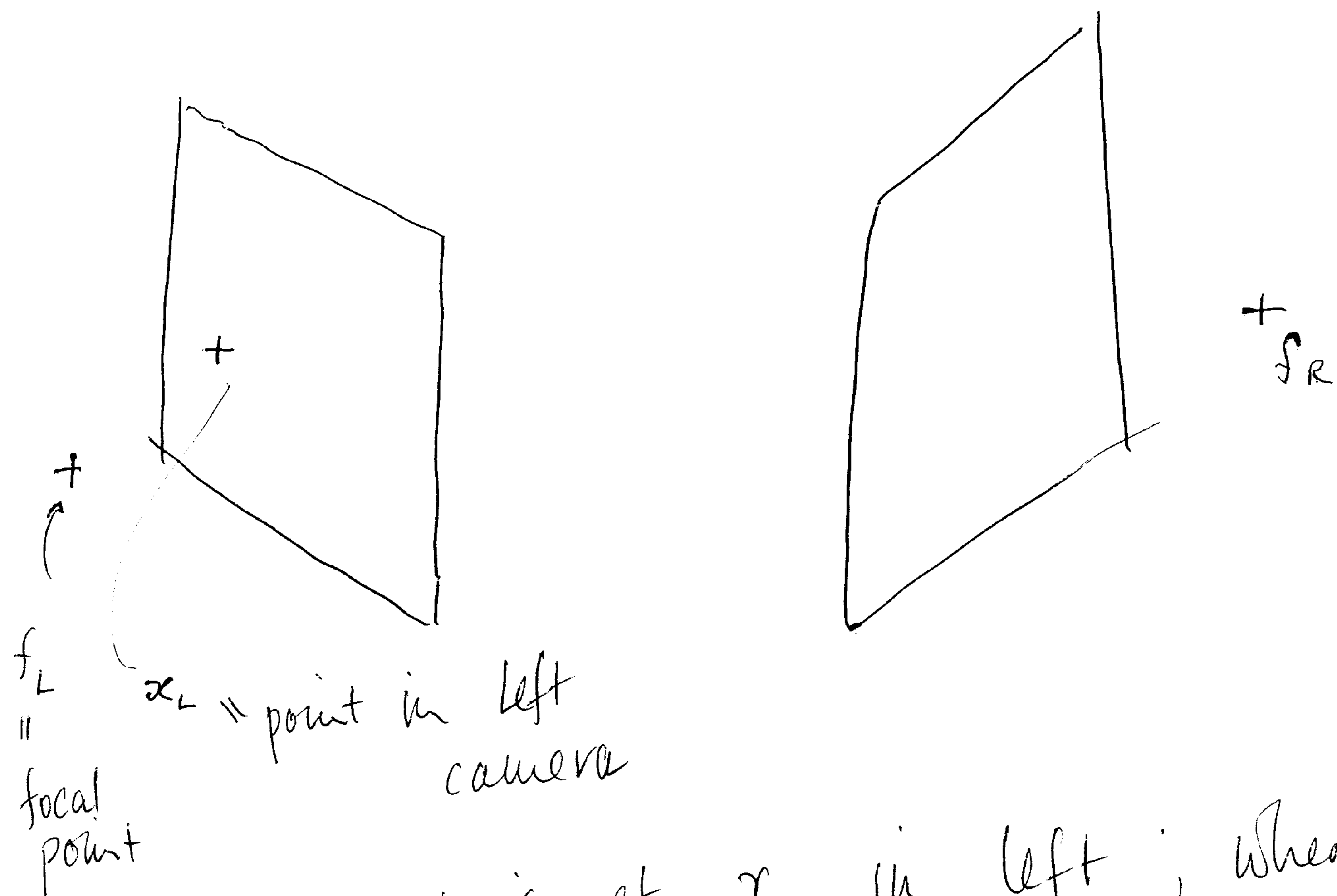
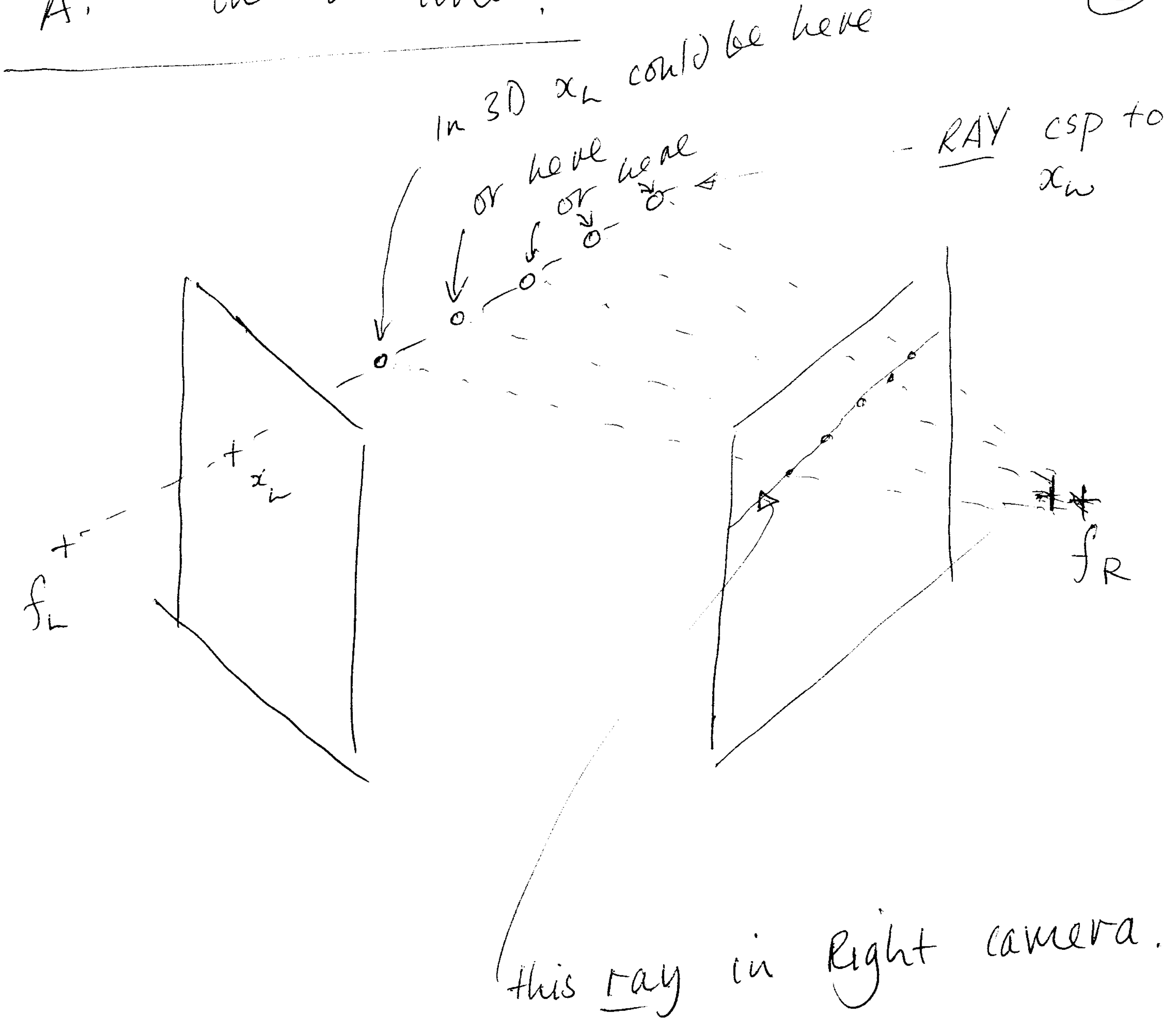
Fundamental matrices, Epipoles, and all that.

