Paths, caching and radiometry

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Light paths

- Rendering studies paths from light to eye
  - classify these by type of bounce (D - diffuse; S - specular/transmissive)
  - different strategies can render different paths
    - Ray tracing
      - LDS*E
    - Finite element radiosity
      - LD*E
      - with some extra work we haven't discussed, some BUT NOT ALL cases of
        - L(D|S)*DE
    - Gather applied to a radiosity solution
      - LD*S*E
  - But there are many more paths
    - model explicitly

Path tracing

- Paths start at eye
  - At diffuse surface, choose ongoing direction uniformly at random across hemisphere
  - Concentrate on diffuse surfaces for the moment
- Value of pixel
  - average of path values leaving pixel
  - value of path
    - accumulated values of reflectance along path times exitance at end
- where does a path stop?
  - when albedo is zero (many luminaires)

Issues

- Severe variance problems as described
  - or very very slow for nice solutions
- Problem:
  - many low value paths wandering around space looking for a luminaire
  - at one pixel, we may end up sampling only low value paths
    - speckles, etc.
  - Strategy:
    - summarize paths with FE method (as above)
    - or do little work on low value paths if possible.
    - or cache path results

Russian roulette

- Prune low weight paths
  - when a path hits a low albedo surface, its value is likely to be small
    - idea:
      - prune with probability $(1 - \rho)$
      - if it survives, then weight path by $\frac{1}{\rho}$
      - expected value is the same
    - Advantages:
      - now don't need to accumulate albedo!
      - auto-pruning by accumulated albedo

Path tracing

- Path tracing
  - Path starts at eye
  - At a diffuse surface with albedo ρ, path is
    - continued with probability ρ
    - absorbed with probability 1- ρ
  - Value of path
    - E(x) if it arrives at luminaire
    - 0 otherwise
  - Direction along which path continues is uniform over exit hemisphere

This is a diffuse surface formulation.

Irradiance cache vs path tracing, from Pharr + Humphreys

Recall gathering

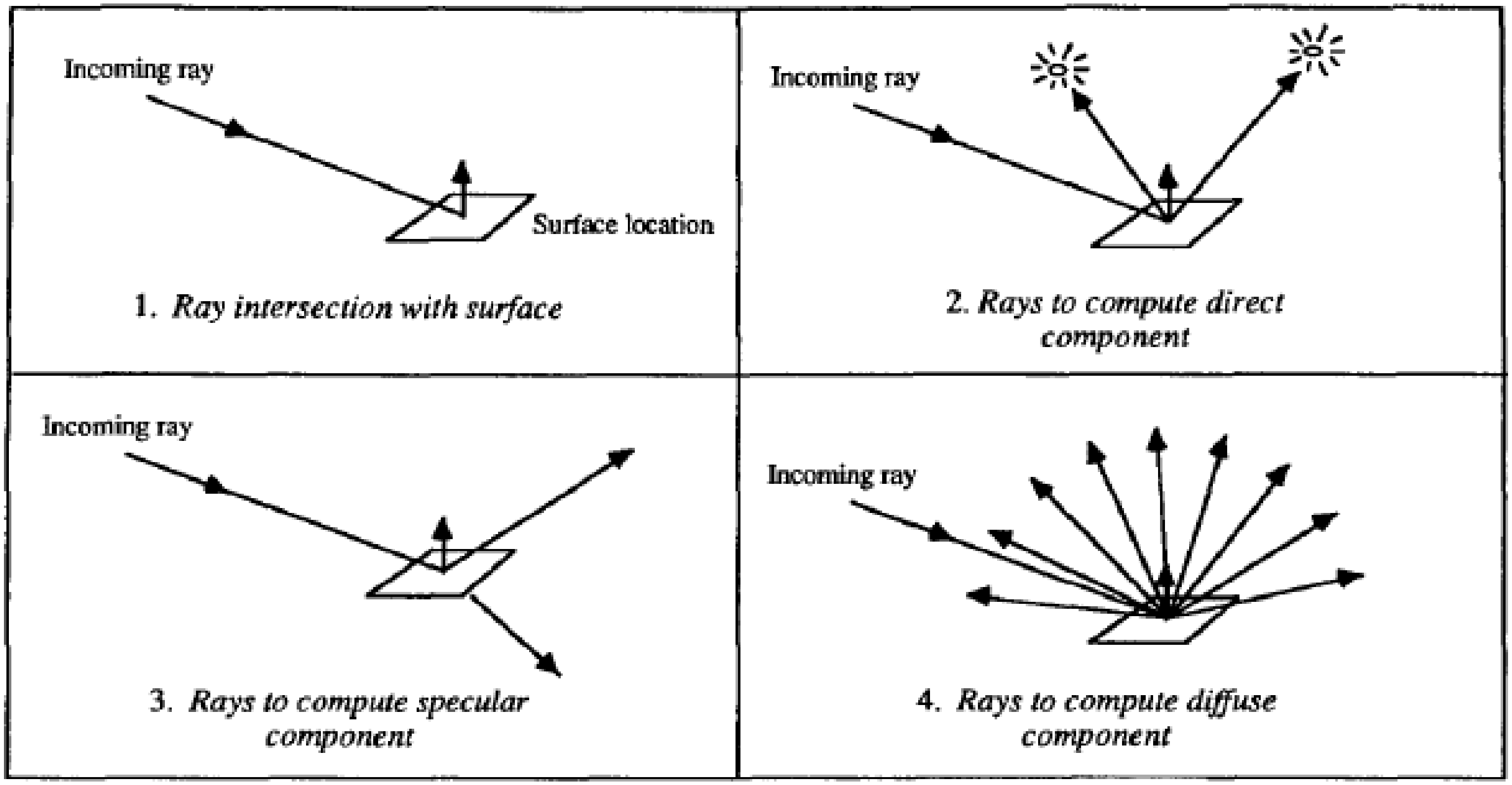


Figure 1: The four steps of ray tracing.

Figure from Ward et al, “A Ray-tracing solution for diffuse interreflection”, 1988

