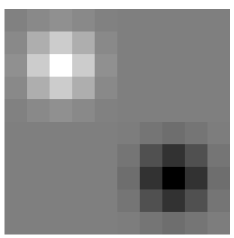
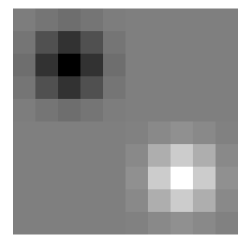
Note: Filtering vs. "convolution"

- In classical signal processing terminology, convolution is filtering with a *flipped* kernel, and filtering with an upright kernel is known as *cross-correlation*
  - Check convention of filtering function you plan to use!

Filtering or "cross-correlation"
(Kernel in original orientation)



"Convolution"
(Kernel flipped in x and y)



Adapted from [D. Fouhey and J. Johnson]

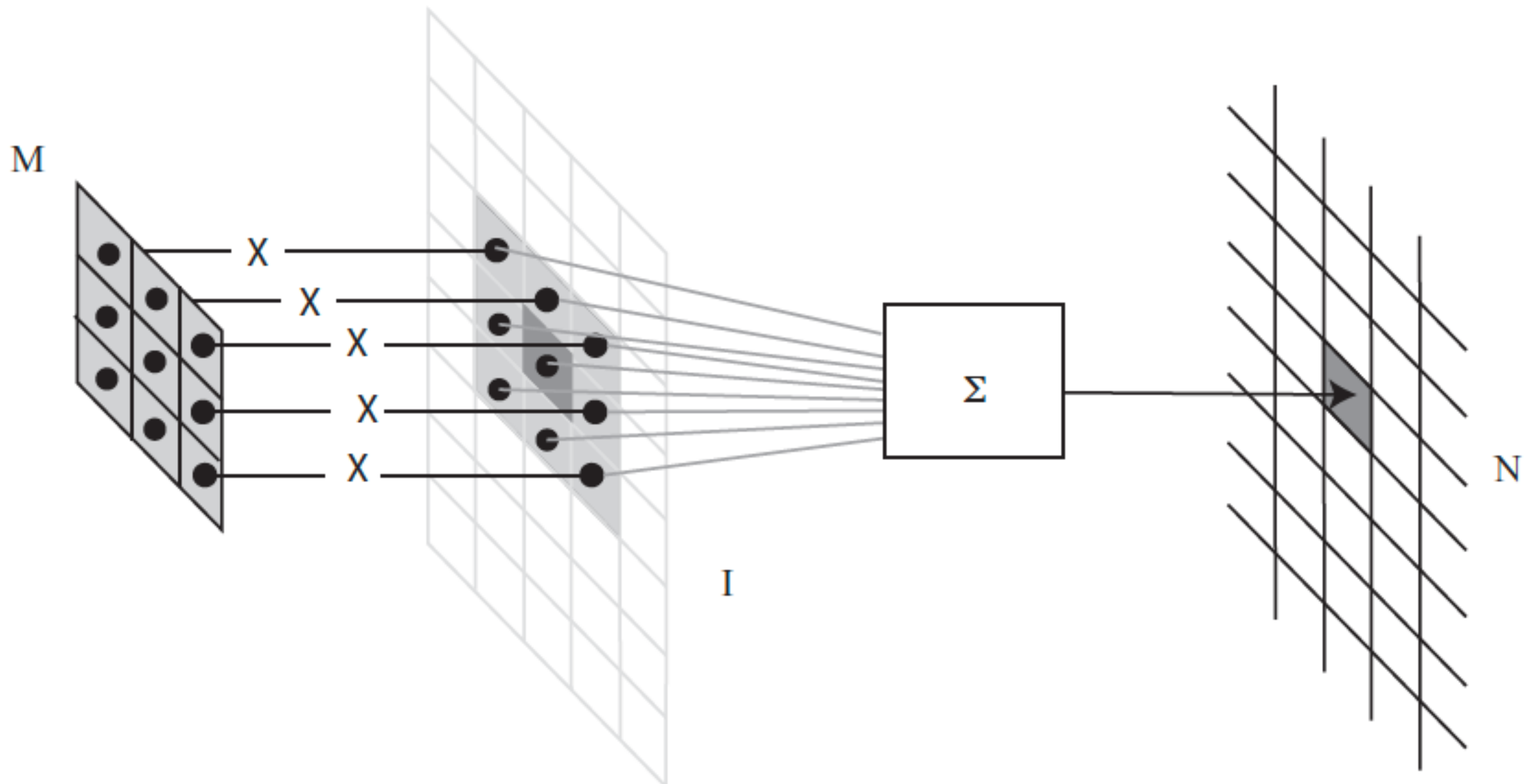


FIGURE 3.1: *To compute the value of $\mathcal{N}$ at some location, you shift a copy of $\mathcal{M}$ (the flipped version of $\mathcal{W}$) to lie over that location in $\mathcal{I}$; you multiply together the non-zero elements of $\mathcal{M}$ and $\mathcal{I}$ that lie on top of one another; and you sum the results.*

Convolution

For the moment, think of an image as a two dimensional array of intensities. Write $\mathcal{I}_{ij}$ for the pixel at position i, j. We will construct a small array (a *mask* or *kernel*) $\mathcal{W}$, and compute a new image $\mathcal{N}$ from the image and the mask, using the rule

$$\mathcal{N}_{ij} = \sum_{uv} \mathcal{I}_{i-u,j-v} \mathcal{W}_{uv}$$

which we will write

$$\mathcal{N} = \mathcal{W} * \mathcal{I}.$$

In some sources, you might see $\mathcal{W} * * \mathcal{I}$ (to emphasize the fact that the image is 2D). We sum over all u and v that apply to $\mathcal{W}$; for the moment, do not worry about what happens when an index goes out of the range of $\mathcal{I}$. This operation is known as *convolution*, and $\mathcal{W}$ is often called the *kernel* of the convolution. You should

Filtering

look closely at the expression; the "direction" of the dummy variable u (resp. v) has been reversed compared with what you might expect (unless you have a signal processing background). What you might expect – sometimes called *correlation* or *filtering* – would compute

$$\mathcal{N}_{ij} = \sum_{uv} \mathcal{I}_{i+u,j+v} \mathcal{W}_{uv}$$

which we will write

$$\mathcal{N} = \texttt{filter}(\mathcal{I}, \mathcal{W}).$$

This difference isn't particularly significant, but if you forget that it is there, you compute the wrong answer.