Estimating the fundamental matrix

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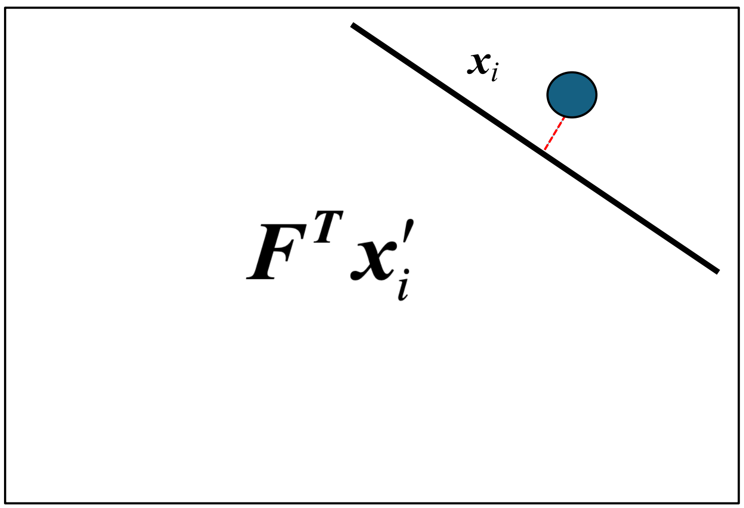
Follows the recipe for fitting

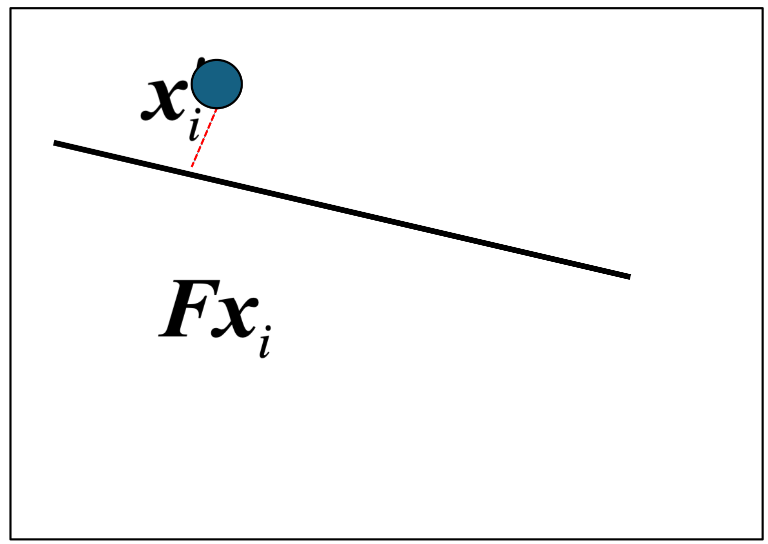
- Find some correspondences
- Set up an optimization problem
- Construct a start point

- In this case, the start point is usually quite good
    - one sometimes doesn’t polish it

