The de Boor algorithm:

- Write a B-spline as

$$X(t) = \sum_{i=0}^{n} p_i \, N_{i,d}(t)$$

now, subs. recursive definition

$$N_{i,d}(t) = \frac{(t - t_i)}{(t_{i+d-1} - t_i)} \cdot N_{i,d-1}(t) +$$

$$\left(\frac{t_{i+d} - t}{t_{i+d} - t_{i+1}} \right) N_{i+1,d-1}(t)$$

$$X(t) = \sum_{i=0}^{n} p_i \left(\frac{t - t_i}{t_{i+d-1} - t_i} \right) N_{i,d-1}(t) +$$

$$\sum_{i=0}^{n} p_i \left(\frac{t_{i+d} - t}{t_{i+d} - t_{i+1}} \right) N_{i+1,d-1}(t)$$

now, rewrite second term

$$u = i + 1$$

gets

$$\sum_{u=1}^{n+1} p_{u-1} \left[\frac{t_{u+d-1} - t}{t_{u+d-1} - t_u} \right] N_{u,d-1}(t)$$

now by convention, set $p_{n+1} = p_{-1} = 0$

then

$$X(t) = \sum_{i=0}^{n+1} \left[\frac{p_i (t - t_i) + p_{i-1} (t_{i+d-1} - t)}{(t_{i+d-1} - t_i)} \right] N_{i,d-1}(t)$$

Now write

$$p_i' = \frac{p_i\,(t-t_i) + p_{i-1}\,(t_{i+d-1}-t)}{(t_{i+d-1}-t_i)}$$

So

$$X(t) = \sum_{i=0}^{n+1} p_i^1 \cdot N_{i,d-1}(t)$$

and we can go through the whole thing again etc.

to get $X(t) = p_r^{d-1}$ for $t \in [t_r, t_{r+1}]$

the recursion has the form

$$p_i^j = (1-\alpha_i^j)\, p_{i-1}^{j-1} + \alpha_i^j\, p_i^{j-1}, \quad j>0$$

$$p_j^0 = p_j$$

$$\alpha_i^j = \frac{(t-t_i)}{(t_{i+d-j}-t_i)}$$