Camera Calibration: Start point

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Camera matrix from linear system

Write $\mathbf{C}_j^T$ for the j'th row of the camera matrix, and $\mathbf{S}_i = [s_{1,i}, s_{2,i}, s_{3,i}, 1]^T$ for homogeneous coordinates representing the i'th point in 3D. Then, assuming no errors in measurement

$$\hat{t}_{1,i} = \frac{\mathbf{C}_1^T \mathbf{S}_i}{\mathbf{C}_3^T \mathbf{S}_i} \text{ and } \hat{t}_{2,i} = \frac{\mathbf{C}_2^T \mathbf{S}_i}{\mathbf{C}_3^T \mathbf{S}_i}, \tag{31.3}$$

which you can rewrite as

$$\mathbf{C}_3^T \mathbf{S}_i \hat{t}_{1,i} - \mathbf{C}_1^T \mathbf{S}_i = 0 \text{ and } \mathbf{C}_3^T \mathbf{S}_i \hat{t}_{2,i} - \mathbf{C}_2^T \mathbf{S}_i = 0. \tag{31.4}$$

x component of location of i'th point in image

One match gives two linearly independent constraints on the camera matrix

Camera matrix from linear system

- N points gives 2N homogeneous equations

$$\begin{pmatrix} -\mathbf{S}_1^T & 0 & \mathbf{S}_1^T t_{x,1} \\ 0 & -\mathbf{S}_1^T & \mathbf{S}_1^T t_{y,1} \\ \dots & \dots & \dots \\ -\mathbf{S}_N^T & 0 & \mathbf{S}_N^T t_{x,N} \\ 0 & -\mathbf{S}_N^T & \mathbf{S}_N^T t_{y,N} \end{pmatrix} \begin{pmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \\ \mathbf{C}_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 0 \end{pmatrix}$$

- The camera matrix is 3 x 4 but scale doesn't matter so there are
- 11 degrees of freedom – we can estimate it with 6 points

Solving

- You have N homogeneous eqns in M<N unknowns

$$\mathcal{M}\mathbf{c} = \mathbf{0}$$

- Solve by minimizing

$$\mathbf{c}^T \mathcal{M}^T \mathcal{M}\mathbf{c}$$

  - subject to

$$\mathbf{c}^T\mathbf{c} = 1$$

Eigenvalue problem:

c is eigenvector of M^T M with smallest eigenvalue

Camera matrix from linear system

- Final linear system:

$$\begin{pmatrix} -\mathbf{S}_1^T & 0 & \mathbf{S}_1^T t_{x,1} \\ 0 & -\mathbf{S}_1^T & \mathbf{S}_1^T t_{y,1} \\ \cdots & \cdots & \cdots \\ -\mathbf{S}_N^T & 0 & \mathbf{S}_N^T t_{x,N} \\ 0 & -\mathbf{S}_N^T & \mathbf{S}_N^T t_{y,N} \end{pmatrix} \begin{pmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \\ \mathbf{C}_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 0 \end{pmatrix}$$

- What if all the n 3D points are *coplanar*,
  - i.e., there exists a set of line parameters $\boldsymbol{\Pi}^T = (a, b, c, d)^T$ such that $\boldsymbol{\Pi}^T \boldsymbol{X}_i = 0$ for all i?
  - Then we will get *degenerate solutions* $(\boldsymbol{\Pi}, \mathbf{0}, \mathbf{0})$, $(\mathbf{0}, \boldsymbol{\Pi}, \mathbf{0})$, or $(\mathbf{0}, \mathbf{0}, \boldsymbol{\Pi})$

Now factor, and get start point

Procedure: 30.1 *Decomposing a general projective camera matrix*

Given a 3×4 camera matrix $\mathcal{C}$ with rank 3, decompose into

$$\mathcal{T}_i \mathcal{C}_p \mathcal{T}_e$$

as follows. Write $\mathcal{C} = [\mathcal{S} \mid \mathbf{p}]$. Now decompose $\mathcal{S}$ into an upper triangular matrix $\mathcal{U}$ and a rotation matrix $\mathcal{R}$. Then

$$\mathcal{T}_i = (1/u_{33})\mathcal{U} \text{ and } \mathcal{T}_e = \begin{bmatrix} \mathcal{R} & \mathcal{T}_i^{-1}\mathbf{p} \\ \mathbf{0}^T & 1 \end{bmatrix}$$

You must use reprojection error

- You might think the start point is good enough
  - it isn’t – it minimizes the wrong error
    - Q: what does the smallest eigenvalue mean?
    - A: who knows?
- If you minimize reprojection error, you’re minimizing something meaningful