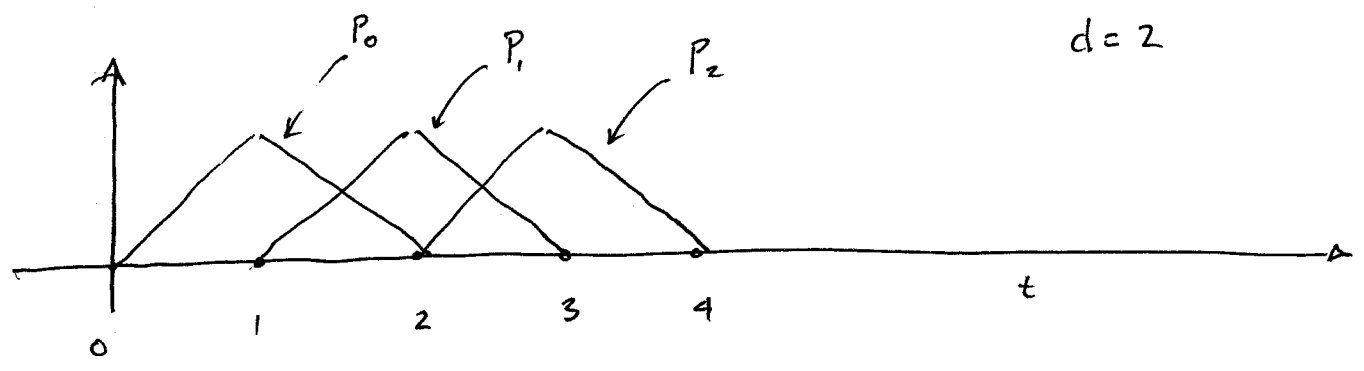
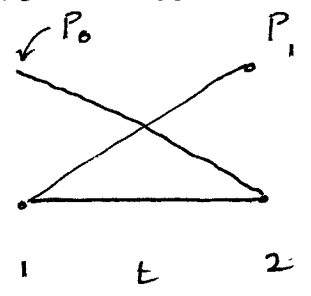


B-Splines and subdivision

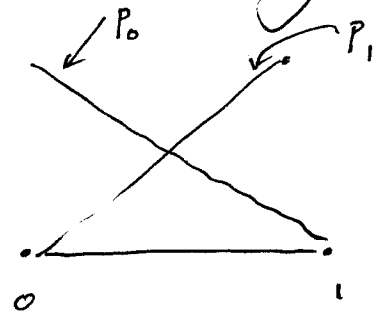
• Assume we have uniform knots



• So if we look at one interval

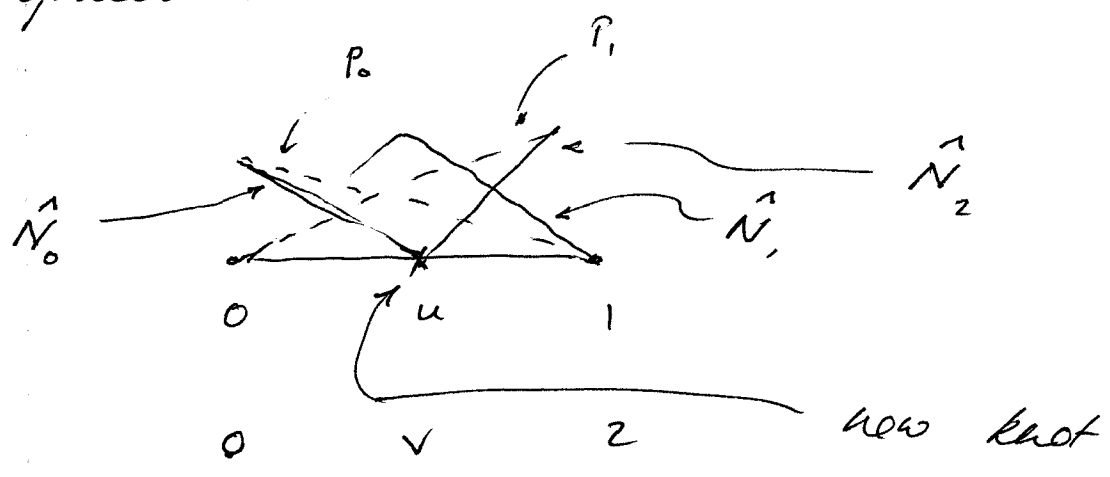


• Reparametrize by $u = t - 1$



Now, we will write another parameter,

$v = 2u$, and make a new set of B splines, w/ uniformly spaced knots



now

$$P_0 N_0 = \frac{1}{2} (2 \hat{N}_0 + \hat{N}_1)$$

$$N_1 = \frac{1}{2} (\hat{N}_1 + 2 \hat{N}_2)$$

(by inspection)