We need to be able to evaluate the far end of each of these rays

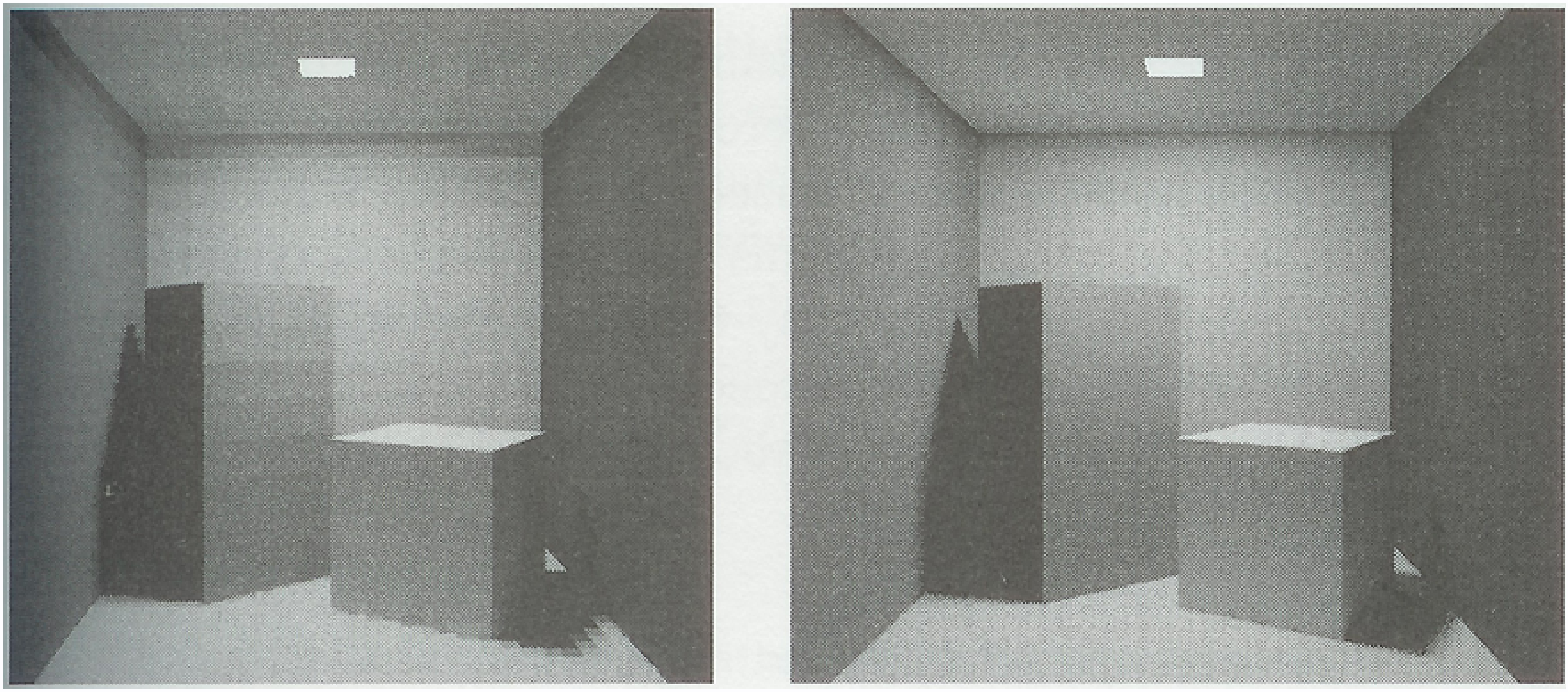
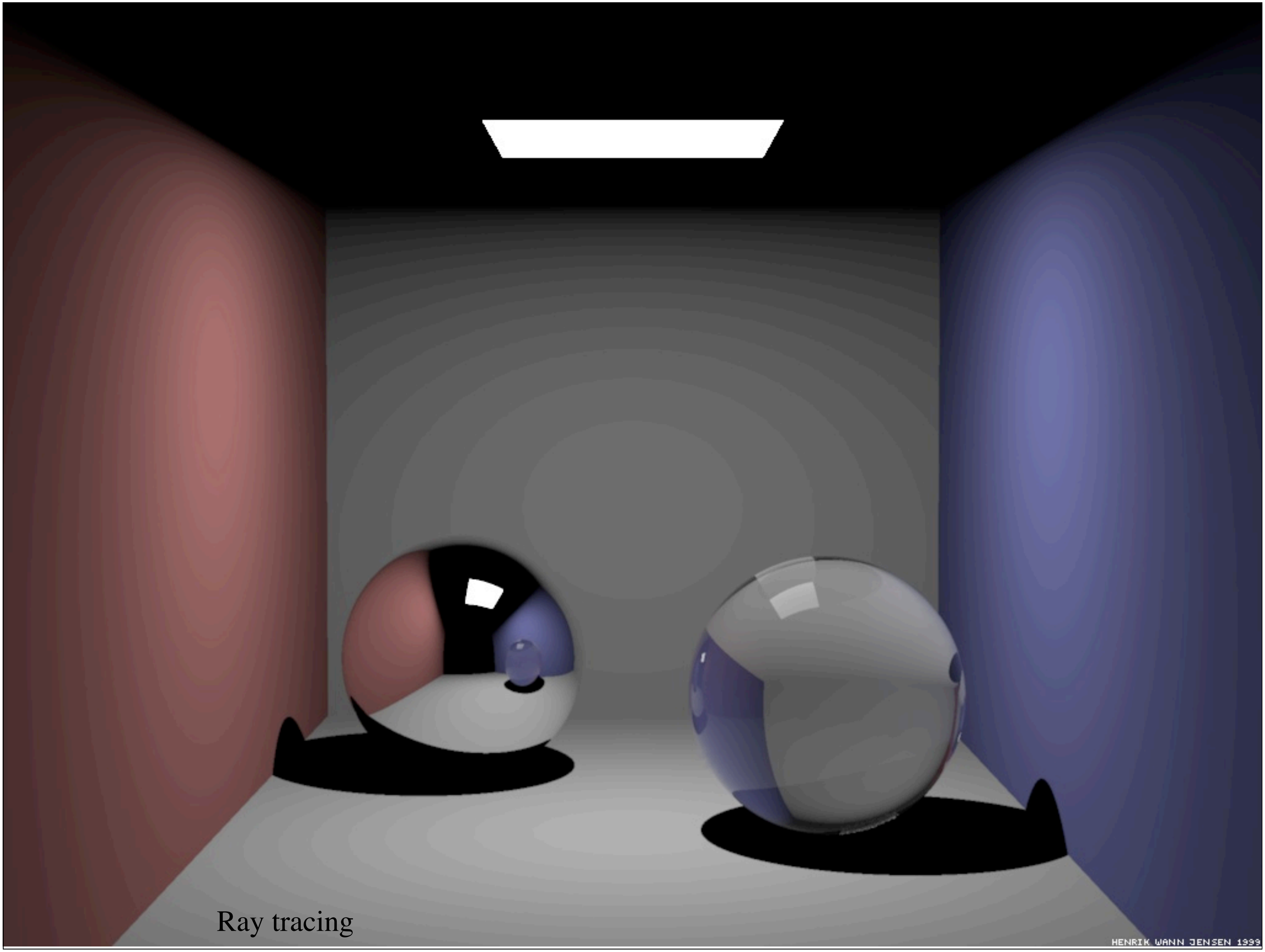
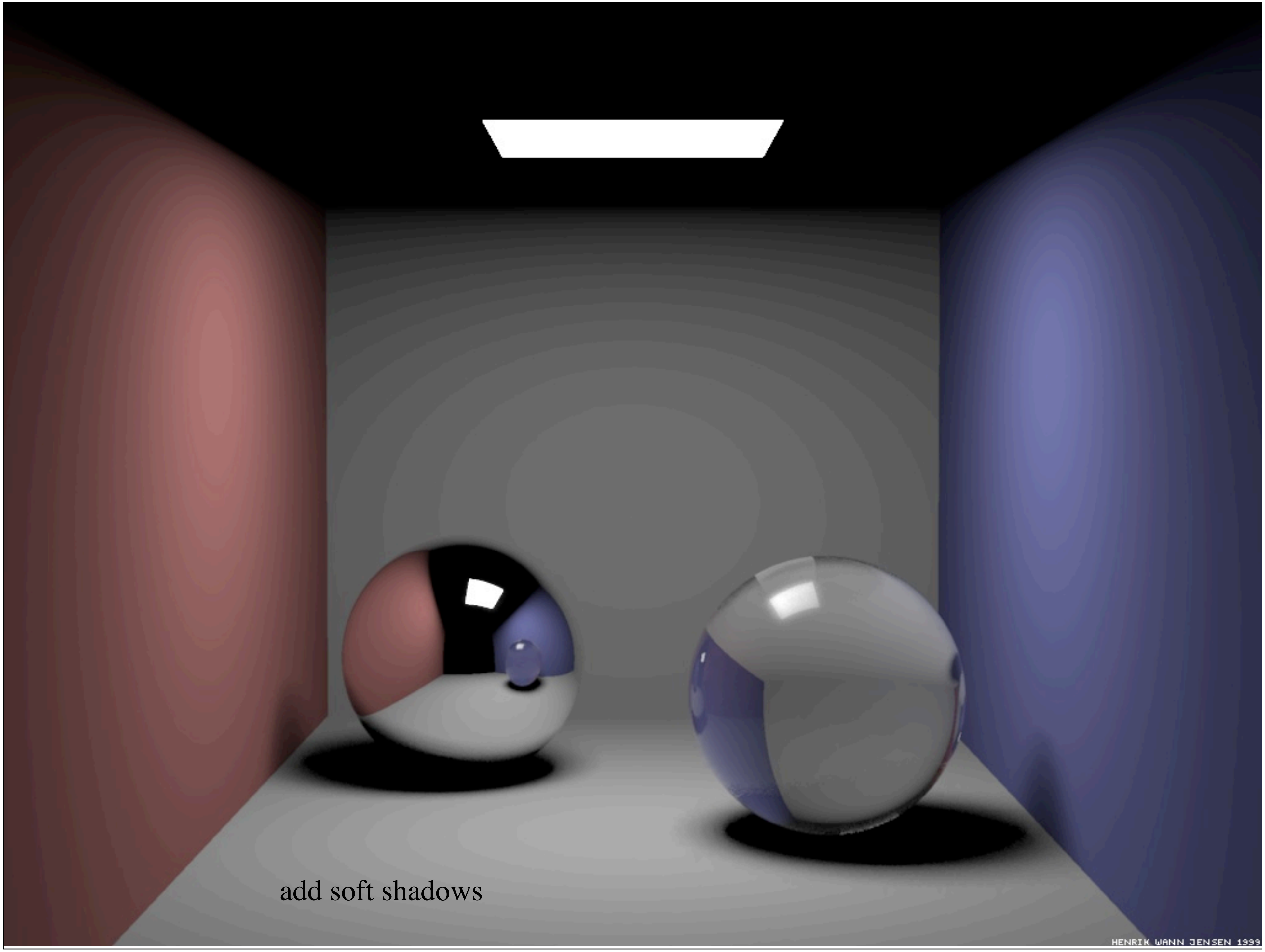
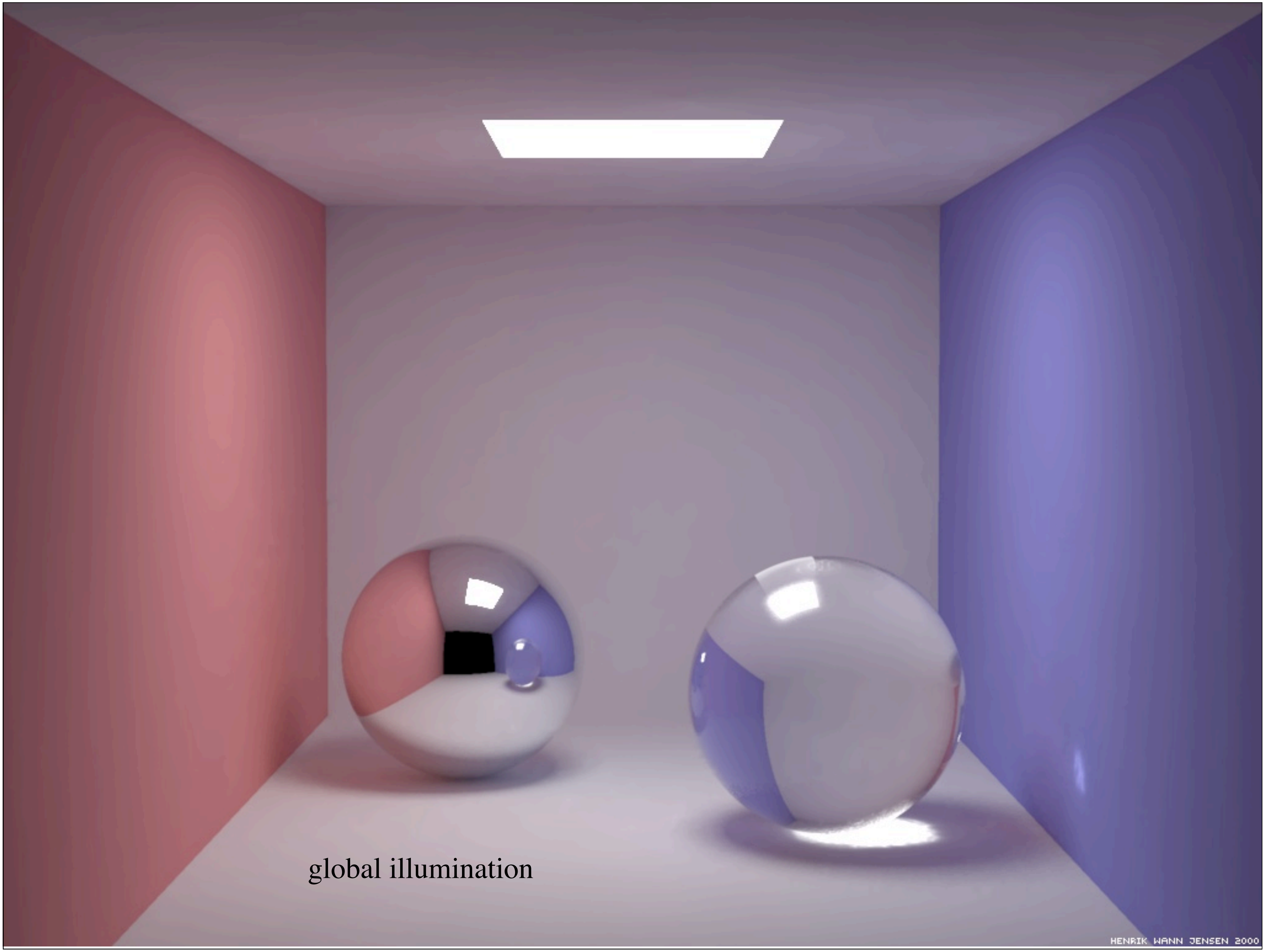


Figure from Dutre, Bekaert, Bala 03; rendered by Suykens-De Laet



Ray tracing


add soft shadows
HENRIK WANN JENSEN 1999

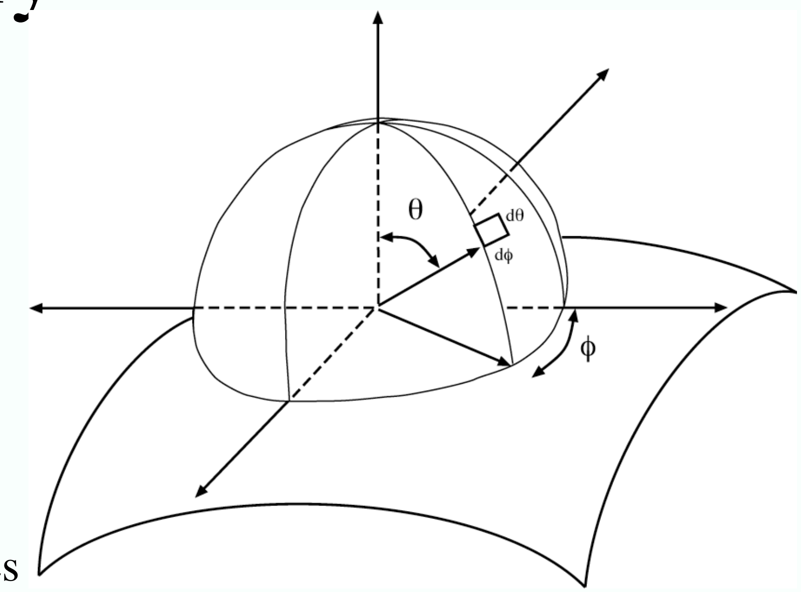

global illumination
HENRIK WANN JENSEN 2000

Caching

- Recall rendering strategy
  - compute approximate radiosity
  - gather it
- Currently, with progressive/hierarchical radiosity