- We have 2 perspective cameras



Q: pt is at x_L in <u>left</u>; where in right?

A: on a line!



Important: • this line follows from left-right camera geometry

• x_L matches to some point on a line in Right cam

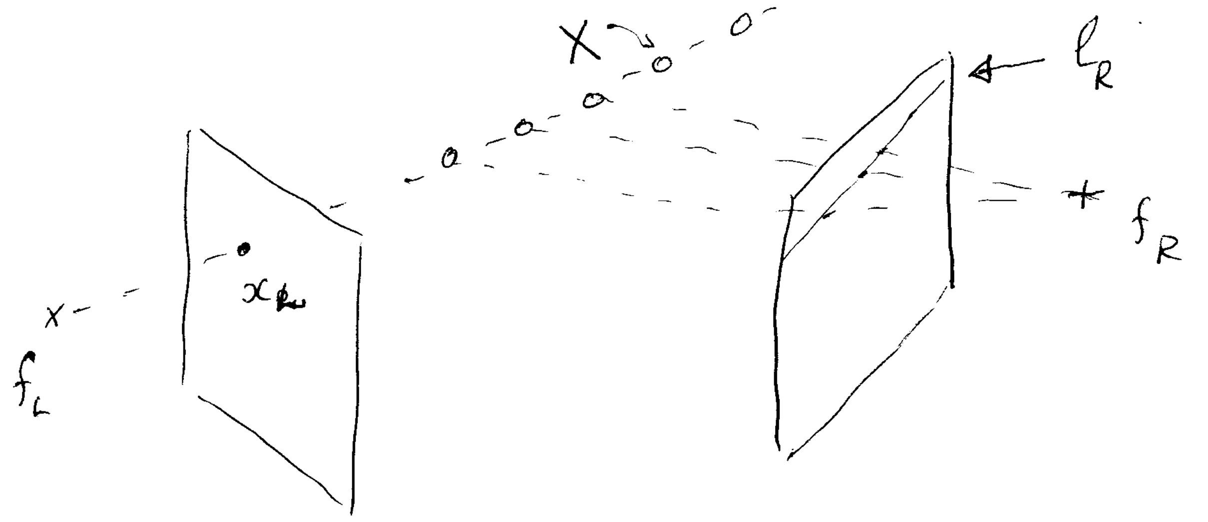
$\Rightarrow$ if you know geom, you only have to search line for match

Also:

- if you move CAMERA from x_L L to R, then x_L turns up somewhere on csp <u>line</u>
- position depends on depth.

Notice:

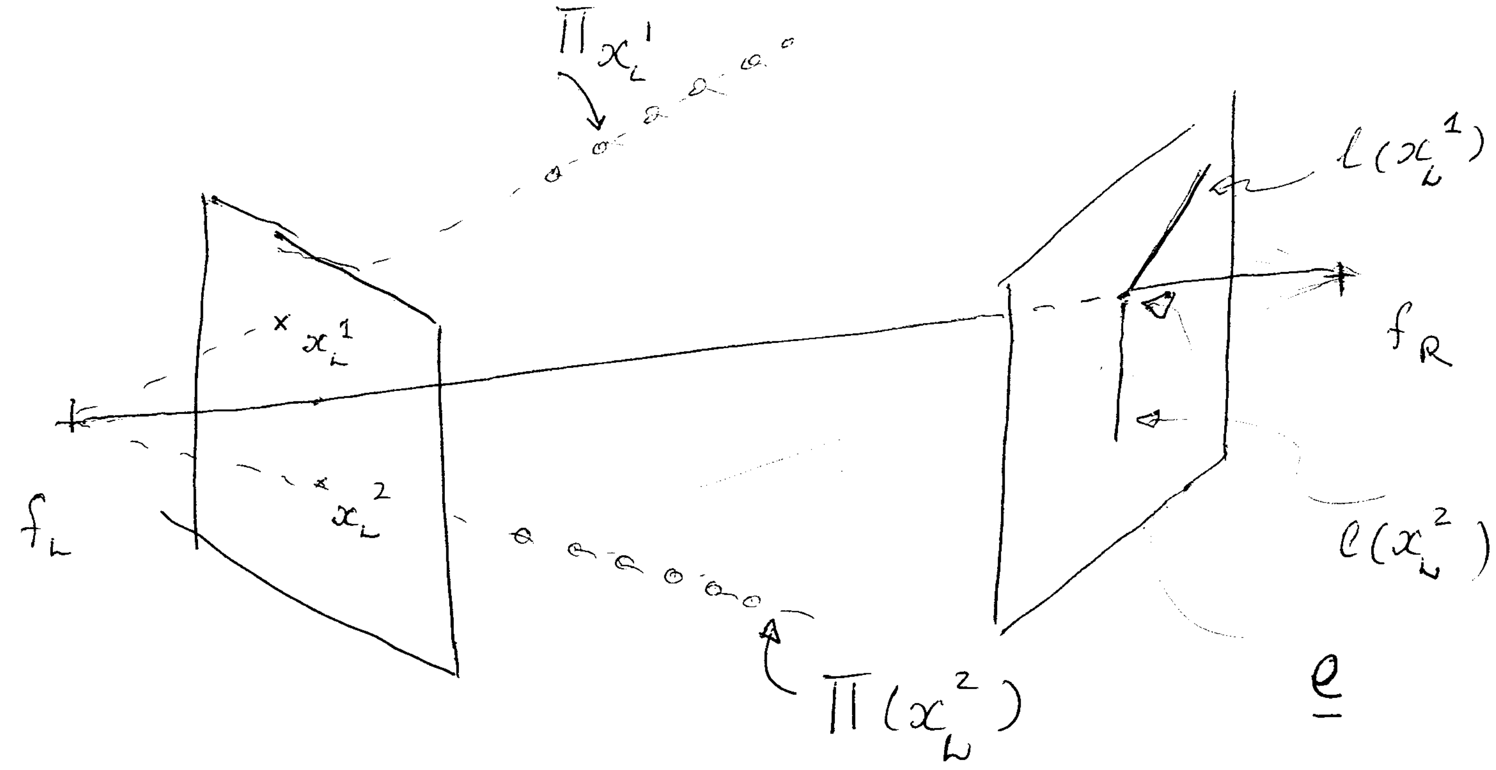
- The line in C RIGHT is special, and easy to predict



NOTICE

- X lies on line $f_L \to x_L$
- f_L, X, f_r define a <u>plane</u> Π
- l_r is $\Pi \cap \{\text{image plane of right cam.}\}$

Now what happens when there is more than one x_L ?



<u>Notice</u> :

