Radiosity estimates via finite elements

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after slides by John Hart

In a world of diffuse surfaces ...

- Recall
  - radiosity is radiated power per unit area, independent of direction
  - we obtained:

$$B(\mathbf{x}) = E(\mathbf{x}) + \rho(\mathbf{x}) \int_S \frac{\cos\theta_i \cos\theta_s}{\pi r^2} Vis(\mathbf{x}, \mathbf{u}) B(\mathbf{u}) dA_s$$

  - which we wrote as:

$$B(\mathbf{x}) - E(\mathbf{x}) - \rho(\mathbf{x}) \int K(\mathbf{x}, \mathbf{u}) B(\mathbf{u}) dA_{\mathbf{u}} = 0$$

Radiosity estimate via finite elements

- Divide domain into patches
- Radiosity will be constant on each patch
  - patch basis function, or element

$$\phi_i(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \text{ in patch } i \\ 0 & \text{otherwise} \end{cases}$$

- Now write
  - B_i for radiosity at patch i
  - E_i for exitance at patch i
  - Substitute into eqn:

$$B(\mathbf{x}) - E(\mathbf{x}) - \rho(\mathbf{x}) \int K(\mathbf{x}, \mathbf{u}) B(\mathbf{u}) dA_{\mathbf{u}} = 0$$

Becomes

$$\left(\sum_i B_i \phi_i(\mathbf{x}) \right) - \left(\sum_i E_i \phi_i(\mathbf{x}) \right) - \left(\rho(\mathbf{x}) \int K(\mathbf{x}, \mathbf{u}) \left(\sum_i B_i(\mathbf{u}) \right) dA_{\mathbf{u}} \right) = R(\mathbf{x})$$

↑

This should be "like zero"

Obtaining an estimate: Finite elements

- But in what sense is it zero?
  - Galerkin method

$$\int R(\mathbf{x})\phi_k(\mathbf{x})dA_x = 0\forall k$$

- Apply to:

$$\left(\sum_i B_i\phi_i(\mathbf{x})\right) - \left(\sum_i E_i\phi_i(\mathbf{x})\right) - \left(\rho(\mathbf{x})\int K(\mathbf{x},\mathbf{u})\left(\sum_i B_i(\mathbf{u})\right)dA_{\mathbf{u}}\right) = R(\mathbf{x})$$

- And get

$$B_k A_k - E_k A_k - \sum_j \left(\int_{\text{patch } k} \rho(\mathbf{x}) \int_{\text{patch } j} K(\mathbf{x},\mathbf{u}) d\mathbf{u} d\mathbf{x}\right) B_j = 0$$

Finite Element Radiosity Equation

- Start with:

$$B_k A_k = E_k A_k + \sum_j \left(\int_{\text{patch } k} \rho(\mathbf{x}) \int_{\text{patch } j} K(\mathbf{x}, \mathbf{u}) d\mathbf{u} d\mathbf{x} \right) B_j$$

- Divide through by A_k, assume constant albedo patches, get

$$B_k = E_k + \sum_j \rho_k F_{jk} B_j$$

- Where geometric effects are concentrated in the form factor

$$F_{jk} = \frac{1}{A_k} \int_{\text{patch } k} \int_{\text{patch } j} K(\mathbf{x}, \mathbf{u}) d\mathbf{u} d\mathbf{x}$$

Finite Element Radiosity

- This is a linear system $$B_k = E_k + \sum_j \rho_k F_{jk} B_j$$
- fold in albedo, write $$B_k = E_k + \sum_j \Gamma_{kj} B_j$$
- or in terms of matrices and vectors $$\mathbf{B} = \mathbf{E} + \Gamma \mathbf{B}$$
- BUT YOU SHOULD NEVER DO: $$\mathbf{B} = (\mathcal{I} - \Gamma)^{-1} \mathbf{E}$$
  - B might have 10^6 elements or more!

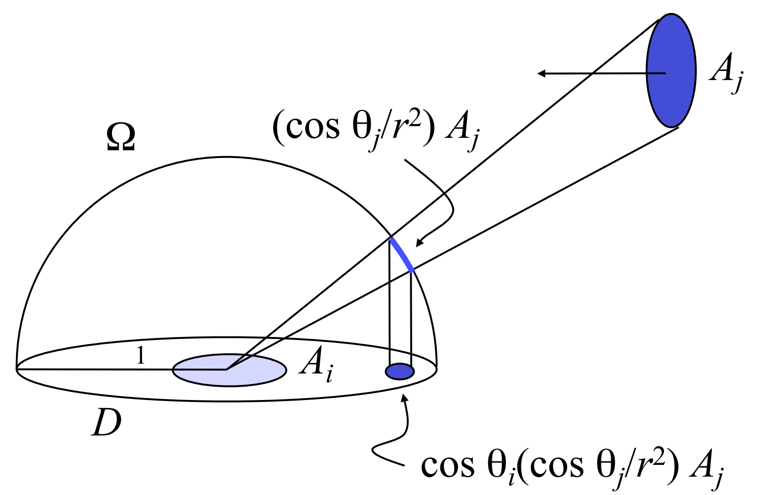
Form factors

- if patches are all flat, then: $F_{ii} = 0$
- if i can't see j at all, then: $F_{ij} = 0$
- reciprocity: $A_k F_{jk} = A_j F_{kj}$
- interpretation:
  - Fjk is percentage of energy leaving k that arrives at j
  - this gives:

$$\sum_j F_{jk} = 1$$

Computing form factors

- Nusselt's analogy

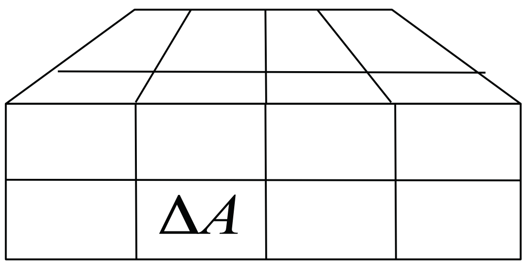


$$F_{ij} = \text{proj}_D(\text{proj}_\Omega(A_j))/\text{Area}(D)$$

The Hemicube

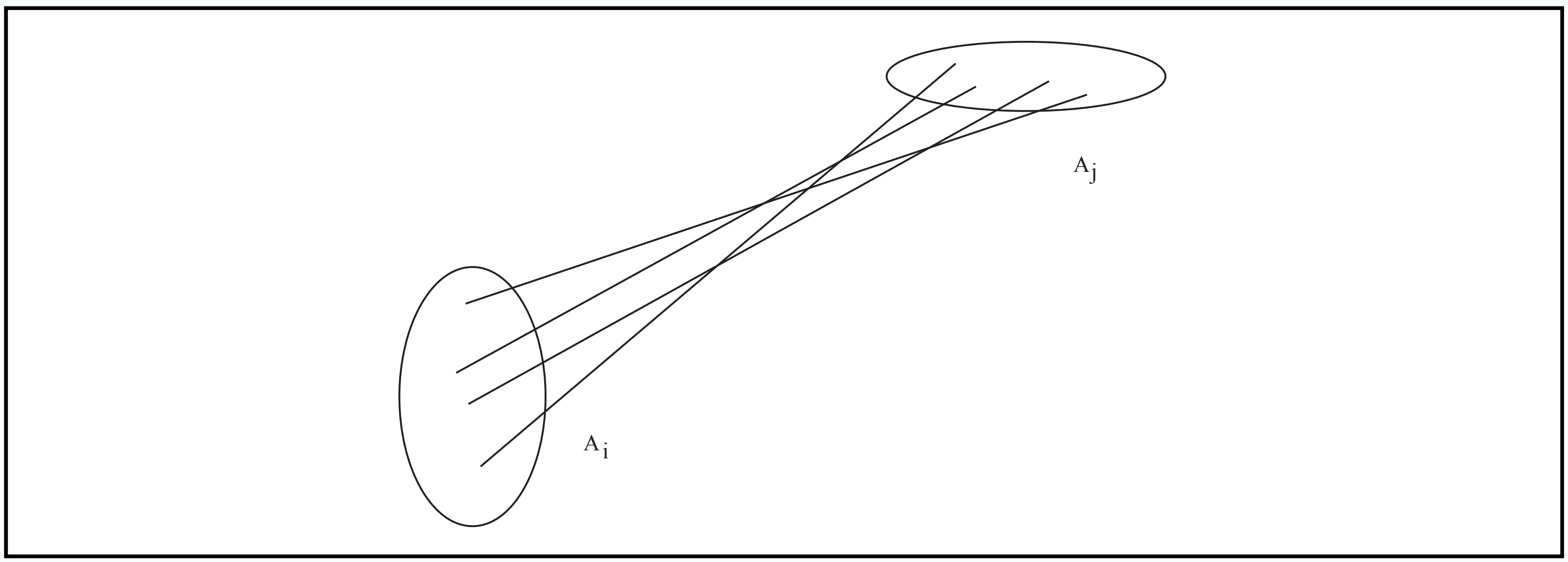
- Render onto faces of cube on receiver

$$\Delta F_{dAiAj} = \frac{\cos\phi_i \cos\phi_j}{\pi r^2} \Delta A$$



Random samples

- with uniform samples



$$A_k F_{jk} = \frac{1}{N} \sum \frac{\cos\theta_i \cos\theta_j Vis(i,j)}{\pi r^2}$$

Solving the radiosity system: Gathering

- Neumann series (again!) $\mathbf{B} = \mathbf{E} + \Gamma\mathbf{E} + \Gamma^2\mathbf{E} + \Gamma^3\mathbf{E} + ...$