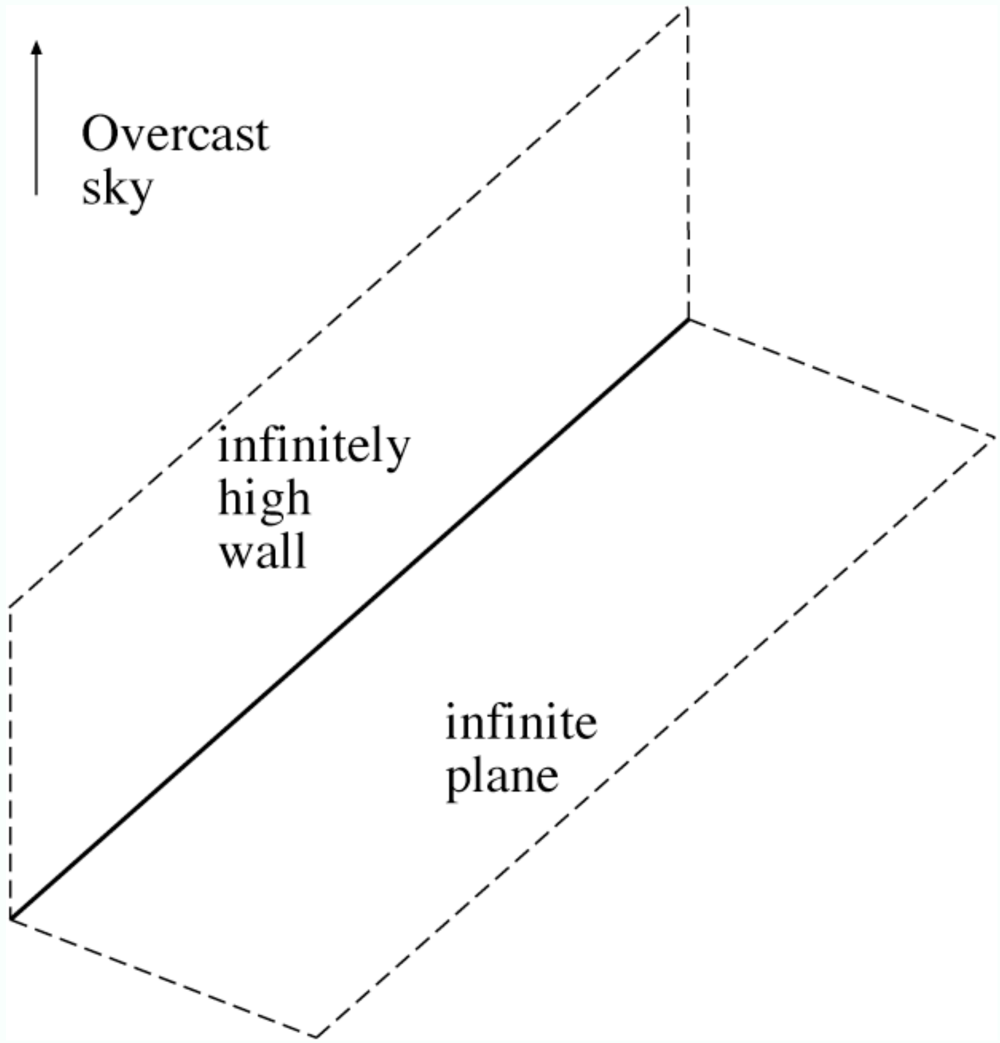
- Alternative:
  - compute by gathering at the far end of the ray
    - unattractive, because we will get a very (infinitely) deep tree
  - compute by interrogating a cache; if there's nothing nearby, gather
    - but what do we put in the cache?
    - a measure of incoming light
      - irradiance (rather than radiosity)

Radiometry

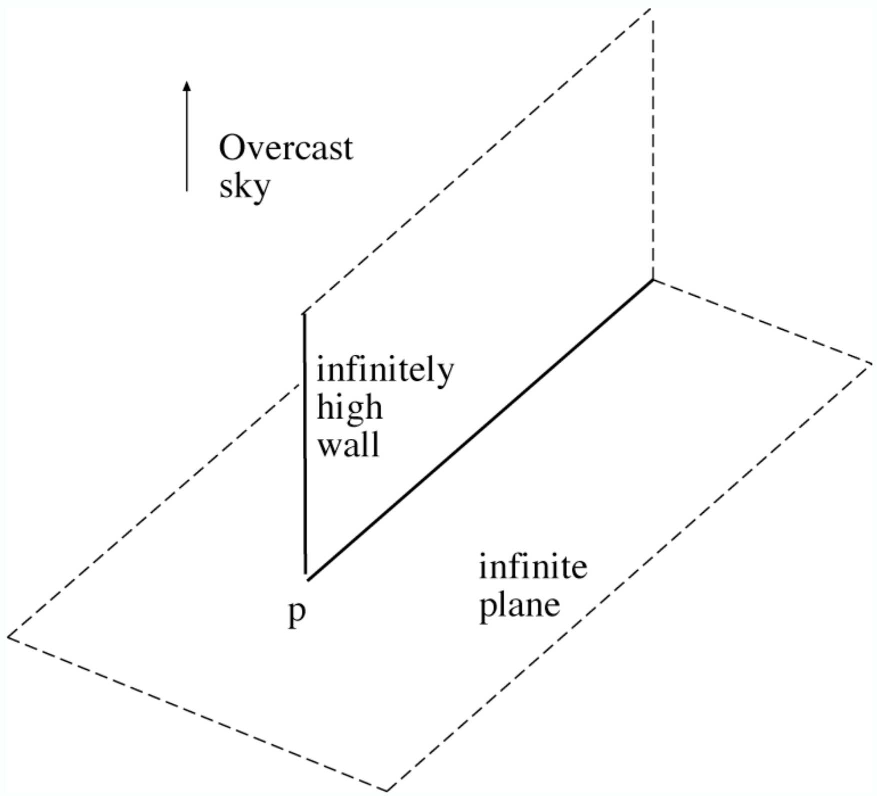


- Questions:
  - how “bright” will surfaces be?
  - what is “brightness”?
    - measuring light
    - interactions between light and surfaces
- Core idea - think about light arriving at a surface
- around any point is a hemisphere of directions
- Simplest problems can be dealt with by reasoning about this hemisphere

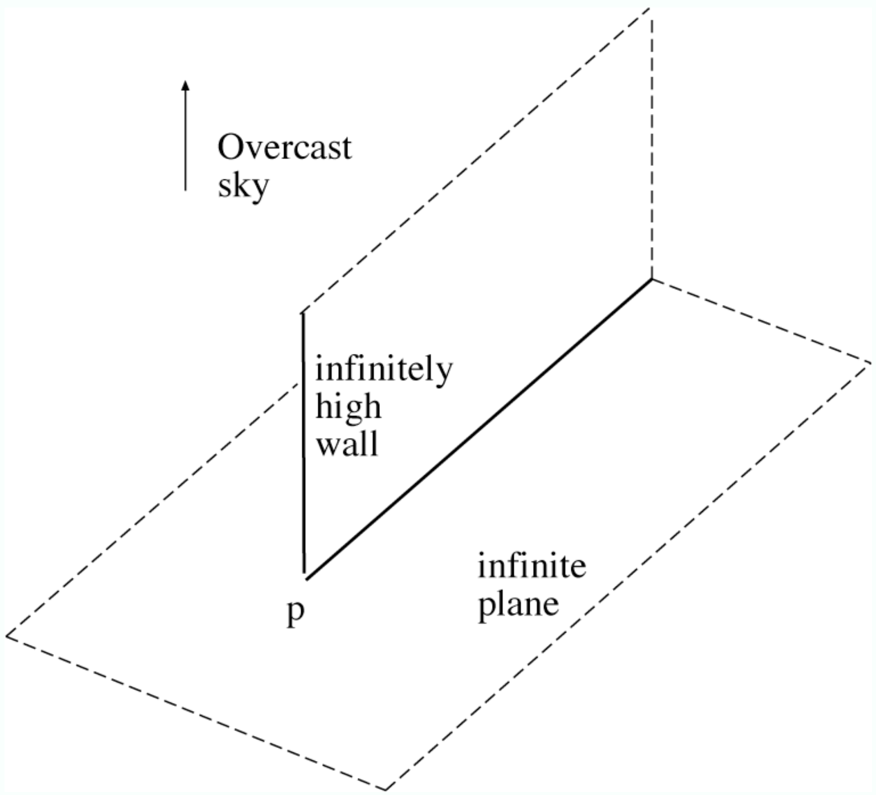
Lambert’s wall

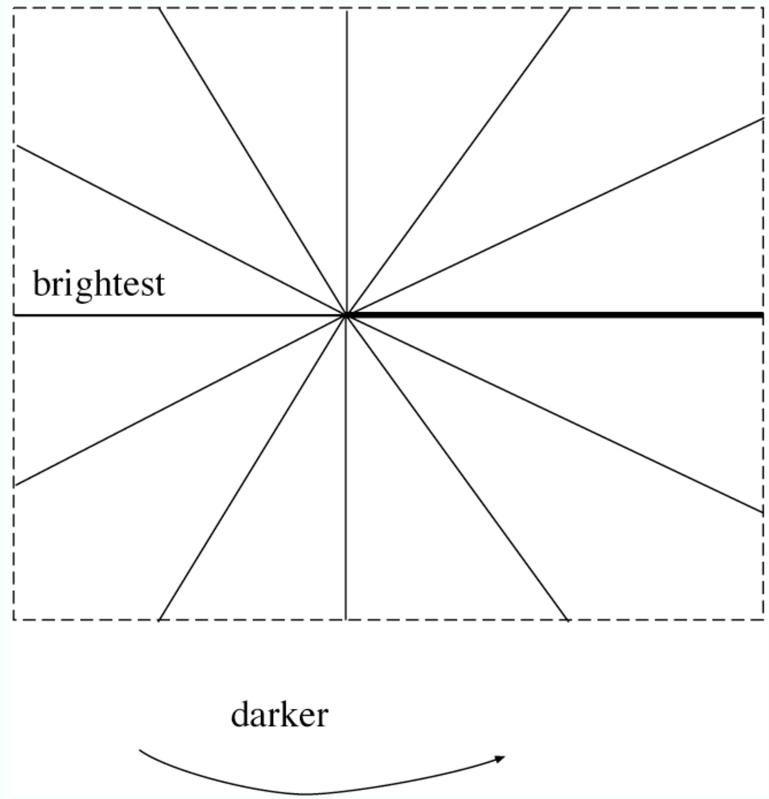


More complex wall



More complex wall

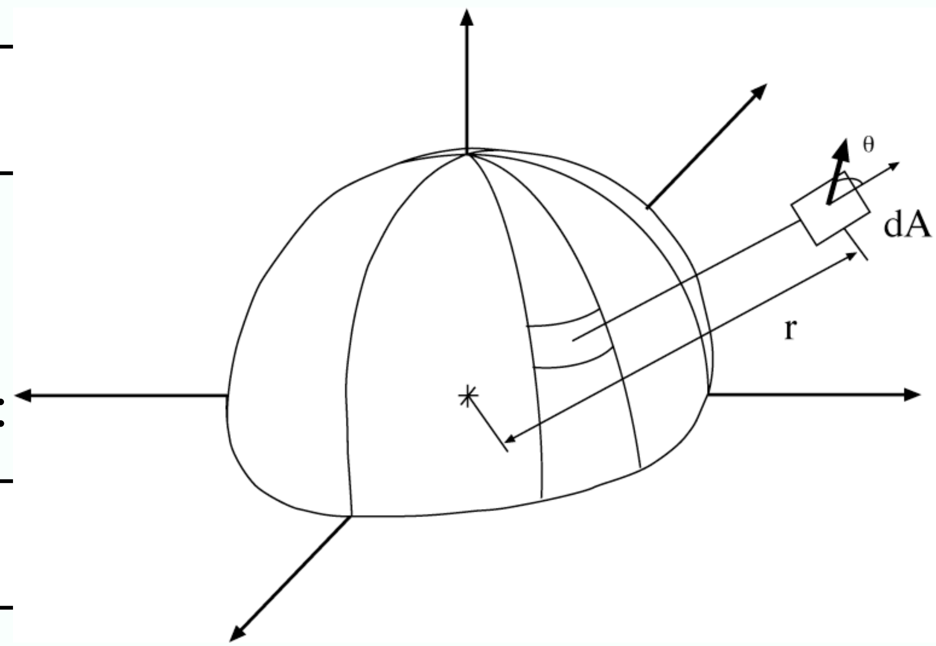




Solid Angle

- By analogy with angle (in radians)
- The solid angle subtended by a patch area dA is given by

