Conservation of energy by particles

~~Notice~~ Write

$$E = \frac{1}{2} m(\underline{v} \cdot \underline{v}) + \phi(x)$$

↑ Kinetic ↑ Potential Energy

Now:

$$\frac{dE}{dt} = m\left(\underline{v} \cdot \frac{d\underline{v}}{dt}\right) + (\nabla\phi) \cdot \frac{d\underline{x}}{dt}$$

$$= m(\underline{v} \cdot \underline{a}) + \nabla\phi \cdot \underline{v}$$

$$= \underline{v} \cdot (m\underline{a} + \nabla\phi)$$

but $\nabla\phi = -m\underline{a}$

so $\frac{dE}{dt} = 0$