f_L and f_R are on $\Pi_{x_L^1}$

AND

on $\Pi_{x_L^2}$

<u>so</u> : • for <u>any</u> x_e , the point $\underline{e}$ is on $l(x_L)$

• this is the <u>epipole</u>

- The lines $l(x_L)$ are called <u>epipolar lines</u>

- Notice we have

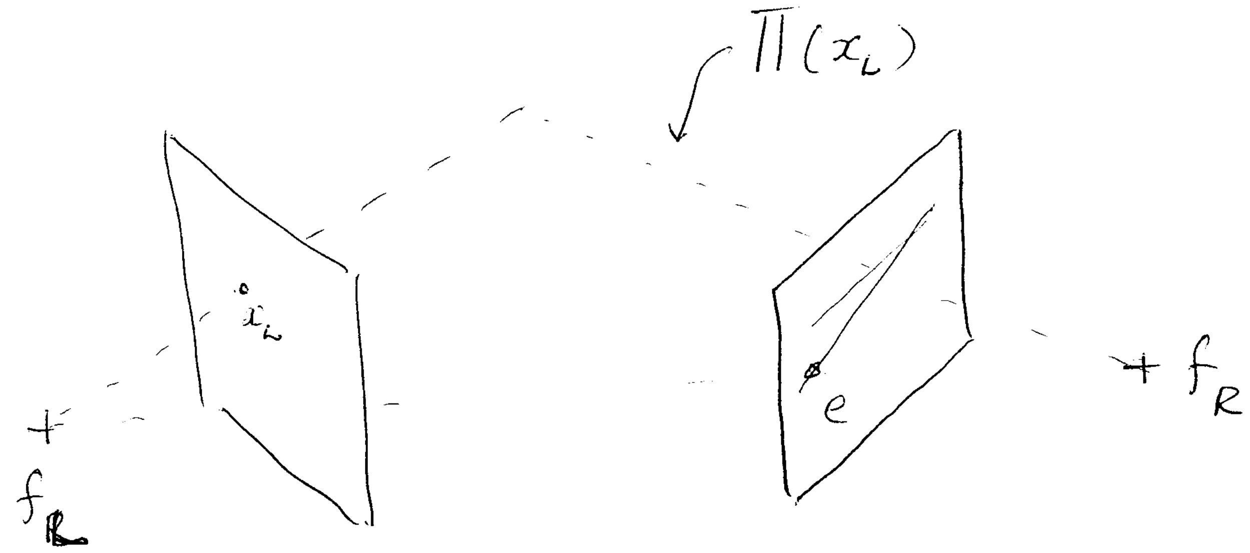
$$[l(x_L)]^T x_R = 0$$

for x_L, x_R corresponding.
(i.e. the point in right image csp to x_L is on the line $l(x_L)$)

- What is the form of the equation for $l(x_L)$?

$$l(x_L) = [a(x_e), b(x_e), c(x_e)]$$

but what are these as functions?



Think about eqn of $\Pi(x_L)$

$$\det\left[\left(f_R,\ f_L,\ x_L,\ \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}\right)\right] = 0$$

(i.e.:

$$\begin{aligned} &(\text{something linear in } x_L)\, X \;+ \\ &(\quad \text{''} \quad)\, Y \;+ \\ &(\quad \text{''} \quad)\, Z \;+ \\ &(\quad\quad)\, T = 0 \end{aligned}$$

And we intersect this with image plane of R camera.

But there aren't any x_L terms (7)
in this intersection.

So $a(x_L) =$ (something linear in x_L)

$b(x_L) =$ "

$c(x_L) =$ "

So $\begin{matrix} a \\ b \\ c \end{matrix} = \overset{3\times 3}{G} \; \overset{3\times \cancel{3} \text{ HCs}}{x_L}$

So there is a matrix F such that

$$\underset{1\times 3}{x_L^T} \; \overset{3\times 3}{F} \; \overset{3\times 1}{x_R} = 0$$

fundamental matrix

The fundamental matrix has
an important property.

$$\left(x_L^T \mathcal{F}\right) e_R = 0$$ for ANY x_L

(why? look at pic; $\underline{e}$ is on any $l(x_L)$.