$$d\omega = \frac{dA\cos\vartheta}{r^2}$$



- Another useful expression:

$$d\omega = \sin\vartheta(d\vartheta)(d\phi)$$

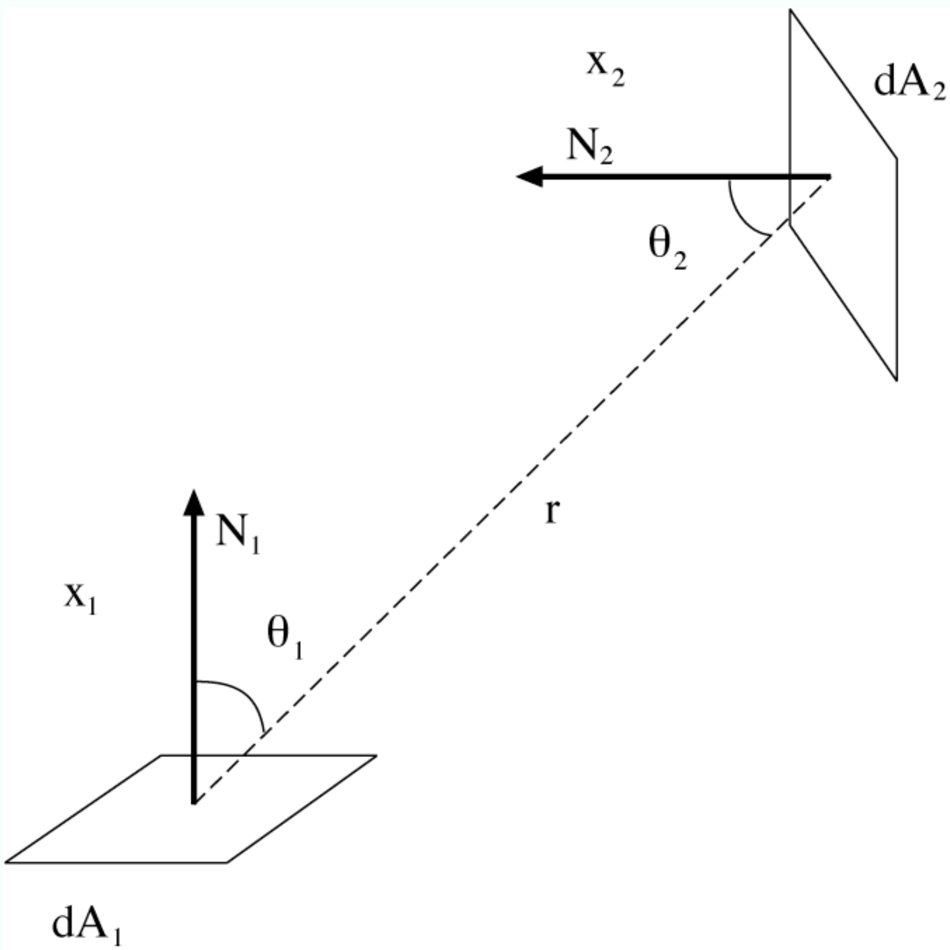
Radiance

- Measure the “amount of light” at a point, in a direction
- Property is:
  Radiant power per unit foreshortened area per unit solid angle
- Units: watts per square meter per steradian (wm-2sr-1)
- Usually written as:

$$\boxed{L(\underline{x}, \vartheta, \varphi)}$$

• Crucial property:
In a vacuum, radiance leaving p in the direction of q is the same as radiance arriving at q from p
– hence the units

Radiance is constant along straight lines



• Power 1->2, leaving 1:

$$\boxed{L(\underline{x}_1,\vartheta,\varphi)(dA_1\cos\vartheta_1)\left(\frac{dA_2\cos\vartheta_2}{r^2}\right)}$$

• Power 1->2, arriving at 2:

$$\boxed{L(\underline{x}_2,\vartheta,\varphi)(dA_2\cos\vartheta_2)\left(\frac{dA_1\cos\vartheta_1}{r^2}\right)}$$

Irradiance

- How much light is arriving at a surface?
- Sensible unit is Irradiance
- Incident power per unit area not foreshortened
- This is a function of incoming angle.
- A surface experiencing radiance L(x,θ,φ) coming in from dω experiences irradiance

$$L(\underline{x},\vartheta,\varphi)\cos\vartheta d\omega$$

$$\int_{\Omega} L(\underline{x},\vartheta,\varphi)\cos\vartheta\sin\vartheta d\vartheta d\varphi$$

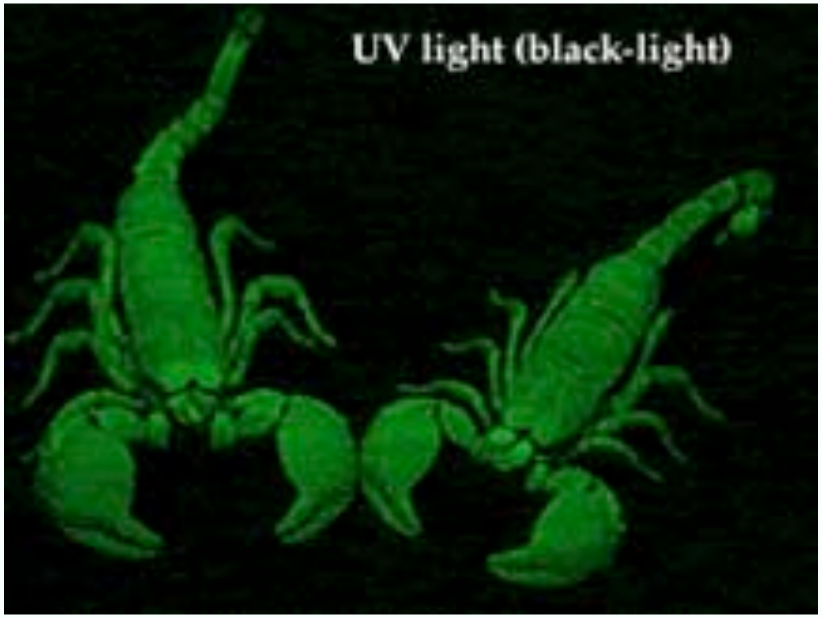
• Crucial property:
Total power arriving at the surface is given by adding irradiance over all incoming angles --- this is why it's a natural unit

Surfaces and the BRDF

- Many effects when light strikes a surface -- could be:
  - absorbed; transmitted. reflected; scattered
- Assume that
  - surfaces don't fluoresce
  - surfaces don't emit light (i.e. are cool)
  - all the light leaving a point is due to that arriving at that point
- Can model this situation with the Bidirectional Reflectance Distribution Function (BRDF)
- the ratio of the radiance in the outgoing direction to the incident irradiance

$$\rho_{bd}(\underline{x}, \vartheta_o, \varphi_o, \vartheta_i, \varphi_i,) = \frac{L_o(\underline{x}, \vartheta_o, \varphi_o)}{L_i(\underline{x}, \vartheta_i, \varphi_i) \cos\vartheta_i d\omega}$$


White light (indoor)


UV light (black-light)

BRDF

- Units: inverse steradians (sr-1)
- Symmetric in incoming and outgoing directions
- Radiance leaving in a particular direction:
  - add contributions from every incoming direction

$$\int_{\Omega} \rho_{bd}\left(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i,\right)L_i\left(\underline{x},\vartheta_i,\varphi_i\right)\cos\vartheta_i d\omega_i$$

Suppressing Angles - Radiosity

- In many situations, we do not really need angle coordinates
  - e.g. cotton cloth, where the reflected light is not dependent on angle
- Appropriate radiometric unit is radiosity
  - total power leaving a point on the surface, per unit area on the surface (Wm-2)
- Radiosity from radiance?
  - sum radiance leaving surface over all exit directions

$$\boxed{B(\underline{x}) = \int_{\Omega} L_o(\underline{x},\vartheta,\varphi)\cos\vartheta\, d\omega}$$

Radiosity

- Important relationship:
  - radiosity of a surface whose radiance is independent of angle (e.g. that cotton cloth)

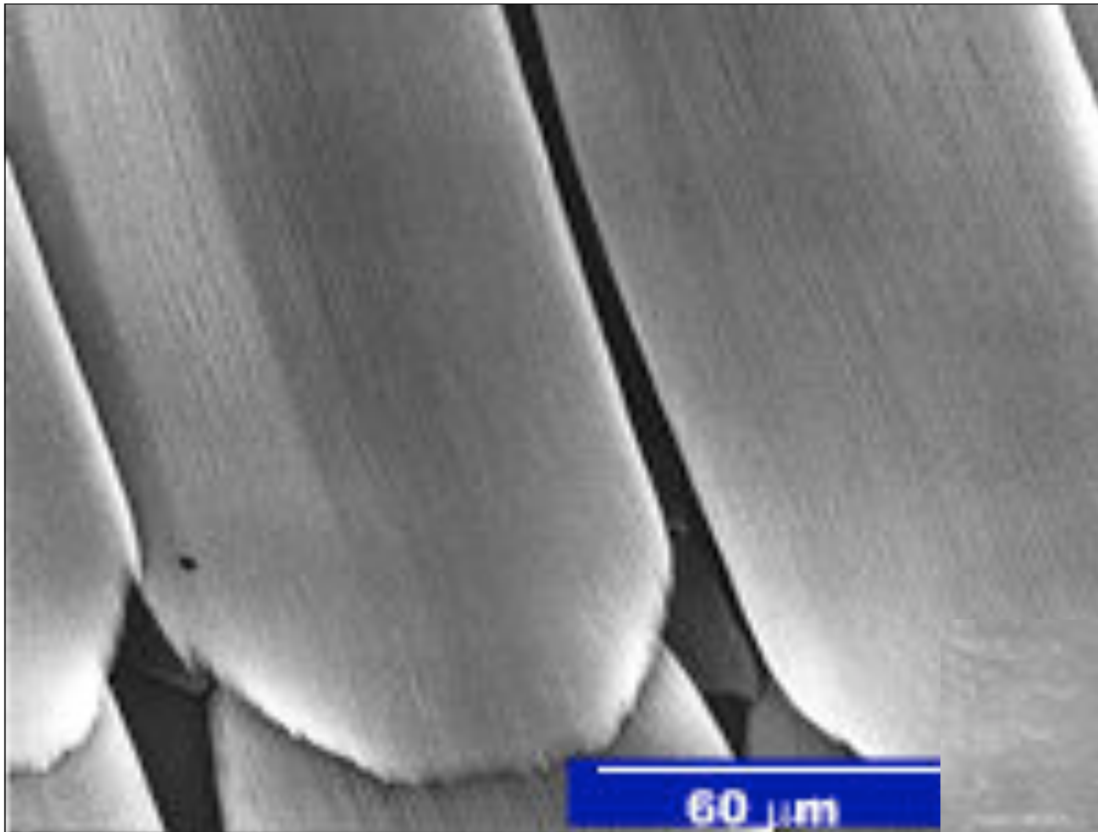
$$
\begin{aligned}
B(\underline{x}) &= \int_{\Omega} L_o(\underline{x},\vartheta,\varphi)\cos\vartheta\, d\omega \\
&= L_o(\underline{x}) \int_{\Omega} \cos\vartheta\, d\omega \\
&= L_o(\underline{x}) \int_0^{\pi/2} \int_0^{2\pi} \cos\vartheta \sin\vartheta\, d\varphi\, d\vartheta \\
&= \pi L_o(\underline{x})
\end{aligned}
$$

