Calibrating Cameras

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Calibration

- Procedure
  - View N known reference points in 3D
  - Obtain locations in image plane
  - Produce
    - Camera intrinsics and extrinsics
- Why
  - recovering intrinsics for future use
  - where is an object/camera in the camera/world frame?
- How
  - Optimization: minimize reprojection error

Notation

The optimization problem is relatively straightforward to formulate. Notation is the main issue. We have N reference points $\mathbf{s}_i = [s_{1,i}, s_{2,i}, s_{3,i}]$ with known position in some reference coordinate system in 3D. The measured location in the image for the i'th such point is $\hat{\mathbf{t}}_i = \left[\hat{t}_{1,i}, \hat{t}_{2,i}\right]$. There may be measurement errors, so the $\hat{\mathbf{t}}_i = \mathbf{t}_i + \xi_i$, where ξ_i is an error vector and $\mathbf{t}_i$ is the unknown true position of the image point. We will assume the magnitude of error does not depend on direction in the image plane (it is *isotropic*), so it is natural to minimize the squared magnitude of the error

$$\sum_i \xi_i^T \xi_i. \tag{31.1}$$

This error is *reprojection error*. The main issue here is writing out expressions for ξ_i in the appropriate coordinates. Write $\mathcal{K}$ for the intrinsic matrix whose u, v'th component will be k_{uv}, and recall this matrix is upper triangular and the bottom right element is 1. Write $\mathcal{T}_e$ for the extrinsic transformation, whose u, v'th component will be e_{uv}. Write

Predicting the points

- Reference points transformed by extrinsics

$$
\begin{aligned}
g_{1,i} &= e_{11}s_{1,i} + e_{12}s_{2,i} + e_{13}s_{3,i} + e_{14} \\
g_{2,i} &= e_{21}s_{1,i} + e_{22}s_{2,i} + e_{23}s_{3,i} + e_{24} \\
g_{3,i} &= e_{31}s_{1,i} + e_{32}s_{2,i} + e_{33}s_{3,i} + e_{34}
\end{aligned}
$$

- Now apply camera transformation and intrinsics

$$
\begin{aligned}
p_{1,i} &= \frac{k_{11}g_{1,i} + k_{12}g_{2,i} + k_{13}g_{3,i}}{g_{3,3}} \\
p_{2,i} &= \frac{k_{22}g_{1,i} + k_{23}g_{3,i}}{g_{3,i}}.
\end{aligned}
$$

Optimization problem

The reprojection error is then

$$\sum_i \xi_i^T \xi_i = \sum_i (t_{1,i} - p_{1,i})^2 + (t_{2,i} - p_{2,i})^2 \tag{31.2}$$

This is a constrained optimization problem, because $\mathcal{T}_e$ is a Euclidean transformation. The constraints here are

$$1 - \sum_v e_{1v}^2 = 0 \text{ and } 1 - \sum_v e_{2v}^2 = 0 \text{ and } 1 - \sum_v e_{3v}^2 = 0$$
$$\sum_v e_{1v} e_{2v} = 0 \text{ and } \sum_v e_{1v} e_{3v} = 0 \text{ and } \sum_v e_{2v} e_{3v} = 0 \qquad .$$

Strategies

- Chuck it into an optimizer and run
  - AAARGH!
- Find a good start point
  - all works out
- Variants
  - Different point locations; points on plane; etc., etc.