- Easy iteration

$$\mathbf{B}^{(0)} = \mathbf{E}$$

$$\mathbf{B}^{(n+1)} = \mathbf{E} + \Gamma\mathbf{B}^{(n)}$$

Gathering with iterative methods

- Linear system   Ax=b

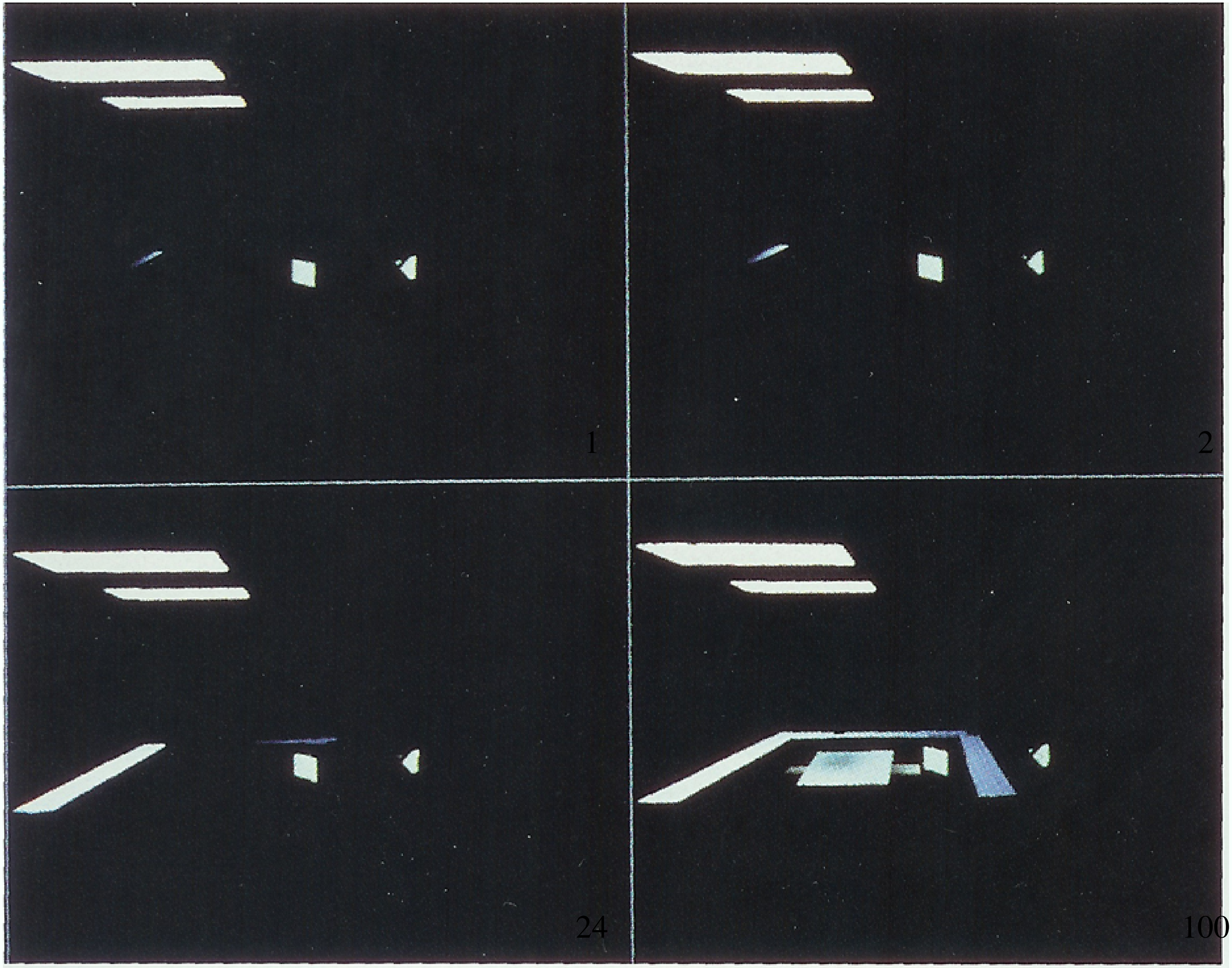
$$\sum_j a_{ij} x_j = b_i$$

- Jacobi iteration
  - reestimate each x

$$x_j^{(n+1)} = \frac{1}{a_{jj}} \left(b_i - \sum_{l \neq j} a_{il} x_l^{(n)} \right)$$

- Gauss-Seidel
  - reuse new estimates

$$x_j^{(n+1)} = \frac{1}{a_{jj}} \left(b_i - \sum_{l < j} a_{il} x_l^{(n+1)} - \sum_{l > j} a_{il} x_l^{(n)} \right)$$



From Cohen, SIGGRAPH 88

Southwell iteration: Progressive radiosity

- Gauss-Seidel, Jacobi, Neumann require us to evaluate whole kernel at each iteration
  - this is vilely expensive 10^6x 10^6 matrix?
  - it's also irrational
    - in G-S, Jacobi, for one pass through the variables,
      - we gather at each patch, from each patch
        - but some patches are not significant sources
    - we should like to gather only from bright patches
      - or rather, patches should "shoot"
- This is Southwell iteration