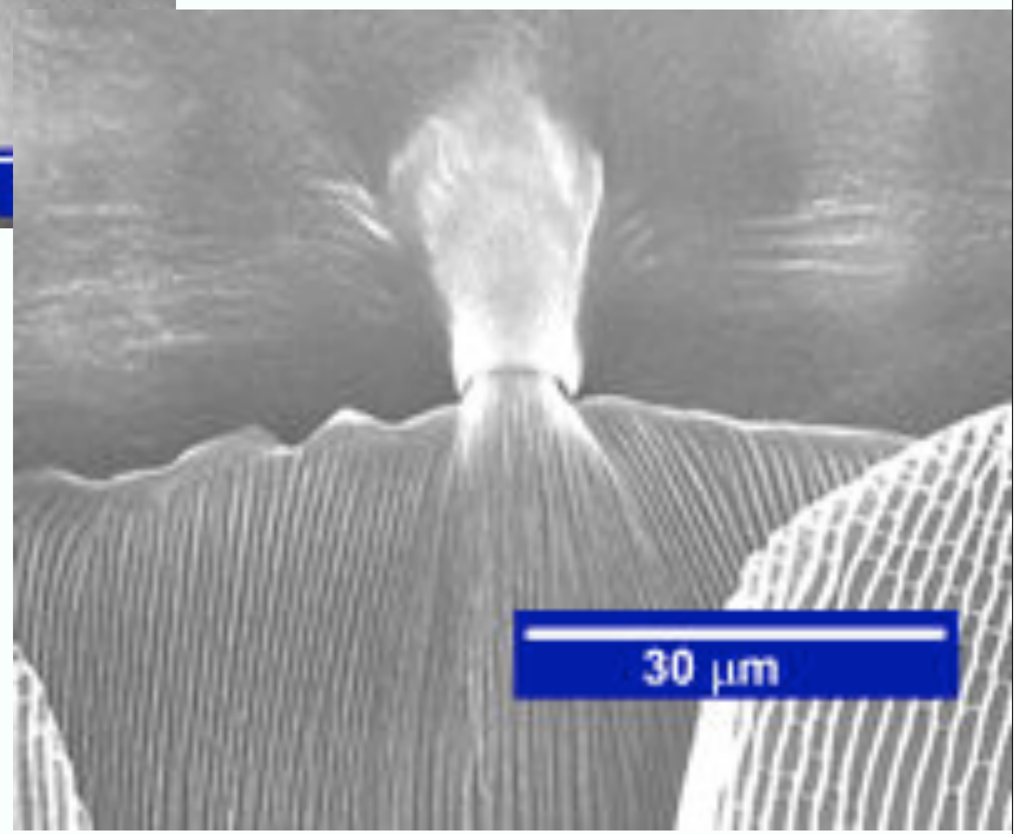
Directional hemispheric reflectance

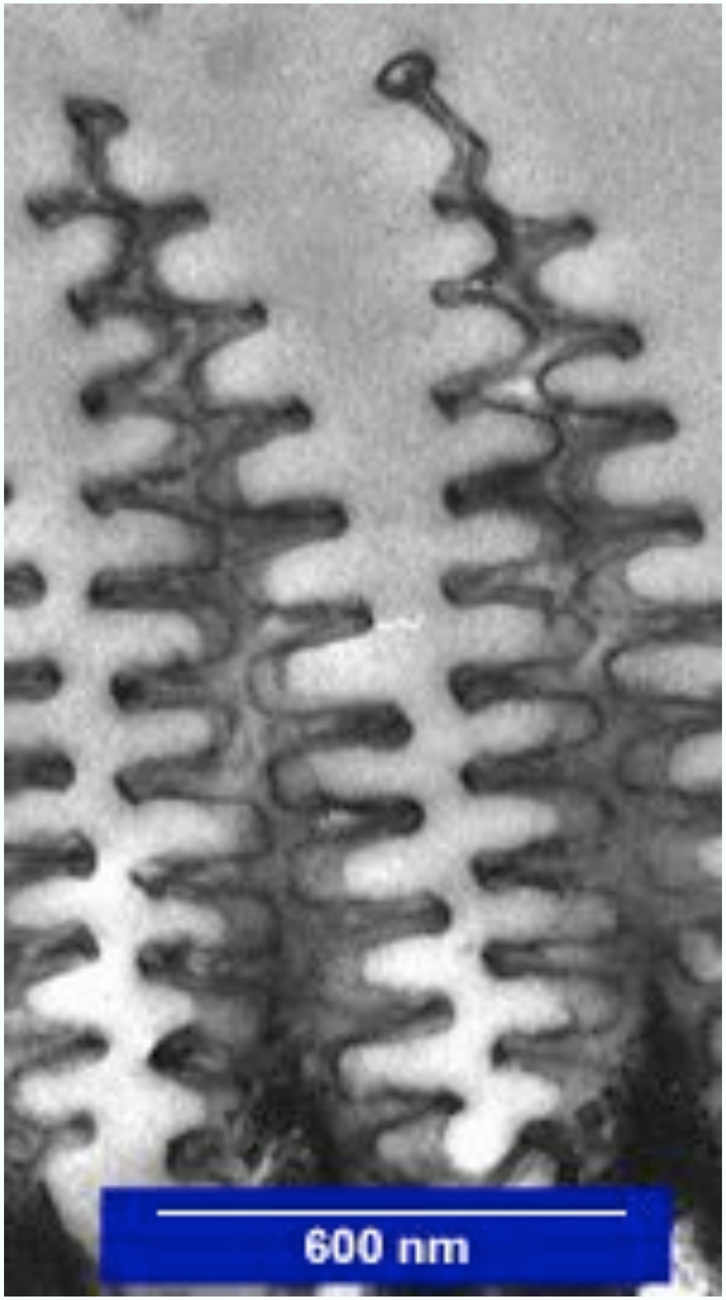
- BRDF is a very general notion
  - some surfaces need it (underside of a CD; tiger eye; etc)
  - very hard to measure and very unstable
  - for many surfaces, light leaving the surface is largely independent of exit angle (surface roughness is one source of this property)
- Directional hemispheric reflectance:
  - the fraction of the incident irradiance in a given direction that is reflected by the surface (whatever the direction of reflection)
  - unitless, range 0-1

$$\rho_{dh}(\vartheta_i,\varphi_i) = \frac{\int_\Omega L_o(\underline{x},\vartheta_o,\varphi_o)\cos\vartheta_o d\omega_o}{L_i(\underline{x},\vartheta_i,\varphi_i)\cos\vartheta_i d\omega_i}$$

$$= \int_\Omega \rho_{bd}(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i,)\cos\vartheta_o d\omega_o$$


60 µm


30 µm


600 nm

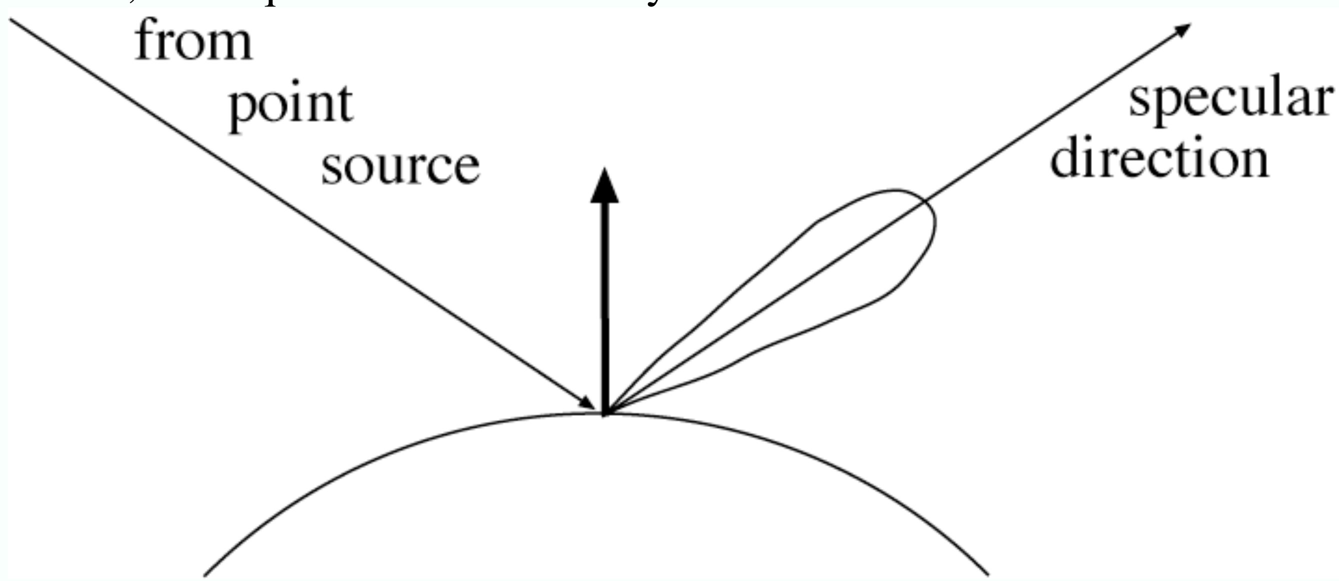
Lambertian surfaces and albedo

- For some surfaces, the DHR is independent of direction
  - cotton cloth, carpets, matte paper, matte paints, etc.
  - radiance leaving the surface is independent of angle
  - Lambertian surfaces (same Lambert) or ideal diffuse surfaces
  - Use radiosity as a unit to describe light leaving the surface
  - DHR is often called diffuse reflectance, or albedo
- for a Lambertian surface, BRDF is independent of angle, too.
- Useful fact:

$$\boxed{\rho_{brdf} = \frac{\rho_d}{\pi}}$$

Specular surfaces

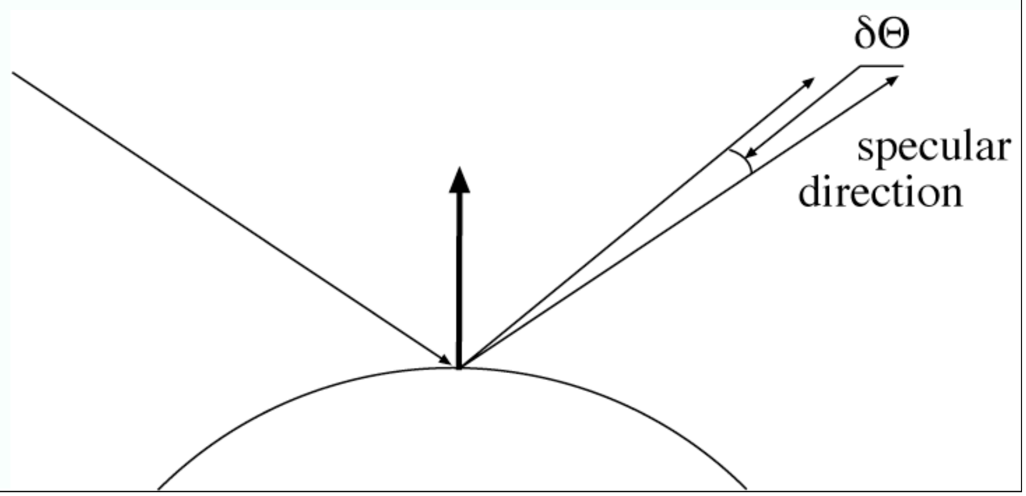
- Another important class of surfaces is specular, or mirror-like.
  - radiation arriving along a direction leaves along the specular direction
  - reflect about normal
  - some fraction is absorbed, some reflected
  - on real surfaces, energy usually goes into a lobe of directions
  - can write a BRDF, but requires the use of funny functions