The optimization problem

- Minimize sum of squared *geometric* distances





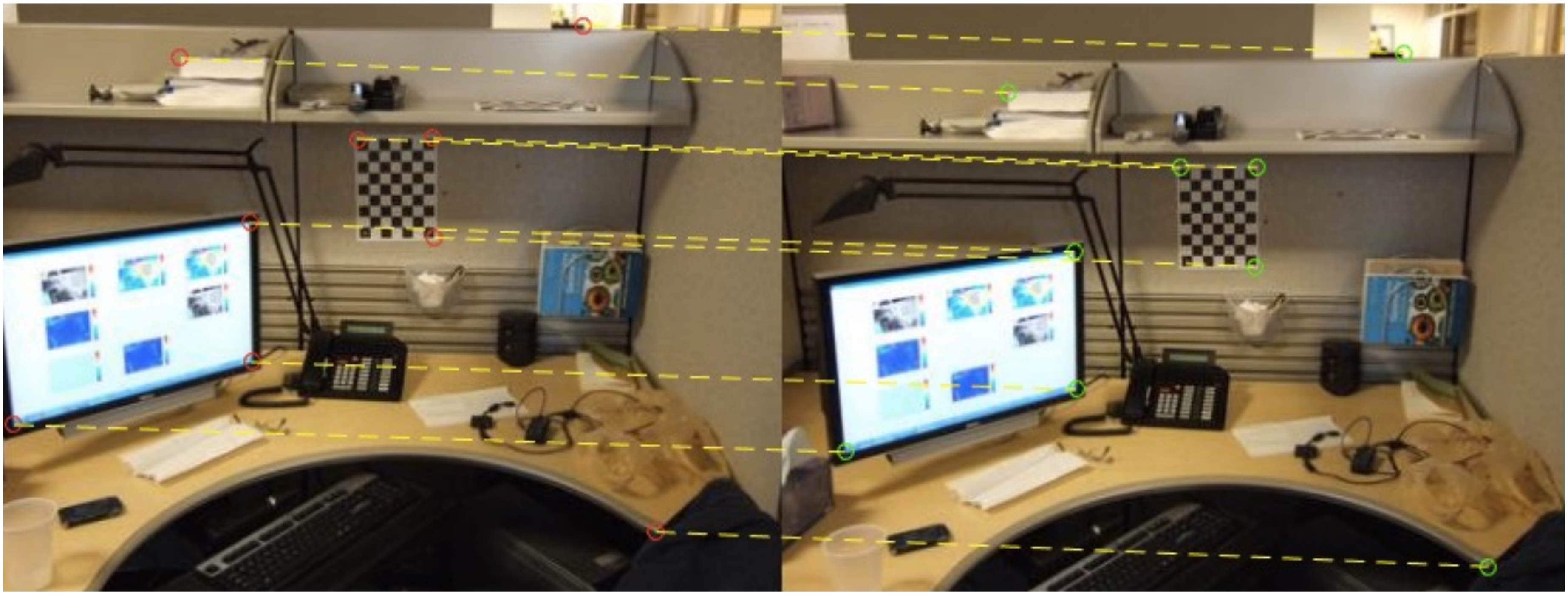
$$\sum_i [\mathrm{dist}(\boldsymbol{x}_i', \boldsymbol{F}\boldsymbol{x}_i)^2 + \mathrm{dist}(\boldsymbol{x}_i, \boldsymbol{F}^T\boldsymbol{x}_i')^2]$$

This is constrained

$$\det(F) = 0$$

Obtaining a start point

- correspondences:
  - $\boldsymbol{x} = (x, y, 1)^T$ and $\boldsymbol{x}' = (x', y', 1)^T$



Obtaining a start point

- correspondences:
  - $\boldsymbol{x} = (x, y, 1)^T$ and $\boldsymbol{x}' = (x', y', 1)^T$
- Constraint: $\boldsymbol{x}'^T \boldsymbol{F} \boldsymbol{x} = 0$

$$(x', y', 1) \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0 \quad \Rightarrow \quad (x'x, x'y, x', y'x, y'y, y', x, y, 1) \begin{pmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{pmatrix} = 0$$

The eight point algorithm

$$\underbrace{\begin{bmatrix} & & & \vdots & & & & & \\ x'x & x'y & x' & y'x & y'y & y' & x & y & 1 \\ & & & \vdots & & & & & \end{bmatrix}}_{\boldsymbol{U}} \begin{pmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{pmatrix} = \mathbf{0}$$