so

$$P_0 N_0 + P_1 N_1$$

$$= P_0 \hat{N}_0 + \frac{1}{2} (P_0 + P_1) \hat{N}_1 + P_1 \hat{N}_2$$

③

We have subdivision

• a $d=2$ spline with cp's

$$P_0, P_1, P_2 \dots P_n$$

is exactly reproduced by a $d=2$

spline with cp's

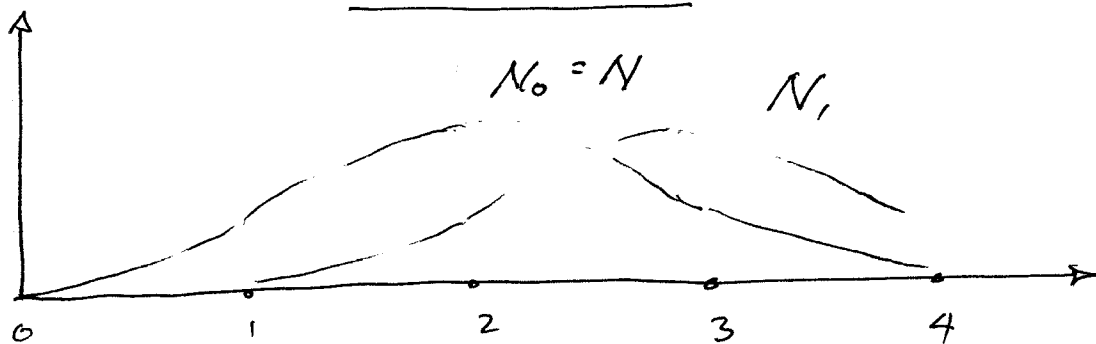
$$P_0, \frac{P_0 + P_1}{2}, P_1 \dots P_{n-1}, \frac{P_{n-1} + P_n}{2}, P_n$$

• Not v. interesting in this form, but works for higher degree:

• $d=4$ is most interesting case

Cubic case

(4)



$$N(t) = \begin{cases} \frac{1}{6} t^3 & t < 1 \\ -\frac{1}{2} t^3 + 2t^2 - 2t + \frac{2}{3} & 1 \leq t < 2 \\ \frac{1}{2} t^3 - 4t^2 + 10t - \frac{22}{3} & 2 \leq t < 3 \\ -\frac{1}{6} t^3 + 2t^2 - 8t + \frac{32}{2} & 3 \leq t < 4 \end{cases}$$

Now

$$\begin{aligned} & N_0 p_0 + N_1 p_1 + N_2 p_2 + N_3 p_3 \\ &= p_0 N(t+3) + p_1 N(t+2) + p_2 N(t+1) \\ & \quad + p_3 N(t) \end{aligned}$$

(Notice how the shift works!)

(5)

Now it is straight forward to verify that

$$N(t) = \frac{1}{8} N(2t) + \frac{1}{2} N(2t-1) + \frac{3}{4} N(2t-2) \\ + \frac{1}{2} N(2t-3) + \frac{1}{8} N(2t-4)$$

(Wade through all the cases!).

Now, we have a curve

$$\frac{p}{3} N(t) + \frac{p}{2} N(t+1) + \frac{p}{1} N(t+2) \\ + \frac{p}{0} N(t+3)$$

We want to write as

$$q(t) \text{ such that } q(2t) = p(t) \\ \text{for } t = [0 \dots 1/2] \\ (\text{this is first half})$$

6

$$p_0 N(t+3) = p_0 \left[\frac{1}{8} N(2t+6) + \frac{1}{2} N(2t+5) + \frac{3}{4} N(2t+4) \right. \\ \left. + \frac{1}{2} N(2t+3) + \frac{1}{8} N(2t+2) \right]$$

$$p_1 N(t+2) = p_1 \left[\frac{1}{8} N(2t+4) + \frac{1}{2} N(2t+3) + \frac{3}{4} N(2t+2) \right] \\ \left. + \frac{1}{2} N(2t+1) + \frac{1}{8} N(2t) \right]$$

Now:

$$q_0(2t) = q_0 N(2t+3) + q_1 N(2t+2) + q_2 N(2t+1) \\ + q_3 N(2t)$$

Match $N(2t+3)$ terms:

$$\left. \begin{aligned} q_0 &= \frac{1}{2}(p_0 + p_1) \\ q_1 &= \frac{1}{8}p_0 + \frac{3}{4}p_1 + \text{others} \dots \end{aligned} \right\} \underline{\text{and so on}}$$

(7)

Now . notice that there is nothing to match the $N(2t+6)$ term which appears in p but not in q .