Phong's model

- There are very few cases where the exact shape of the specular lobe matters.
- Typically:
  - very, very small --- mirror
  - small -- blurry mirror
  - bigger -- see only light sources as "specularities"
  - very big -- faint specularities
- Phong's model
  - reflected energy falls off with

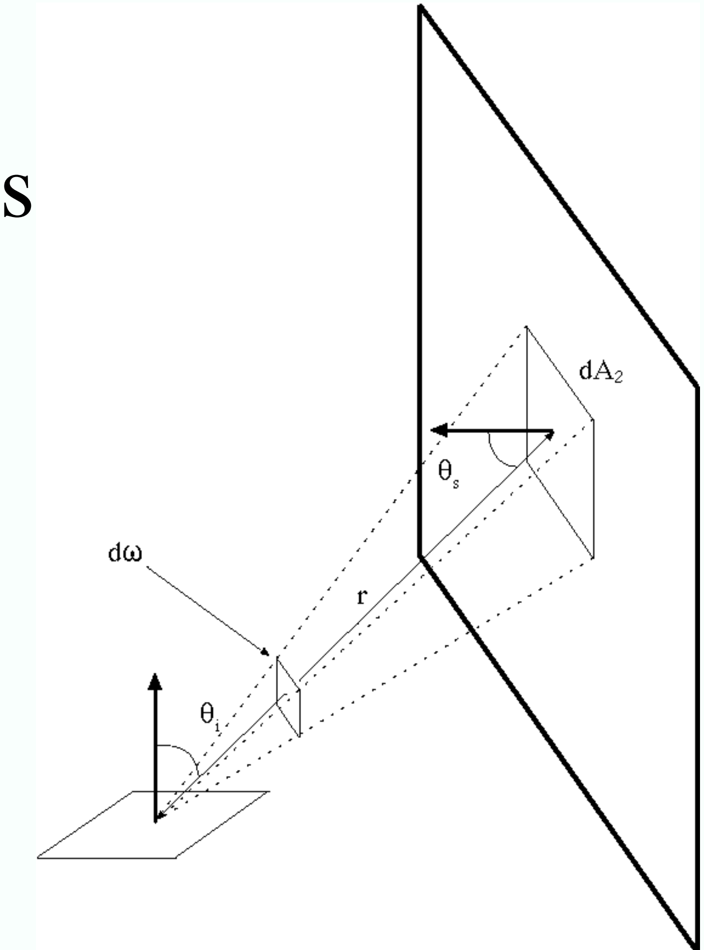
$$\boxed{\cos^n(\delta\vartheta)}$$



Lambertian + specular

- Widespread model
  - all surfaces are Lambertian plus specular component
- Advantages
  - easy to manipulate
  - very often quite close true
- Disadvantages
  - some surfaces are not
    - e.g. underside of CD’s, feathers of many birds, blue spots on many marine crustaceans and fish, most rough surfaces, oil films (skin!), wet surfaces
  - Generally, very little advantage in modelling behaviour of light at a surface in more detail -- it is quite difficult to understand behaviour of L+S surfaces

Area sources



- Examples: diffuser boxes, white walls.
- The radiosity at a point due to an area source is obtained by adding up the contribution over the section of view hemisphere subtended by the source
    - change variables and add up over the source

Radiosity due to an area source

- rho is albedo
- E is exitance
- r(x, u) is distance between points
- u is a coordinate on the source

$$
\begin{aligned}
B(x) &= \rho_d(x)\int_{\Omega} L_i(x, u \to x)\cos\theta_i d\omega \\
&= \rho_d(x)\int_{\Omega} L_e(x, u \to x)\cos\theta_i d\omega \\
&= \rho_d(x)\int_{\Omega} \left(\frac{E(u)}{\pi}\right)\cos\theta_i d\omega \\
&= \rho_d(x)\int_{source} \left(\frac{E(u)}{\pi}\right)\cos\theta_i \left(\cos\theta_s \frac{dA_u}{r(x,u)^2}\right) \\
&= \rho_d(x)\int_{source} E(u)\frac{\cos\theta_i \cos\theta_s}{\pi r(x,u)^2} dA_u
\end{aligned}
$$

Caching

- What should we cache?
  - radiosity (but surface might not be diffuse)
  - radiance (tricky to represent)
  - irradiance
    - incoming light
    - because we can turn it into outgoing light easily

Irradiance caching

- The indirect term varies slowly over space
  - cache and interpolate
- Cache by
  - storing irradiance samples in octree with normal
- Interpolate by
  - obtaining all samples with error smaller than
    - error is:
      - (distance term)+(normal term)
    - not enough samples?
      - generate new ones and cache them
  - forming weighted sum using extrapolated illumination values

Powerful standard method in
almost every modern rendering system

Irradiance cache vs path tracing, from Pharr + Humphreys

Cache sample locations from Pharr + Humphreys

Irradiance caching: samples

- Obtaining samples:
  - evaluate irradiance at sample point by:
    - direct term:
      - sample each source directly, as before
    - indirect term:
      - sample non-source directions with probability $P(\omega)$
      - form estimate

$$\frac{1}{N}\sum \frac{L(x,\omega_j)\cos\theta_j}{P(\omega_j)}$$

      - Notice that the incoming radiance might be computed from the cache, if there are samples

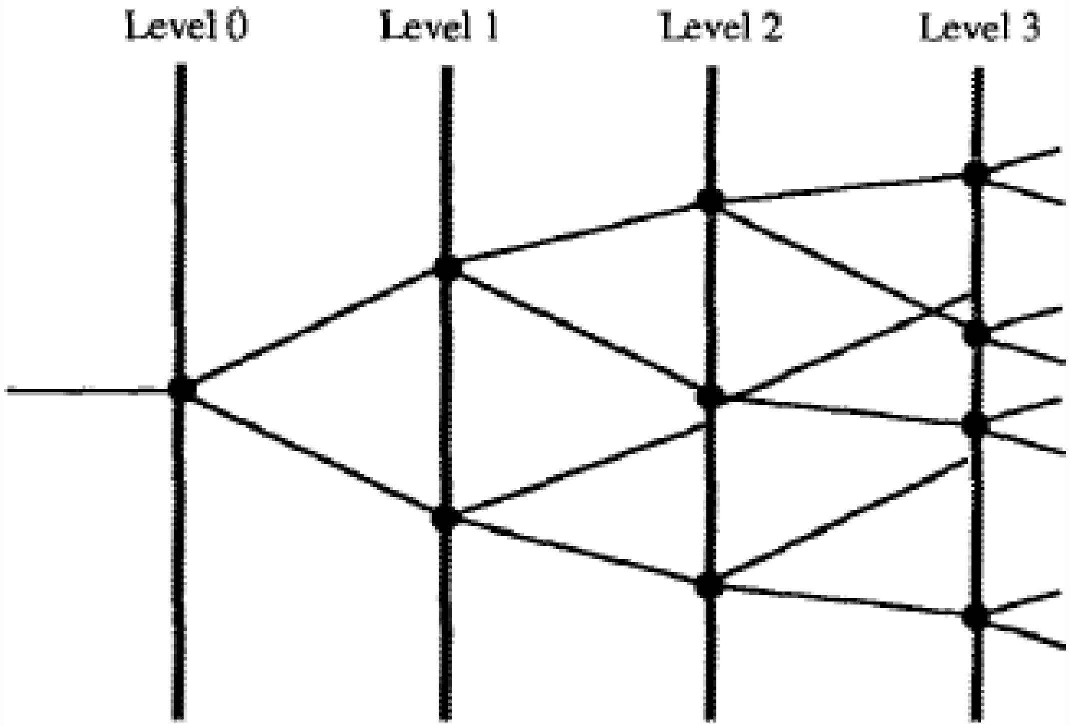


Figure 7: The lines represent rays, and the points represent primary evaluations. The rays that reuse computed values do not propagate.

Note:
Russian roulette prevents the tree getting out of hand
Fairly quickly, the cache fills up

Figure from Ward et al, "A Ray-tracing solution for diffuse interreflection", 1988

Irradiance caching: reconstruction

- Query octree for possible samples
  - Do not want to use:
    - samples that are too far away
      - this is a function of how samples were obtained
    - samples with a bad normal
    - samples that lie closer to the eye than current point
- Reconstruct by
  - weighted sum of samples
  - interpolation process can use:
    - distances
    - gradients

