Fundamental matrices, epipoles, and all that.

We have 2 perspective cameras

\[ + \]
\[ + \]
\[ s_L \]
\[ x_L \] point in left camera

Q: pt is at \( x_L \) in left, where in right?
A: on a line!

In 3D, \( x_w \) could be here

\( \text{RAY} \ cs \ to \ x_w \)

or here or here

\( x_l \)

\( f_l \)

this ray in right camera.

**Important:** this line follows from left-right camera geometry.

\( x_l \) matches to some point on a line in right cam.

\( \Rightarrow \) if you know geom, you only have to search line for match.
Also: if you move CAMERA from L to R, then $x_L$ turns up somewhere on CSP line. Position depends on depth.

Notice: The line in CRIGHT is special, and easy to predict.

Notice: $x$ lies on line $f_L \rightarrow x_L$.

$f_L$, $x$, $f_R$ define a plane $\Pi$.

$e_R$ is $\Pi \cap \{\text{image plane of right cam}\}$.
Now what happens when there is more than one \( x_e \)?

Notice:

- \( f_L \) and \( f_R \) are on \( \Pi x_e^1 \)
- AND
- on \( \Pi x_e^2 \)

So: for any \( x_e \), the point \( e \) is on \( l(x_e) \)

...this is the epipole...
The lines $l(x_e)$ are called epipolar lines.

Notice we have

$$\begin{bmatrix} l(x_e) \end{bmatrix}^T x_R = 0$$

for $x_e$, $x_R$ corresponding.

(i.e. the point in right image corresponding to $x_e$ is on the line $l(x_e)$)

What is the form of the equation for $l(x_e)$?

$$l(x_e) = \begin{bmatrix} a(x_e), b(x_e), c(x_e) \end{bmatrix}$$

but what are these as functions?
Think about eqn of $\Pi(x_L)$

$$\det \begin{bmatrix} f_R & f_L & x_L & x \\ y & z & T \end{bmatrix} = 0$$

(i.e.)

$$\begin{pmatrix} \text{something linear in } x_L \end{pmatrix} X + \begin{pmatrix} \text{something linear in } x_L \end{pmatrix} Y + \begin{pmatrix} \text{something linear in } x_L \end{pmatrix} Z + \begin{pmatrix} \text{something linear in } x_L \end{pmatrix} T = 0$$

And we intersect this with image plane of $R$ camera.
But there aren't any $x_k$ terms.

In this intersection:

\[ a(x_k) = \text{(something linear in } x_k) \]

\[ b(x_k) = \text{''} \]

\[ c(x_k) = \text{''} \]

So

\[ a = E \]

\[ b = \begin{bmatrix} 3 \\ x_k \end{bmatrix} \]

\[ c \]

So there is a matrix $F$ such that

\[ \begin{bmatrix} x_k \end{bmatrix}^T F \begin{bmatrix} x_k \end{bmatrix} = 0 \]

\[ \begin{bmatrix} 3 \\ x_k \end{bmatrix} F \begin{bmatrix} 3 \\ x_k \end{bmatrix} = 0 \]

\[ \begin{bmatrix} 3 \\ x_k \end{bmatrix} \]

fundamental matrix
The fundamental matrix has an important property.

\[(x_k^T F) e_R = 0 \quad \text{for ANY } x_k\]

(why? look at pic; \(e\) is on any \(e(x_k)\).)

\[\text{so} \quad x_k^T F e_R = 0\]

\[\text{so} \quad \det F = 0\]
Reasons to care

It tells us how points move when camera moves.

\[ \begin{align*}
\text{Translate} & \quad \rightarrow \\
+ & \quad + \\
\infty & \quad \infty
\end{align*} \]

\( e \) is at infinity.

It's enough to draw epipolar lines, epipoles on R camera.

\[ \text{Diagram} \]
Rotate + translate
Importance:

1) Tells us where to look for matches.