• but $t \in [0, 1/2]$

$$\therefore 2t+6 \in [6, 7]$$

and N is zero in this range!

we get

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/8 & 3/4 & 1/8 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 1/8 & 3/4 & 1/8 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

Now q is the first half of the p curve

8

second half is

$$N(2t-1) = r_0 N(2t+2) + r_1 N(2t+1) + r_2 N(2t) \\ t \in \left[\frac{1}{2}, 1\right] + r_3 N(2t-1)$$

Again, match terms

→ Notice that $N(2t-3)$ doesn't match anything

OK, because $t \in \left[\frac{1}{2}, 1\right] \therefore 2t-3 \in [-2, -1]$ and

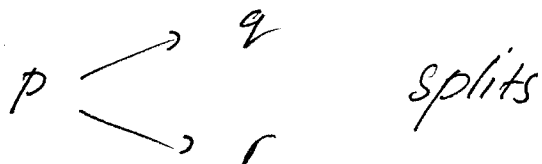
so N is zero

get

$$\begin{matrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{matrix} = \begin{bmatrix} 1/8 & 3/4 & 1/8 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 1/8 & 3/4 & 1/8 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

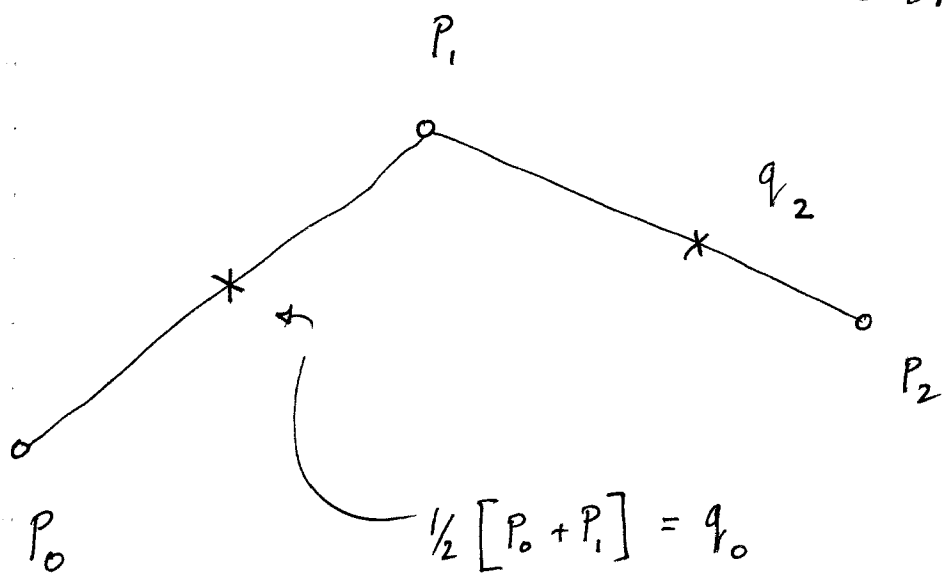
Two ways to read this

Simple:



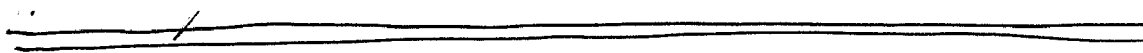
More interesting:

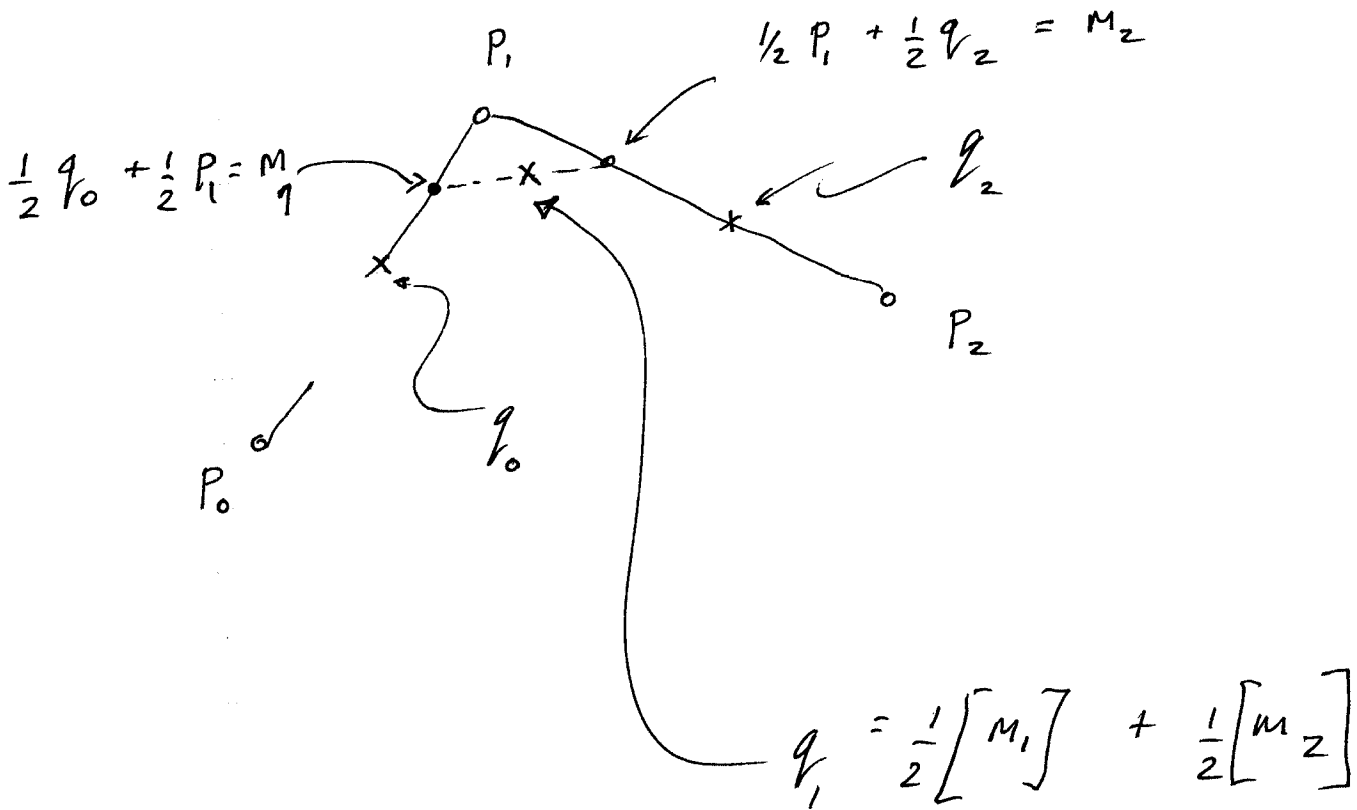
we have added pts to control structure, to get cs for same code



$$q_1 = \frac{1}{8} p_0 + \frac{3}{4} p_1 + \frac{1}{8} p_2$$

$$= \frac{1}{2} \left[\left(\frac{1}{2} \left[\frac{1}{2} p_0 + \frac{1}{2} p_1 \right] + \frac{1}{2} p_1 \right) \right] + \frac{1}{2} \left[\left(\frac{1}{2} p_1 + \frac{1}{2} \left[\frac{1}{2} p_1 + \frac{1}{2} p_2 \right] \right) \right]$$

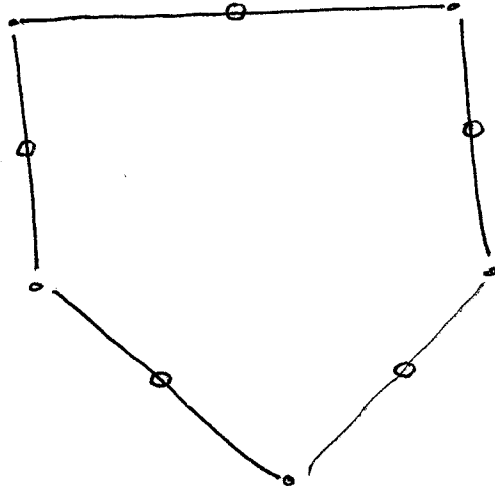




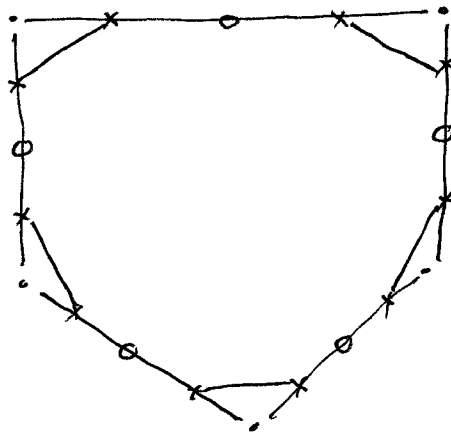
Procedure :

- take closed polygon
- I insert midpoints
- II ~~insert~~ mark midpoints ^{M_i} of ~~edges~~ and join with struts
- III midpoints of these struts taken w/ points in I, give polygon.

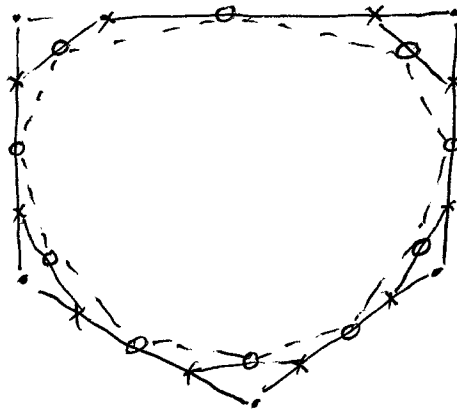
I



II



III



dashed line is
new polygon