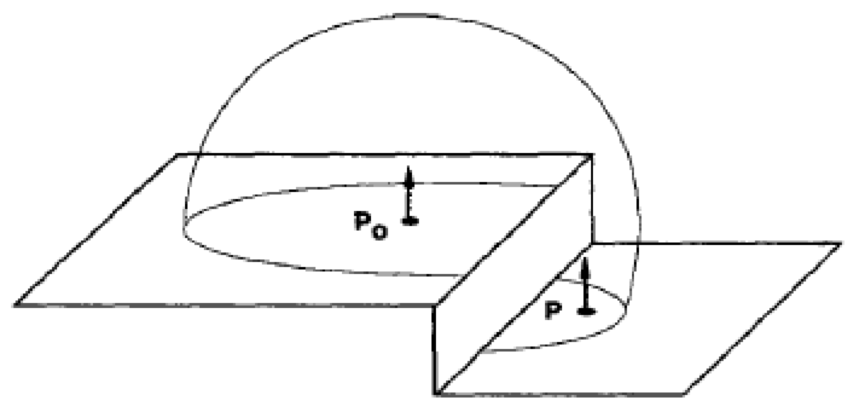
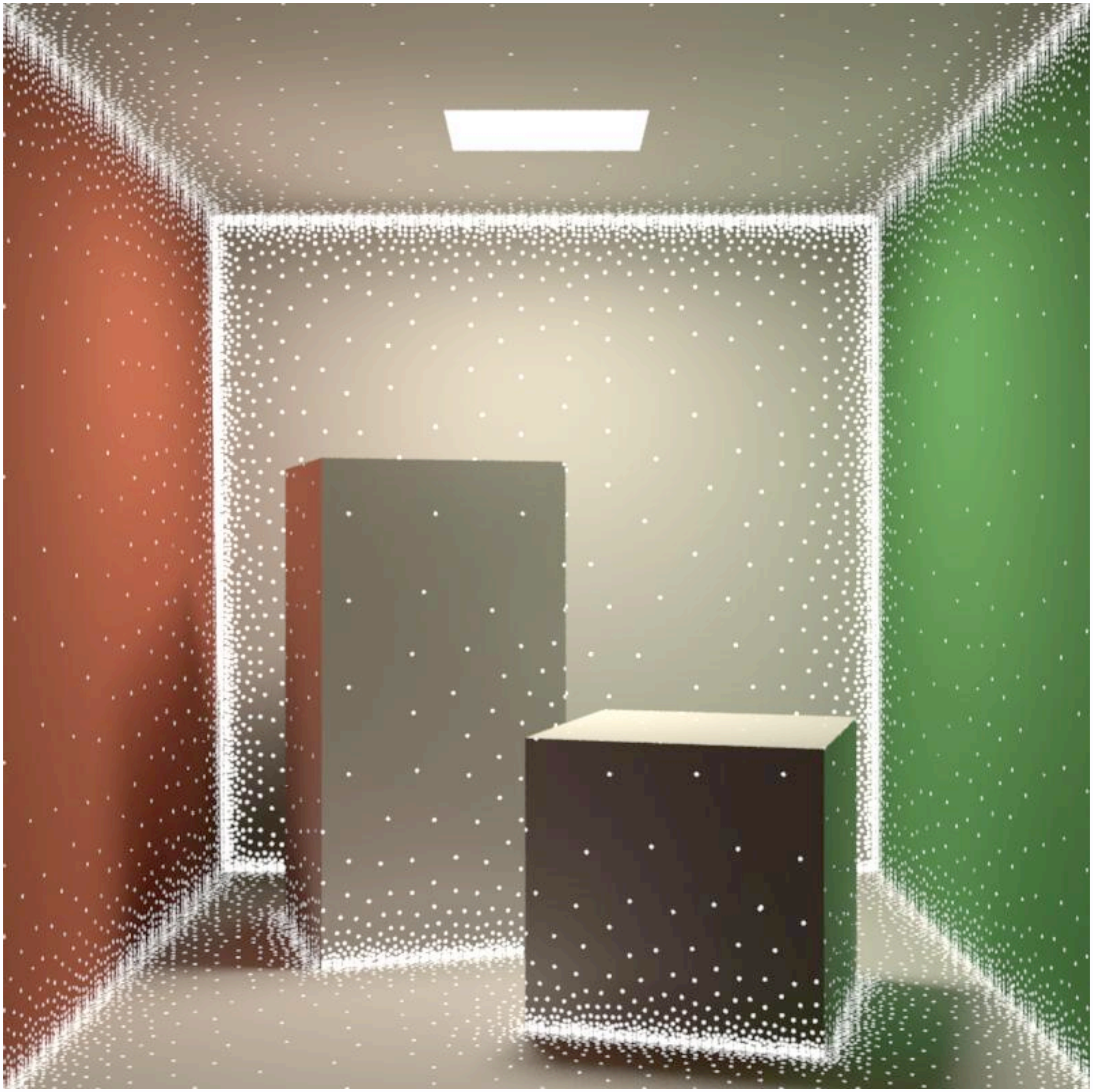
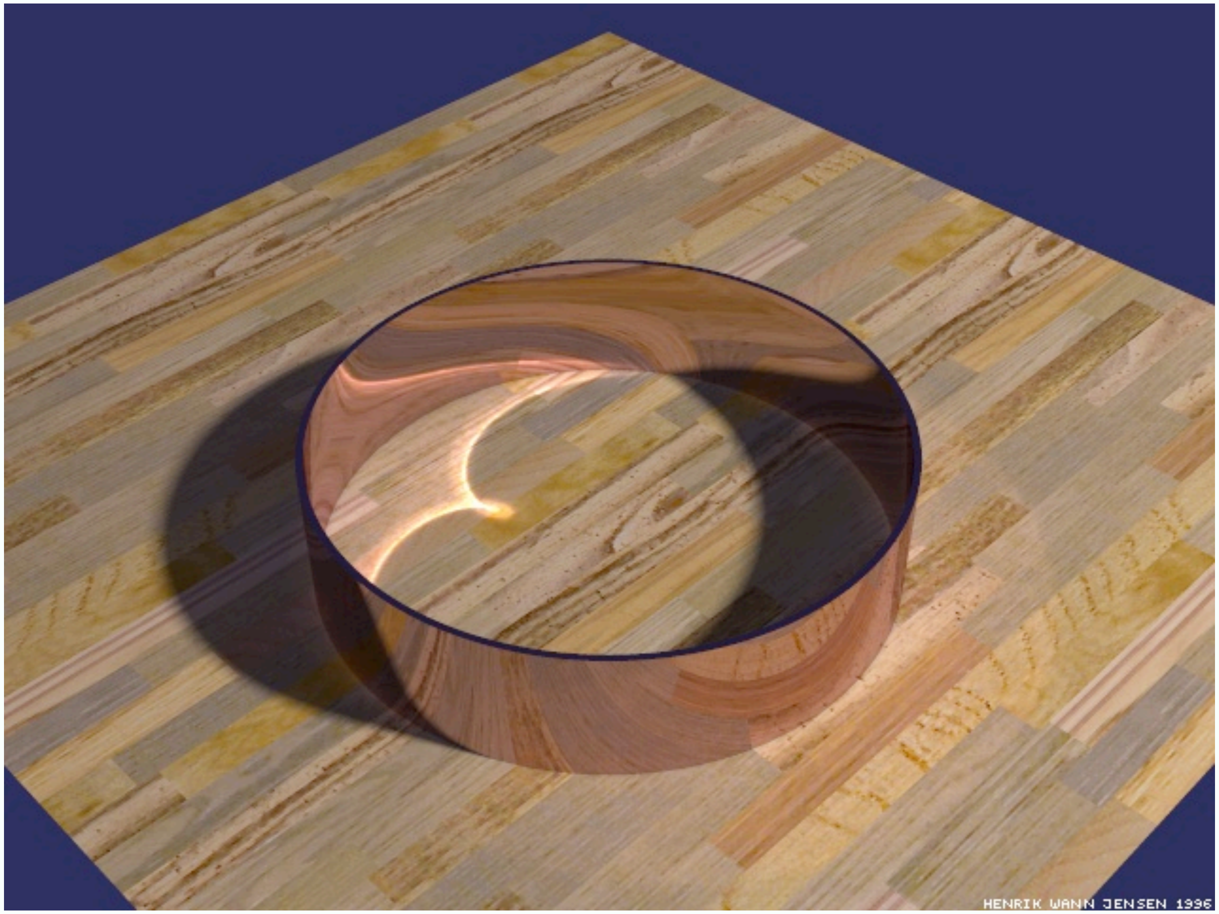


Figure 4: $\vec{P}_0$ sees few close-by surfaces, so its estimated error at $\vec{P}$ is small. But $\vec{P}$ is shadowed by the surface under $\vec{P}_0$, and the true illuminance is different.

Figure from Ward et al, "A Ray-tracing solution for diffuse interreflection", 1988

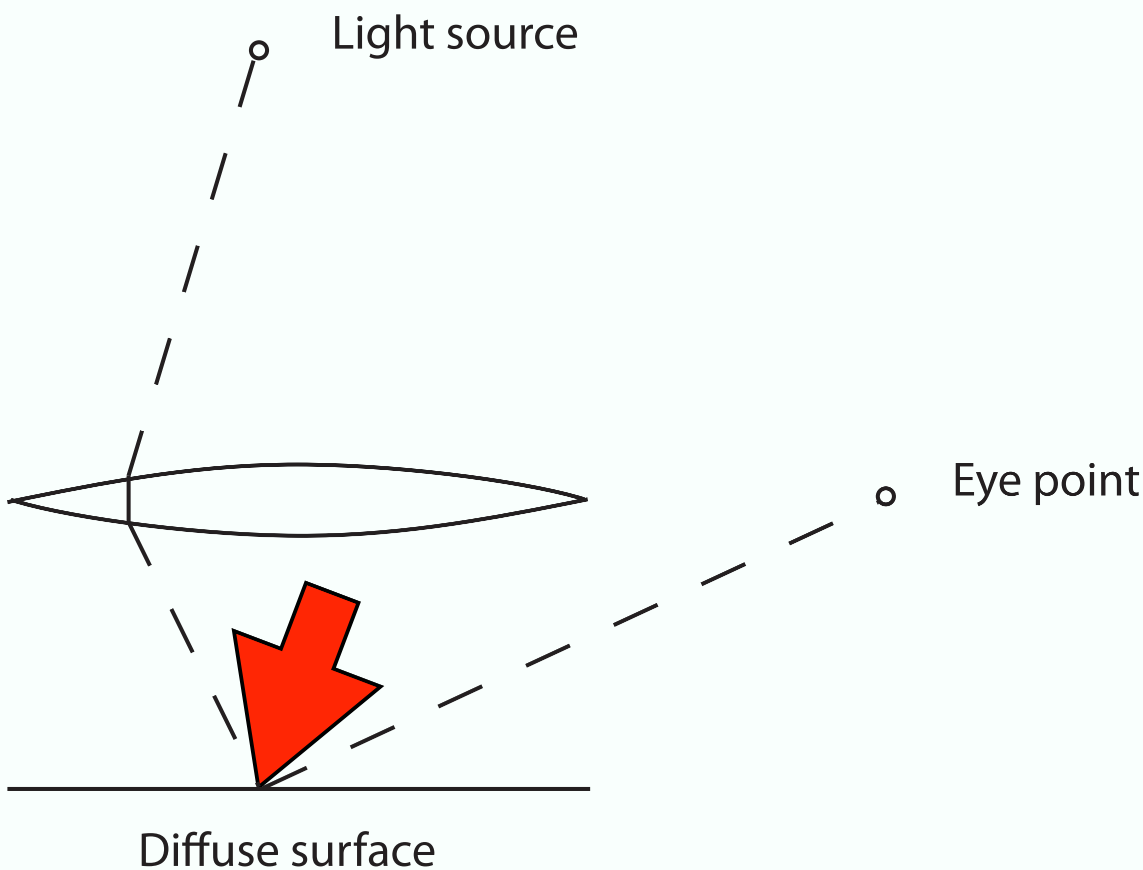


Typical sample locations for
irradiance cache

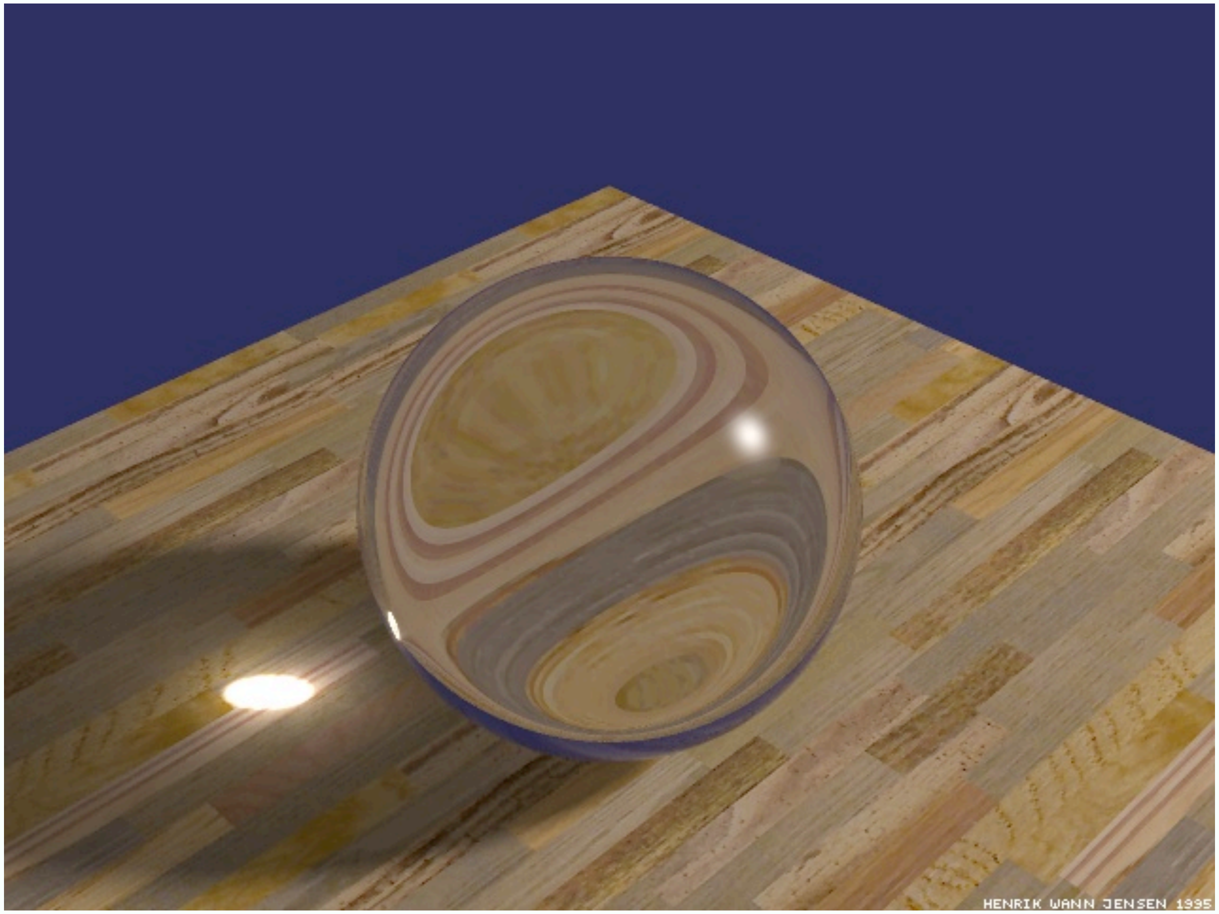


Specular-diffuse transfer creates important effects;
curved surfaces can collect light into caustics