so $x_L^T \mathcal{F} e_R = 0$

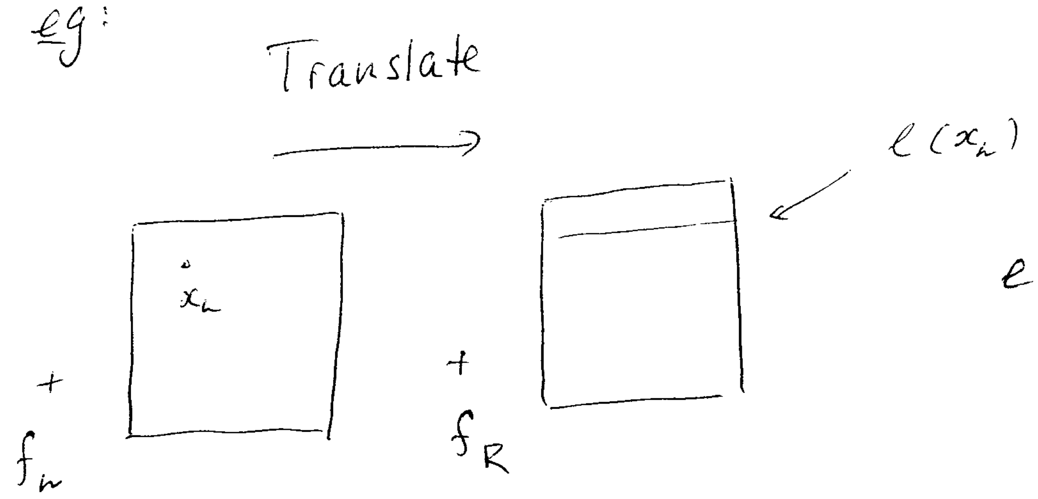
so

$$\det \mathcal{F} = 0 .$$

Reasons to care

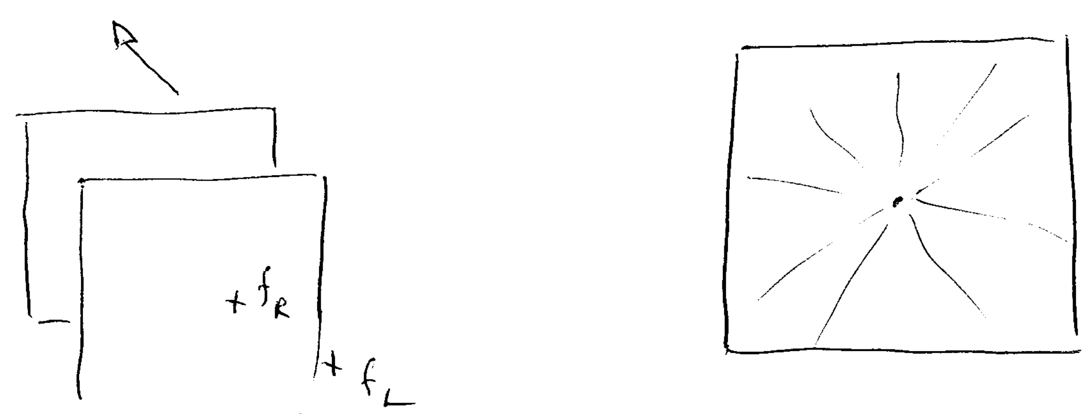
- F tells us how points move when camera moves

eg:

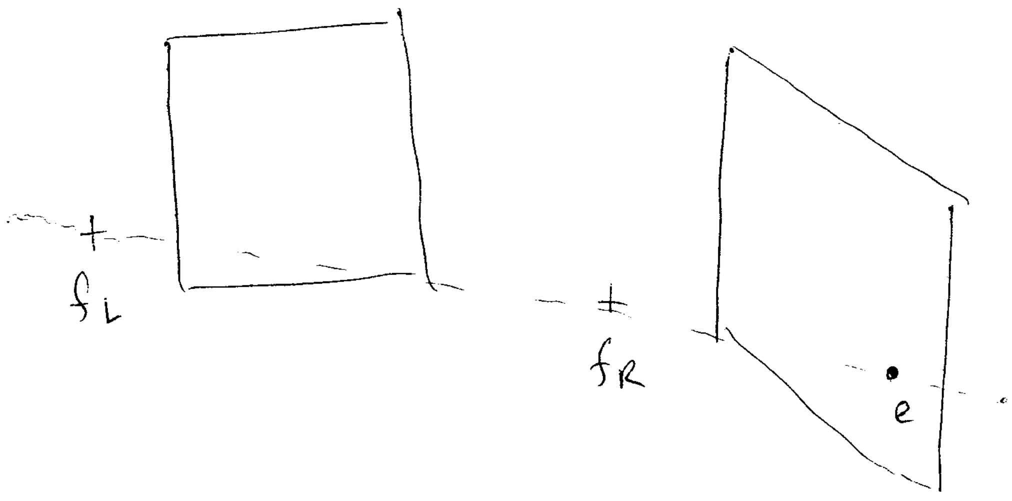


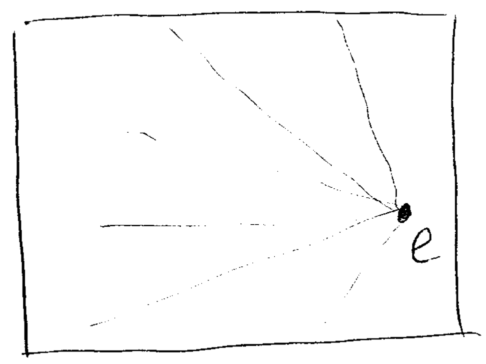
e is at infinity.

It's enough to draw epipolar lines, epipoles on R camera.



Rotate + translate





C right view

Importance:

1) Tells us where to look for matches.