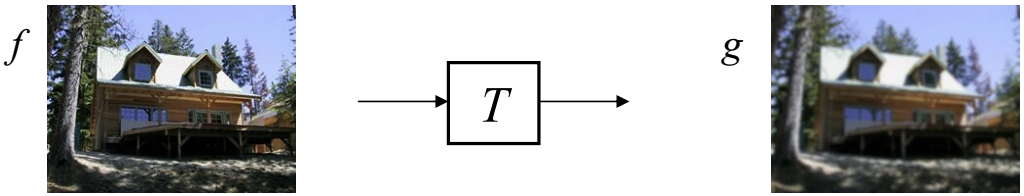
Image filtering

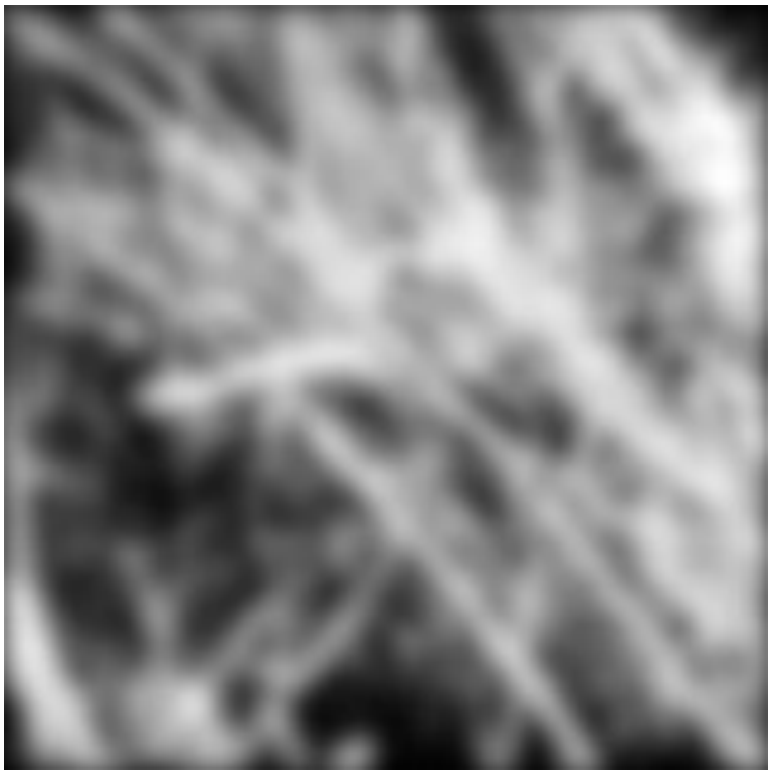
- Roughly speaking, replace image value at x with some function of values in its spatial neighborhood $N(x)$:

$$g(x) = T(f(N(x)))$$



- Examples: smoothing, sharpening, edge detection, etc.

Image filtering



Recall: Image transformations

- What are different kinds of image transformations?
  - Range transformations or point processing
  - Image warping
  - Image filtering

Image filtering: Outline

- Linear filtering and its properties
- Gaussian filters and their properties
- Nonlinear filtering: Median filtering
- Fun filtering application: Hybrid images

Sliding window operations

- Let's slide a fixed-size window over the image and perform the same simple computation at each window location
- Example use case: how do we reduce image noise?
  - Let's take the *average* of pixel values in each window
  - More generally, we can take a *weighted sum* where the weights are given by a *filter kernel*