Transmissive - diffuse transfer



Again, hard to find the ray leaving the patch
that finds the light source



Transmissive-diffuse transfer creates important effects;
curved surfaces can collect light into caustics

Transmissive-diffuse transfer creates important effects;
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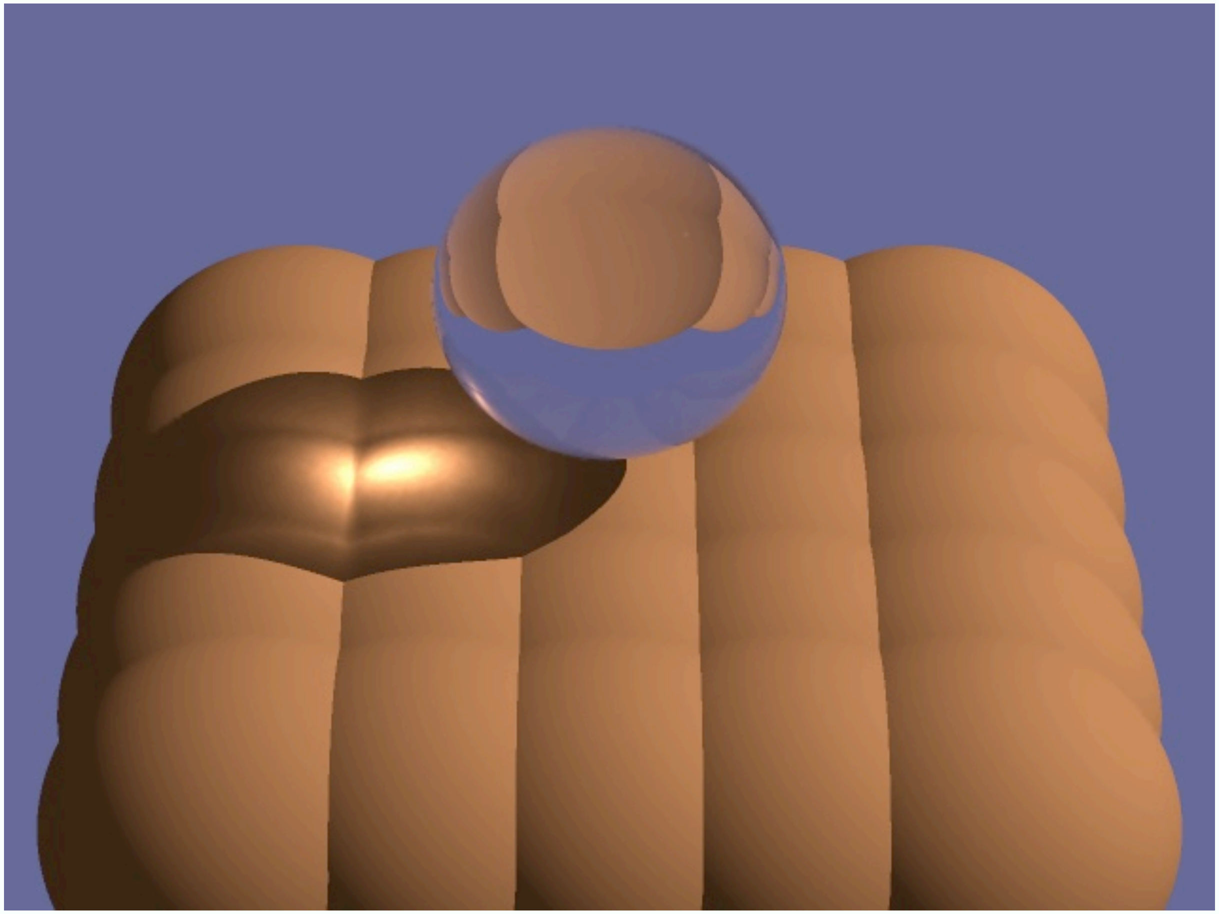

Trasmissive - diffuse transfer creates important effects;
curved surfaces can collect light into caustics

Particle tracing

- Particle starts at source
- At a diffuse surface with albedo ρ, path is
  - continued with probability ρ
  - absorbed with probability 1- ρ
- In either case, it deposits power in a texel at that point
  - particle with power ϕ arriving at texel with area A_t
    - deposits power $\frac{\phi}{A_t}$
    -
- Direction along which path continues is uniform over exit hemisphere

This is a diffuse surface formulation.

More complex surface types

- Specular, transmissive, glossy, etc
  - Must now work with the radiance along path segments, brdf

$$L(\mathbf{x}, \mathbf{x} \to \mathbf{y}) = L_e(\mathbf{x}, \mathbf{x} \to \mathbf{y}) + \int \text{radiance due to irradiance} d\omega$$

- Expand to

$$L(\mathbf{x}, \mathbf{x} \to \mathbf{y}) = L_e(\mathbf{x}, \mathbf{x} \to \mathbf{y}) + \int \rho_{bd}(\mathbf{x} \to \mathbf{y}, \mathbf{u} \to \mathbf{x}) L(\mathbf{x}, \mathbf{u} \to \mathbf{x}) \cos\theta d\omega$$


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Path tracing for general surfaces

- Path tracing
  - Path starts at eye
  - At a surface, path is
    - continued with probability α
    - absorbed with probability 1- α
  - Value of path
    - $L_e(\mathbf{x}_n, \mathbf{x}_n \to \mathbf{x}_{n-1})$ if it arrives at luminaire
    - 0 otherwise
  - Direction along which path continues
    - a draw from P(ω)
  - Weight path segment by

$$\frac{\rho_{bd}(\mathbf{x}_{n-1} \to \mathbf{x}_n, \mathbf{x}_n \to \mathbf{x}_{n+1}) \cos\theta}{P(\omega)\alpha}$$

    - and accumulate these weights

This will do anything,
but with very serious variance problems

Variance problems

- Paths may not find the light often
  - this could be fixed by clever choice of P to heavily emphasize directions toward the source
- Caustics will be poorly rendered, because the path to the source is obscure

Bidirectional path tracing

- Start paths at both eye and light and join them
- Notice:
  - a pair of eye-light paths generates many possible transfer paths
  - we can use each of these, if we compute weights correctly to get integral estimate right

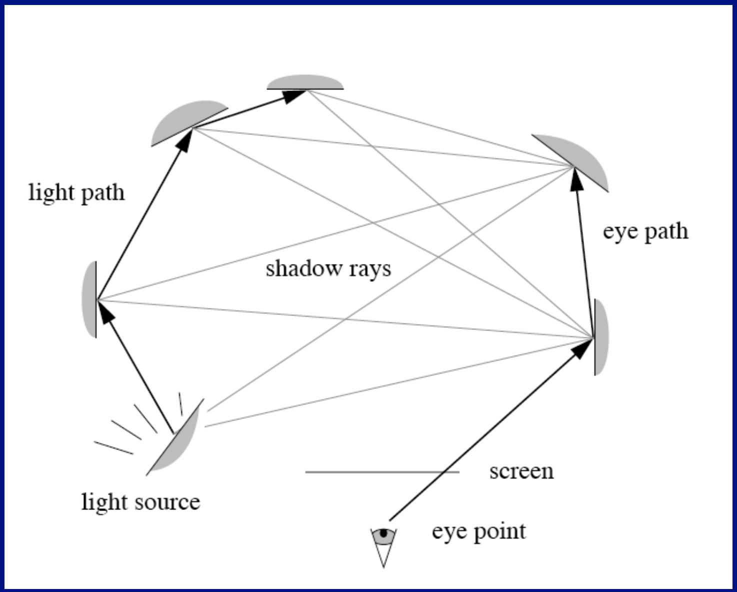

Figure from "Bidirectional path tracing." Lafortune and Willems, 1993

From the eye | From the source | Bidirectional

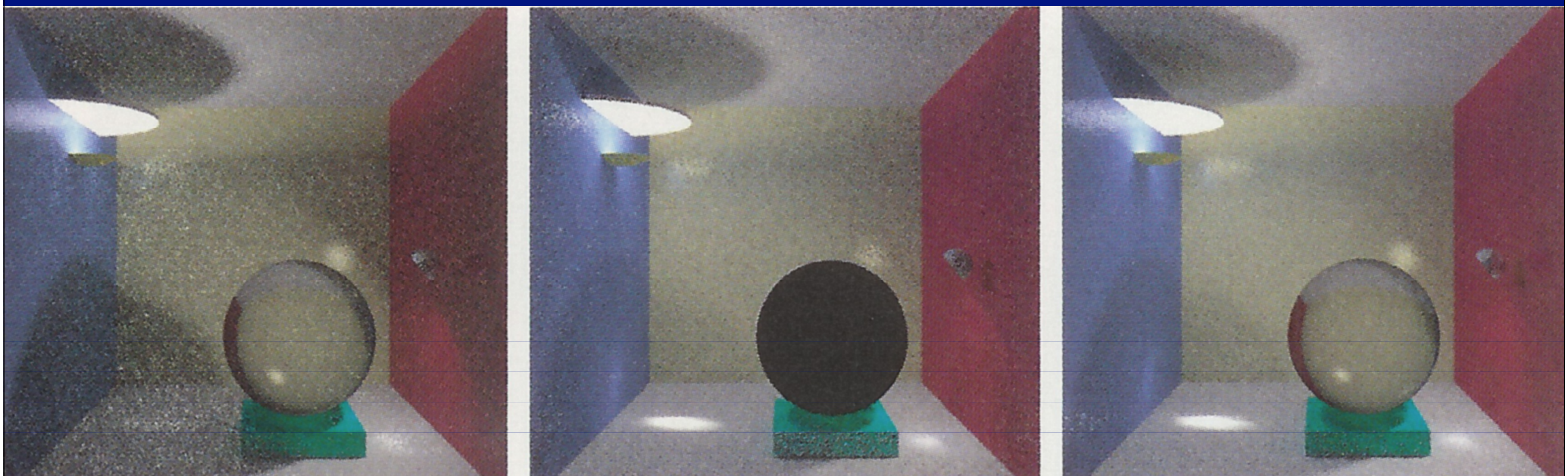

Figure from Dutre, Bekaert, Bala 03; rendered by Suykens-De Laet