Rendering diffuse interreflections

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with slides from John Hart
Diffuse-diffuse transfer

Light

○ Focal point
Interreflections are significant

From Koenderink slides on image texture and the flow of light
Radiosity and diffuse interreflections

- Assume we’re in a world of diffuse surfaces
- Rendering
  - cast eye rays
  - evaluate radiosity at first hit
  - average, stick into pixel
- Not practical --- we don’t know radiosity
- Model
Interreflection model

\[ B(x) = E(x) + \int_{\Omega} \left\{ \text{radiosity due to incoming radiance} \right\} d\omega \]

Integral over all incoming directions

For the moment, read this as incoming light
All diffuse surfaces are area sources!

- Receiver can’t tell whether light is created or reflected at source
- If receiver is luminaire (makes light), that just adds
- Diffuse interreflection equation
  - \(\text{vis}(x, u) = \begin{cases} 1 & \text{if they can see each other}, \ 0 & \text{otherwise} \end{cases}\)
  - Notice nasty property
    - B (unknown) is inside the integral!
    - Fredholm equation of the second kind

\[
B(x) = E(x) + \rho(x) \int_S \frac{\cos \theta_i \cos \theta_s}{\pi r^2} \text{vis}(x, u) B(u) dA_s
\]
Evaluating the radiosity

- cast eye rays
- evaluate radiosity at first hit
- average, stick into pixel

- Not practical --- we don’t know radiosity
- But

\[ B(x) = E(x) + \rho(x) \int_S \frac{\cos \theta_i \cos \theta_s}{\pi r^2} \text{Vis}(x, u) B(u) dA_s \]

This is an average over all light coming from somewhere else; this average smoothes
Rewrite model

\[ B(x) = E(x) + \int_{\Omega} \left\{ \text{radiosity due to incoming radiance} \right\} d\omega \]

\[ B(x) = E(x) + \rho(x) \int_{D} \left\{ \text{power due to radiosity at } u \right\} du \]

Here \( D \) is every point that can be seen from \( x \)

\[ B(x) = E(x) + \rho(x) \int_{D} \left\{ \text{power arriving due to } B(u) \right\} du \]
Rewrite model

\[ B(x) = E(x) + \rho(x) \int_{D} \{ \text{power arriving due to } B(u) \} \, du \]

We know an expression for this

\[ B(x) = E(x) + \rho(x) \int_{D_x} \{ \text{power due to } \left[ E(u) + \rho(u) \int_{D_u} \{ \text{power due to } B(v) \, dv \} \right] \} \, du \]

Here \( D_x \) is every point that can be seen from \( x \),
\( D_u \) is every point that can be seen from \( u \)
Notation

- We know form of “Power arriving due to $B(u)$”
  - but it’s tediously long
  - rewrite

From our work on area sources

$$\rho(x) \int_S \frac{\cos \theta_i \cos \theta_s}{\pi r^2} \text{Vis}(x, u) B(u) du_s = \rho(x) \int_S K(x, u) B(u) du_s$$
Notation

- Think of
  - functions as very long vectors
  - $K(x, u)$ as a matrix
  - write

$$\rho(x) \int_{S} K(x, u) B(u) du_s = \rho KB$$
Core idea: Neumann series

- We have

\[ B(x) = E(x) + \rho(x) \int_{S} \frac{\cos \theta_i \cos \theta_s}{\pi r^2} \text{Vis}(x, u) B(u) dA \]

- Can write:

\[ B = E + \rho K B \]

- Which gives

\[ B = E + (\rho K)E + (\rho K)(\rho K)E + (\rho K)^3 E + \ldots \]

Exitance

Source term

One bounce \hspace{1cm} Two bounces
The terms

\[ B = E + (\rho K)E + (\rho K)(\rho K)E + (\rho K)^3 E + \ldots \]

- Exitance
- Source term
  - mostly zero
  - Can change fast - shadows, etc.
  - Changes much more slowly, because K smoothes
- One bounce
- Two bounces
  - Changes even more slowly, because K smoothes
Using an estimate

• Notice:

\[ B = E + (\rho K) B \]

• Assume that I have a very rough estimate of B
  • I could render this using

\[ B = E + (\rho K) \hat{B} \]

• This isn’t such a good idea, because our shadows will be mangled
Lischinski ea 93
The right way

\[ B = E + (\rho K)E + (\rho K)(\hat{B} - E) \]

- Exitance
  - Mostly zero
  - Can change fast - shadows, etc.
- Source term
- One or more bounces
  - Changes much more slowly, because K smoothes, so we should approximate this
Computing the integrals

- Two terms
  - source term
    - we expect to need multiple samples, some large values, large changes over space
    - large variance will be ugly - should compute this term carefully at each point to render
  - indirect term
    - this term should change slowly over space, and should be smaller in value
    - large variance less ugly - we can use fewer samples and pool samples

\[
\rho(x) \int K(x, u) E(u) du
\]

\[
\rho(x) \int K(x, u)(\hat{B}(u) - E(u)) du
\]
Integrals with importance sampling

- Recall definition: \( \rho(x)K F = \rho(x) \int K(x, u) F(u) du \)

- How to evaluate this integral at a point?
  - obtain \( u_i \sim p(u) \)
  - Form:
    \[
    \frac{1}{N} \sum_{i=1}^{N} \frac{K(x, u_i) F(u_i)}{p(u_i)}
    \]
- Similar to evaluating illumination from area source
Importance sampling

• What is a good $p(u)$?
  • $p(u)$ should be big when $K(x, u) F(u)$ is big
  • this helps to control variance
  • known as importance sampling
  • Significant considerations:
    • fast variation in $F(u)$
    • fast variation in $K$
      • usually due to visibility

• How many samples?
  • fixed number
    • may be expensive, ineffective
  • by estimate of variance
    • this goes down as $1/N$, which is very bad news
Computing the direct term

- We know where E is non-zero
  - luminaires
  - zero at most points
- Treat these as area sources
  - ie samples randomly distributed across area
  - number of samples prop to intensity, total energy
  - or stratified sampling
  - use visibility considerations to choose which sources are sampled

\[ \rho(x) \int K(x, u) E(u) du \]
Computing the indirect term

• Small (ish)
• Varies relatively slowly across space
• Non-zero at most points
• Don’t really know where it will be large
• Strategies
  • choose directions on the input hemisphere uniformly at random
  • make an importance map for input hemisphere, reuse

\[ \rho(x) \int K(x, u)(\hat{B}(u) - E(u)) du \]