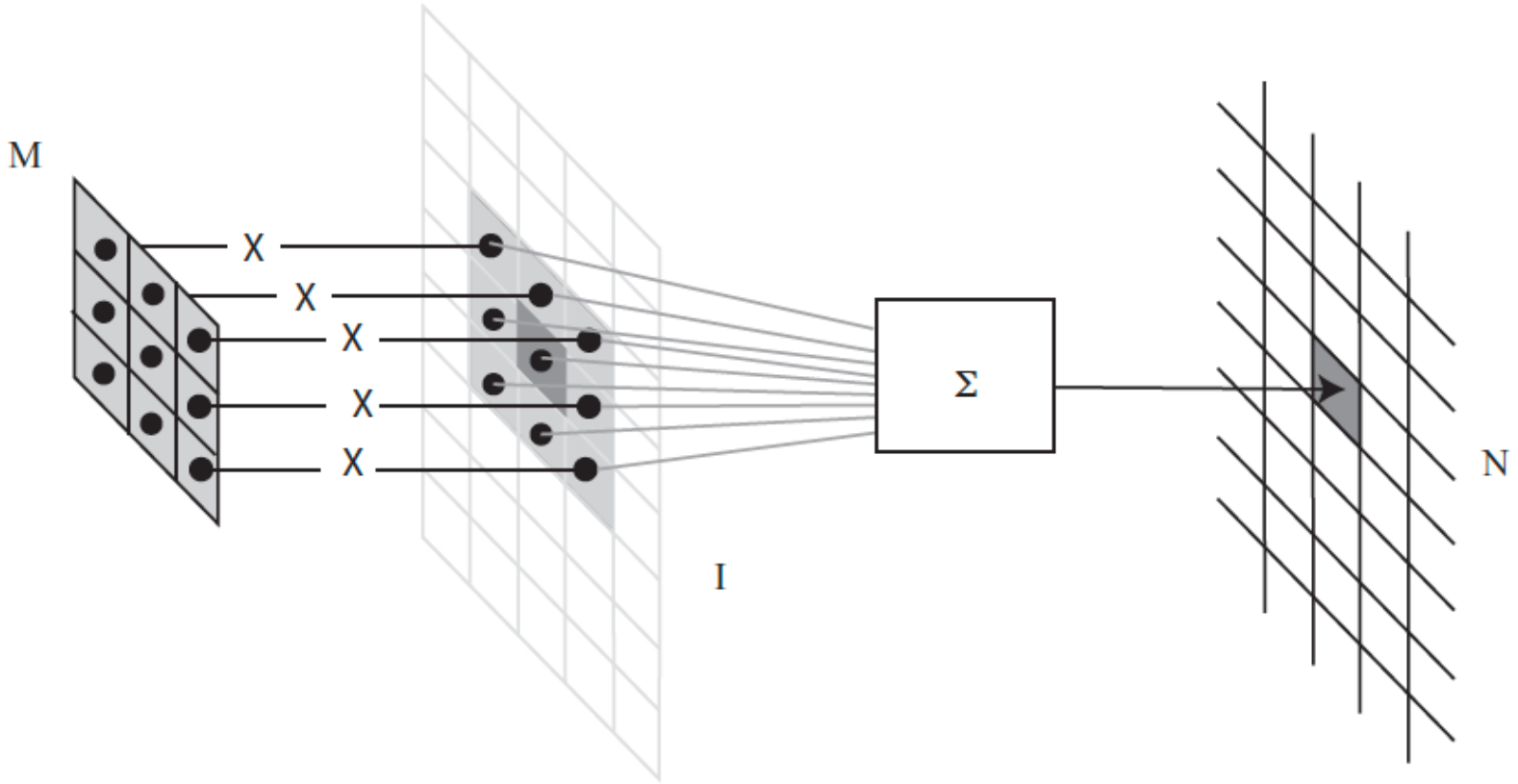
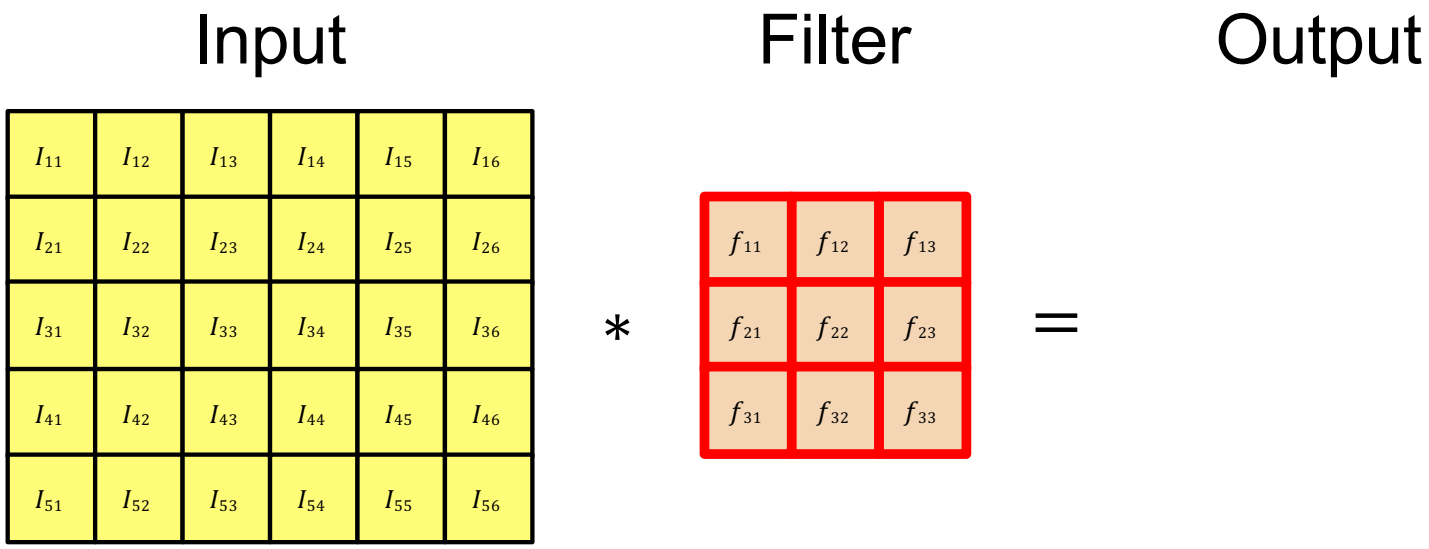