- Homogeneous least squares to find $\boldsymbol{f}$:

$$\arg\min_{\|\boldsymbol{f}\|=1} \|\boldsymbol{U}\boldsymbol{f}\|_2^2$$

➡ Eigenvector of $\boldsymbol{U}^T\boldsymbol{U}$ with smallest eigenvalue

Enforcing rank-2 constraint

- $\boldsymbol{F}$ needs to be singular/rank 2.
  - Enforce this: take SVD of the initial estimate and throw out the smallest singular value

$$\boldsymbol{F}_{\text{init}} = \boldsymbol{U\Sigma V}^T$$

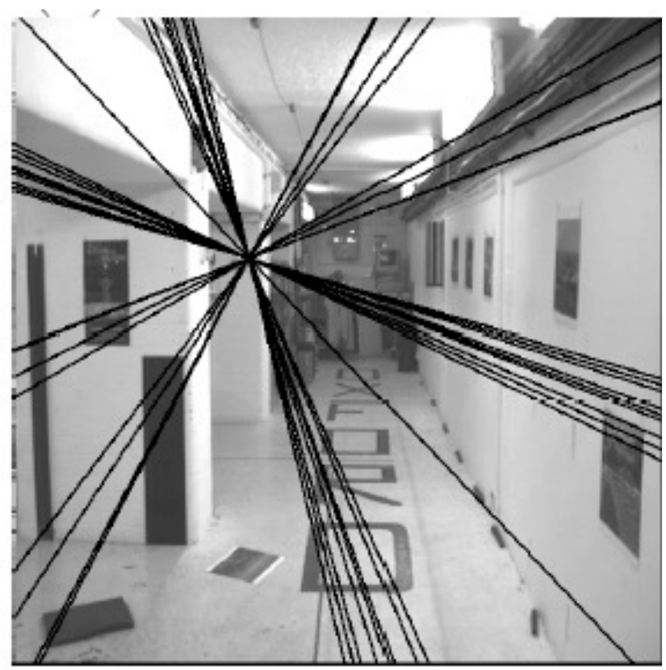
$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \longrightarrow \boldsymbol{\Sigma}' = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{F} = \boldsymbol{U\Sigma' V}^T$$

Enforcing rank-2 constraint

Initial $\boldsymbol{F}$ estimate

Rank-2 estimate



Normalized eight point algorithm

- $$\overset{10^6}{} \quad \overset{10^6}{} \quad \overset{10^3}{} \quad \overset{10^6}{} \quad \overset{10^6}{} \quad \overset{10^3}{} \quad \overset{10^3}{} \quad \overset{10^3}{} \quad \overset{1}{}$$
$$\underbrace{\begin{bmatrix} x'x & x'y & x' & y'x & y'y & y' & x & y & 1 \\ & & & \vdots & & & & & \end{bmatrix}}_{\boldsymbol{U}} \begin{pmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{pmatrix} = \mathbf{0}$$

- Recall that x, y, x', y' are pixel coordinates. What might be the order of magnitude of each column of $\boldsymbol{U}$?
- This causes numerical instability!

The normalized eight-point algorithm

- In each image, center the set of points at the origin, and scale it so the mean squared distance between the origin and the points is 2 pixels
- Use the eight-point algorithm to compute $\boldsymbol{F}$ from the normalized points
- Enforce the rank-2 constraint
- Transform fundamental matrix back to original units: if $\boldsymbol{T}$ and $\boldsymbol{T}'$ are the normalizing transformations in the two images,
  then the fundamental matrix in original coordinates is $\boldsymbol{T}'^{T}\boldsymbol{F}\boldsymbol{T}$

R. Hartley. In defense of the eight-point algorithm. TPAMI 1997

Comparison of estimation algorithms

	8-point	Normalized 8-point	Nonlinear least squares
Av. Dist. 1	2.33 pixels	0.92 pixel	0.86 pixel
Av. Dist. 2	2.18 pixels	0.85 pixel	0.80 pixel

Seven-point algorithm

- Set up least squares system with seven pairs of matches and solve for null space (two vectors) using SVD
- Solve for polynomial equation to get coefficients of linear combination of null space vectors that satisfies $\det(\boldsymbol{F}) = 0$

Source: e.g., M. Pollefeys tutorial (2000)