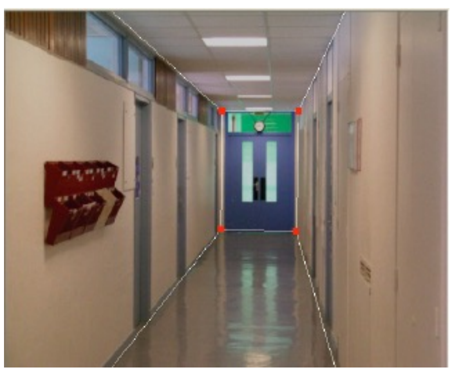
Calibration from vanishing points

- $\boldsymbol{v}_i^T \boldsymbol{K}^{-T} \boldsymbol{K}^{-1} \boldsymbol{v}_j = 0$

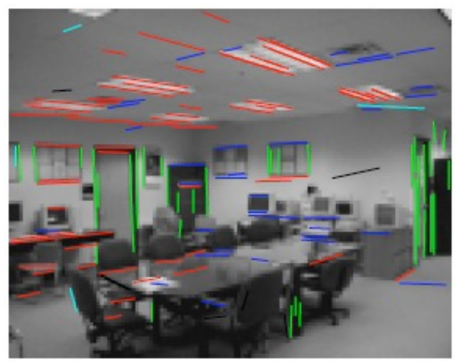
- Three constraints (one per pair)
- $\boldsymbol{K}$ (simplified) has three parameters
  - f, p_x, p_y
    - you can’t get aspect ratio or skew
- Complications:
  - constraints are nonlinear, but it’s not hard to do the algebra
  - At least two *finite* vanishing points are needed to solve for both focal length and principal point

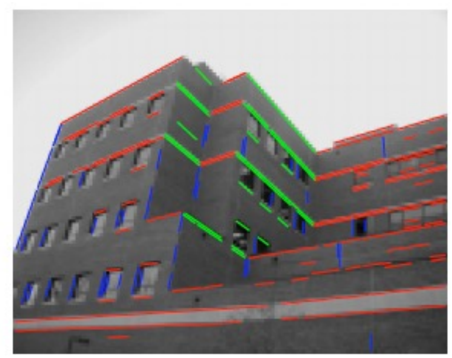
Calibration from vanishing points





Cannot recover focal length, principal point is the third vanishing point





Can solve for focal length, principal point

Rotation from vanishing points

- Constraints on vanishing points: $\boldsymbol{v}_i \cong \boldsymbol{K}\boldsymbol{R}\boldsymbol{e}_i$
- We just used orthogonality constraints to solve for $\boldsymbol{K}$
- Now we have:

•$\boldsymbol{K}^{-1}\boldsymbol{v}_i \cong \boldsymbol{R}\boldsymbol{e}_i$

•Notice: $\boldsymbol{R}\boldsymbol{e}_1 = [\boldsymbol{r}_1 \quad \boldsymbol{r}_2 \quad \boldsymbol{r}_3]\begin{pmatrix}1\\0\\0\end{pmatrix} = \boldsymbol{r}_1$

•Thus, $\boldsymbol{r}_i \cong \boldsymbol{K}^{-1}\boldsymbol{v}_i$

- The scale ambiguity goes away since we require $\|\boldsymbol{r}_i\|^2 = 1$

Calibration from vanishing points: Summary

1. Solve for intrinsic parameters (focal length, principal point) using three orthogonal vanishing points
2. Get extrinsic parameters (rotation) directly from vanishing points once calibration matrix is known

- Advantages
  - No need for calibration chart, 2D-3D correspondences
  - Could be completely automatic
- Disadvantages
  - Only applies to certain kinds of scenes
  - It is tricky to accurately localize vanishing points
  - Need at least two finite vanishing points