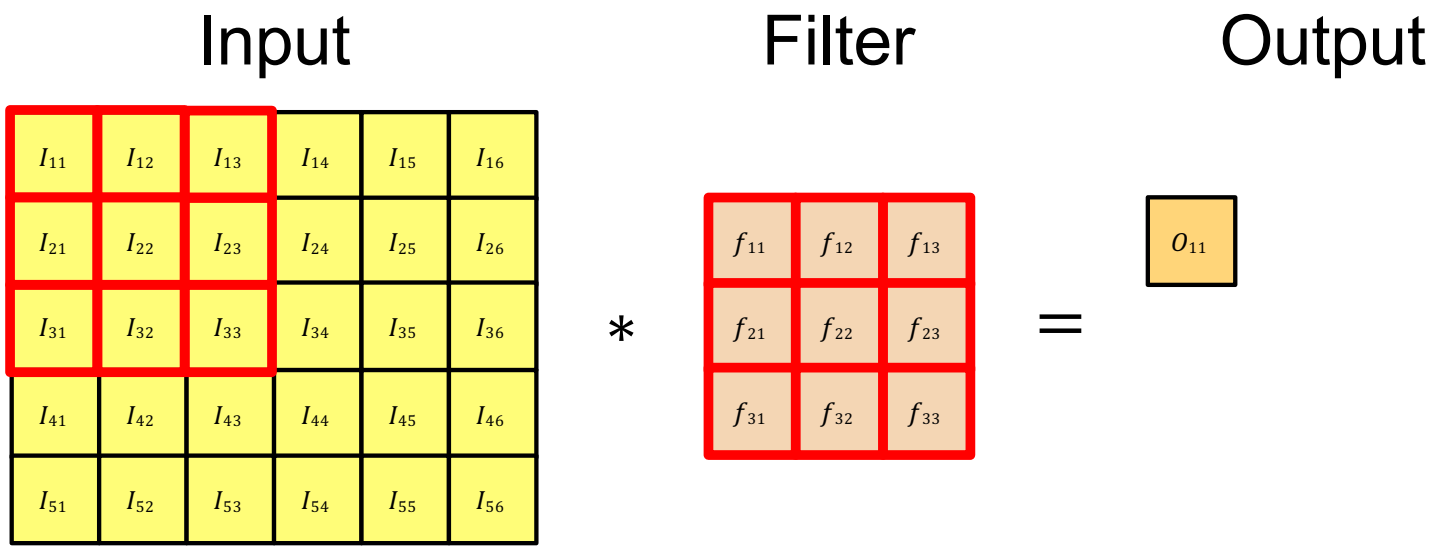
FIGURE 3.1: *To compute the value of $\mathcal{N}$ at some location, you shift a copy of $\mathcal{M}$ (the flipped version of $\mathcal{W}$) to lie over that location in $\mathcal{I}$; you multiply together the non-zero elements of $\mathcal{M}$ and $\mathcal{I}$ that lie on top of one another; and you sum the results.*

