Taking curves into account:

- we have \( K \) curves, each has \( n \) faces.
- each face is a "point".
- particularly interesting cases:
  - faces that are tangent (ish) to many convexes
  - faces that are "far" from other faces

Examples of \( I \), 2D

\[ \text{\#-\#} \text{ are the convexes} \]
\[ \text{/} \text{ are bitangent planes} \]
notice that for the first geometry, no face tangent to $\Sigma$; second, there is one. This difference is worth knowing about.

Examples of $\Pi$:

This pair of planes is interesting.

As is this
we need to extend the graph construction.

there are multiple pairs like this...

there is one example construction...

- Think of each convex as a "bag" of points (= planes)
- Each convex has a different color
- Choose a distance $d$
represent the axes with a collection of sets.

\[ K = \{ \Sigma c_1, \ldots, c_{a-3}, \Sigma c_6, \ldots, c_{bs3}, \ldots \} \]

where \[ S = \Sigma c_1, c_2 \ldots c_{a-3} \] is in \( K_a \)

iff \[ \min_{p_i \in c_i, p_j \in c_j} d(p_i, p_j) \leq \alpha \] for all \( i, j \).

C.equivalently, \( c_1, \ldots, c_r \) are in \( S \) if they "almost" share a plane.

Q: What is the resulting object?

Q: How to represent it?
Abstract simplicial complexes

A collection $A$ of subsets of a set $A$ is an abstract simplicial complex if every $\sigma \in A$ has all subsets $\sigma \subseteq \sigma' \in A$. The elements of $A$ are the vertices of $A$. Each set in $A$ is a simplex whose dimension is its cardinality.

Eg: $A = \{ r, g, b, c, m, y \}$

$A = \{ e_r g, e_r g_b, e_g b, e_r g_b, e_g b, e_r g_b \}$

\[ \text{"vertex"} \quad \text{edge} \quad \text{face} \]
An Analogy with Simplicial Complex

- $k$ simplex is hull $(k+1)$ affine (indep. points).
- $0$ simplex $\equiv$ vertex
- $1$ simplex $\equiv$ edge
- $2$ simplex $\equiv$ triangle.
- A face of a simplex is convex hull of a non-empty subset of pts.
- A Simplicial Complex $K$ is a finite set of simplices such that
K contains every face of every simplex of K

- for \( \sigma, \tau \in K \), either \( \sigma \cup \tau = \emptyset \) or one is a face of \( \sigma \) and of \( \tau \)

← simplicial complex

← NO
The graph construction (above) was a family of simplicial complexes.

- Assume vertices go in only when connected to an edge.

Here:

\[ \phi \in = K \circ C \circ K' \cdots K'x \]

\[ \text{no edges} \]

- Increase dist a bit
- Insert one edge

Q: how can we describe such a complex?
Simple + Topological

cf. graphs.

This complex has a cycle that is not a boundary.

This has at least two

3D pix, too
(2) An \( n \)-chain is a weighted sum of \( n \)-simplexes. We will use \( \text{mod 2} \) weights (but others are possible).

The boundary of a \( k \)-simplex is the sum of all \( k-1 \) faces.

\[ \text{[need to adjust this for \( \text{weights not \ mod 2} \)}} \]

Examples:

\[ \begin{array}{c}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{array} \]

\[ \begin{array}{c}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{array} \]
The boundary operator is **linear** (mod 2 helps here!)

1-chain

2-chain

This one disappears (mod 2!)
Notice that

\[ \partial_k \text{ takes } k\text{-chains to } (k-1)\text{ chains} \]

AND

\[ \partial_{k-1} \circ \partial_k = 0 \]

(The boundary of a boundary is empty)

CHECK THIS:

Now consider:

\[ \text{This has } \partial_1 = 0 \]

BUT it isn't a boundary.
We define

\[ H_k = \text{(space of } k\text{-chains } \sigma) \, \text{s.t. } \partial_k(\sigma) = 0 \, \text{over } \mathbb{Z}/2\mathbb{Z} \mod 2 \) \]

This is the \( k \)-th simplicial homology group (over \( \mathbb{Z}/2\mathbb{Z} \mod 2 \)).

\[ B_k = \dim (H_k) = \# \text{ of linearly independent generators} \]

\( k \)-th Betti number
we have already seen $B_0$, $B$, in action.

Examples in Figures

There are

Reps of $\mathbf{CV}_n$ not objects

- compute Abstract Simplicial complex as above
- compute $B_0$, $B_k$ for various $n$'s
- construct barcode.