UNIVERSITY OF ILLINOIS, URBANA-CHAMPAIGN

Department of Computer Science **CS 419 : Production Computer Graphics**

Professor: David Forsyth

Final Examination

NAME:——————

Fill the answers and your name in on the exam, and return it. You have 180 mins.

There are a total of 56 marks available, and full marks is 56 marks.

Good luck, and be careful.

Important note: by submitting your exam for grading, you are certifying that you referred only to your single page of notes in preparing your answer, and not to any other source of reference, including the papers of other students.

Question	Marks	Out of
Rendering		10
Bezier Curves		5
B-Splines		7
Animation		11
Tensor Products		8
More Animation		10
Bits and Pieces		13

Rendering

- A focal point is at $(0, 0, 0)$. An image plane is at $z = 1$. Write the points on the image plane $(x, y, 1)$. A sphere with unit radius has center at $(2, 3, 4)$. Consider the family of rays from the focal point to some point on the image plane; each ray is given by a choice of (x, y). Write an equation for the family of rays that are *tangent* to this sphere. (4)

- What is a caustic? (briefly) (1)

- What is the difference between a refraction caustic and a reflection caustic? (briefly) (1)

- Give one example of a refraction caustic and one of a reflection caustic? (briefly) (1)

- In the space below, sketch a geometry consisting of an eye point, a mirror, a point light source, and two diffuse surfaces where backward ray tracing (i.e. tracing rays from the light source) is required to get a correct image. Why is backward ray tracing necessary? (briefly) (3)

Bezier Curves

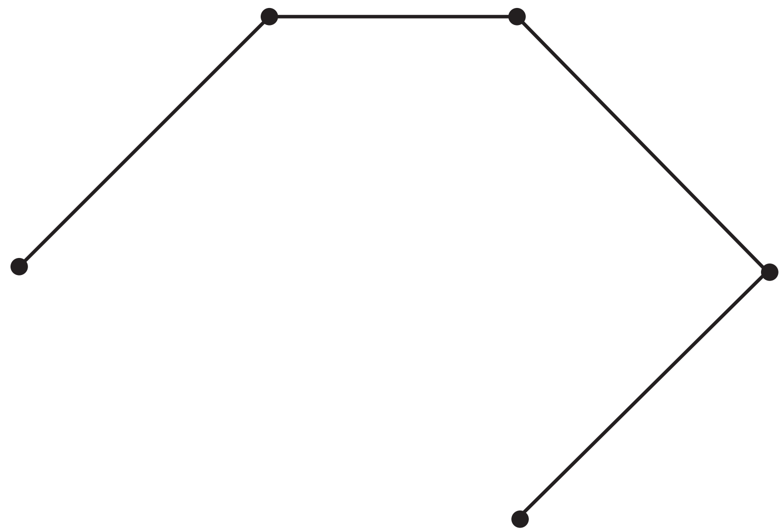

Figure 1:

- Figure 1 shows the control structure for a Bézier curve. Sketch the curve. (3)
- Figure 2 shows the control structure for a Bézier curve. Draw the line tangent to the curve at the point $t = 0.5$, showing your working. (2)

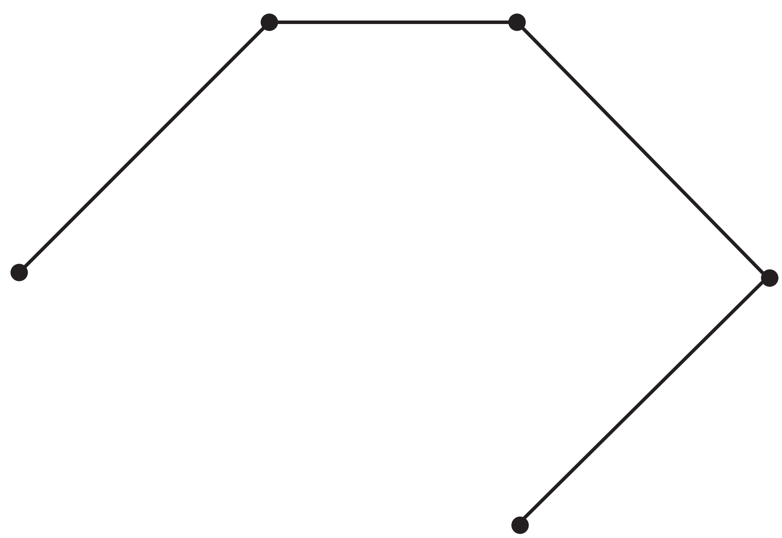

Figure 2:

B-Splines

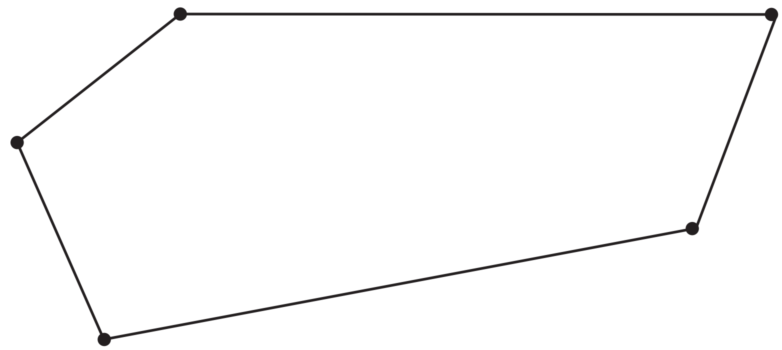

Figure 3:

- Figure 3 shows the control structure for a periodic B-spline curve of order three (degree two). Sketch the curve. (3)
- Figure 4 shows the control structure for a cubic B-spline curve (order four, degree three). Show the control points for the two cubic B-spline curves produced by one step of B-spline subdivision, as described in class. Show your working. (4)

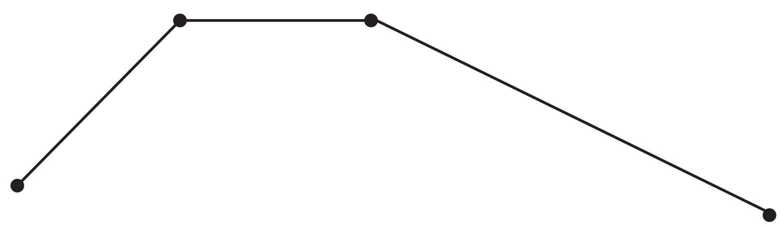

Figure 4:

Animation

- A particle of mass m is at position $\mathbf{x}_0$ at time t in a potential field $\phi(\mathbf{x})$ and has velocity $\mathbf{v}_0$.
  1. Write an expression for its acceleration. (2)
  2. Write an expression for its position and velocity at time $t + \Delta t$. (2)
- We could use a potential field to resolve collisions between particles and rigid objects.
  1. How? (briefly) (2)
  2. Explain what might go wrong, briefly. (2)
- Why are particle systems so widely used in animation? (briefly; 3 reasons) (3)

Tensor Product Surfaces

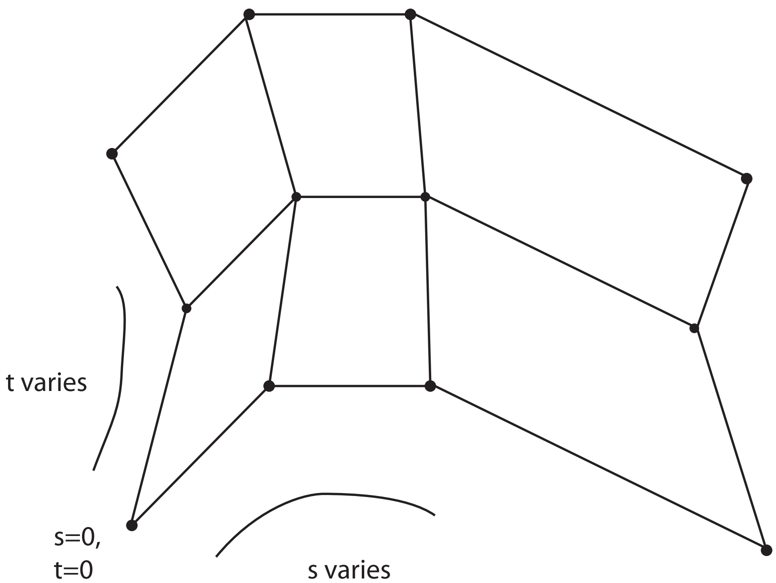

Figure 5:

- Figure 5 shows the control structure for a tensor product Bézier surface. Mark the point $(s, t) = (0.2, 0.8)$ on the drawing, showing your working. (4)
- Name an important advantage of the B-spline surface representation over the Bézier surface representation. (1)
- Describe (briefly) an easy way to compute an approximate intersection between two B-spline surfaces. (3)

More Animation

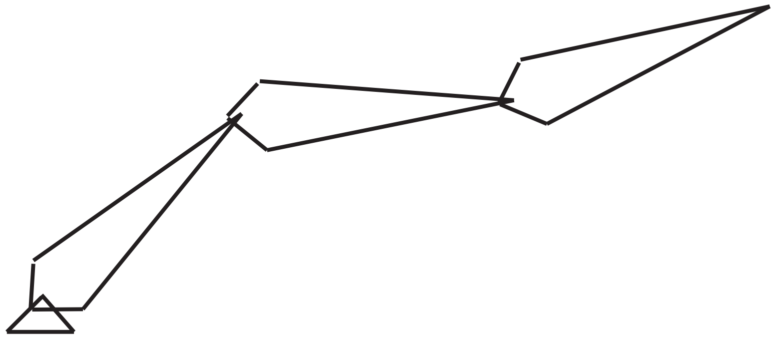

Figure 6:

Figure 6 shows a simple kinematic chain in 2D. All three joints are rotational joints. The endpoint is currently at $\mathbf{x}$ and the angles are currently in state θ.

- What is the difference between forward and inverse kinematics? (2)

- Write $\mathbf{x}(\theta)$ for the endpoint of the chain as a function of the angles and write $\mathcal{J}$ for the matrix of partial derivatives (Jacobian) of this function. I would like to move the endpoint from $\mathbf{x}$ to $\mathbf{x} + \delta\mathbf{x}$. Write an equation for the change in angles that would be required to move the endpoint (you do not need to solve the equation, and you do not need to compute the elements of the Jacobian). (2)

- Is there a unique solution to this equation in this case (i.e. for the manipulator shown here)? Why? (2)

- Are there manipulators for which no solution would be possible? Explain why? (2)

- Briefly explain what kinematic redundancy is; are humans kinematically redundant. (2)

Bits and Pieces

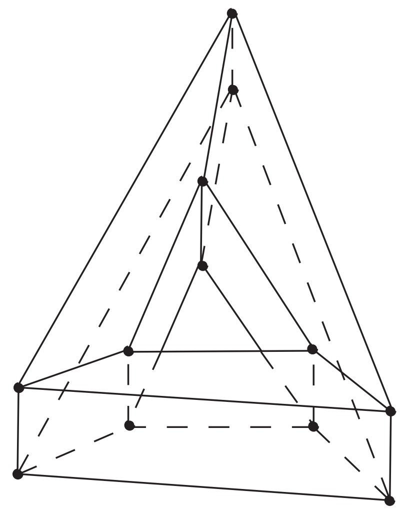

Figure 7:

- Figure 7 shows a simple surface mesh (vertices are filled circles; hidden lines are dashed).
  - Are there any extraordinary vertices? (1)
  - There are 12 faces, 24 edges and 12 vertices in this mesh. We apply one pass of Catmull-Clark subdivision. How many faces, edges and vertices are there in the resulting mesh? (3)

- Can any tensor product Bézier surface also be represented as a B-spline surface? (1)
- Does a photon map give an unbiased estimate of intensity? (1)
- I wish to estimate $\int_{10^{-5}}^{1} \frac{1}{x^2} dx$ using random samples. I draw 10 independent identically distributed samples x_i from the uniform distribution on $[10^{-5}, 1]$. If it helps, $\frac{d}{dx} x^{-1} = -x^{-2}$
  1. I estimate the integral as $\sum_i \frac{1}{x_i}$, but this is wrong. What is right? (2)

  2. I now make two different estimates, using two different sets of ten samples. Are they the same? why? (2)

  3. I would like to improve my estimate, but find that increasing the number of samples is not particularly helpful. Compute $\int_{10^{-5}}^{0.1} \frac{1}{x^2} dx$ and $\int_{0.1}^{1} \frac{1}{x^2} dx$ and use these two values to explain why. (2)

  4. What can I do to improve the estimate without increasing the number of samples (and without looking the answer up in tables, knowing the integral, etc.)? (1)