Applying a linear filter



Adapted from D. Fouhey and J. Johnson

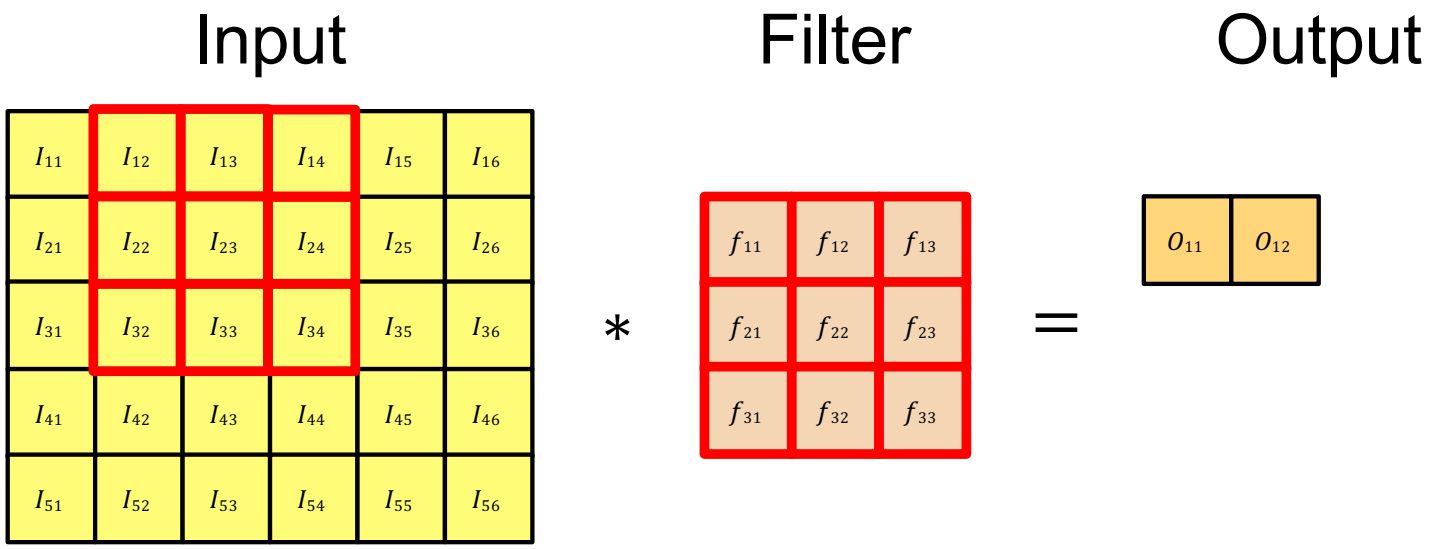
Applying a linear filter



$$O_{11} = I_{11} \cdot f_{11} + I_{12} \cdot f_{12} + I_{13} \cdot f_{13} + \ldots + I_{33} \cdot f_{33}$$

Adapted from D. Fouhey and J. Johnson

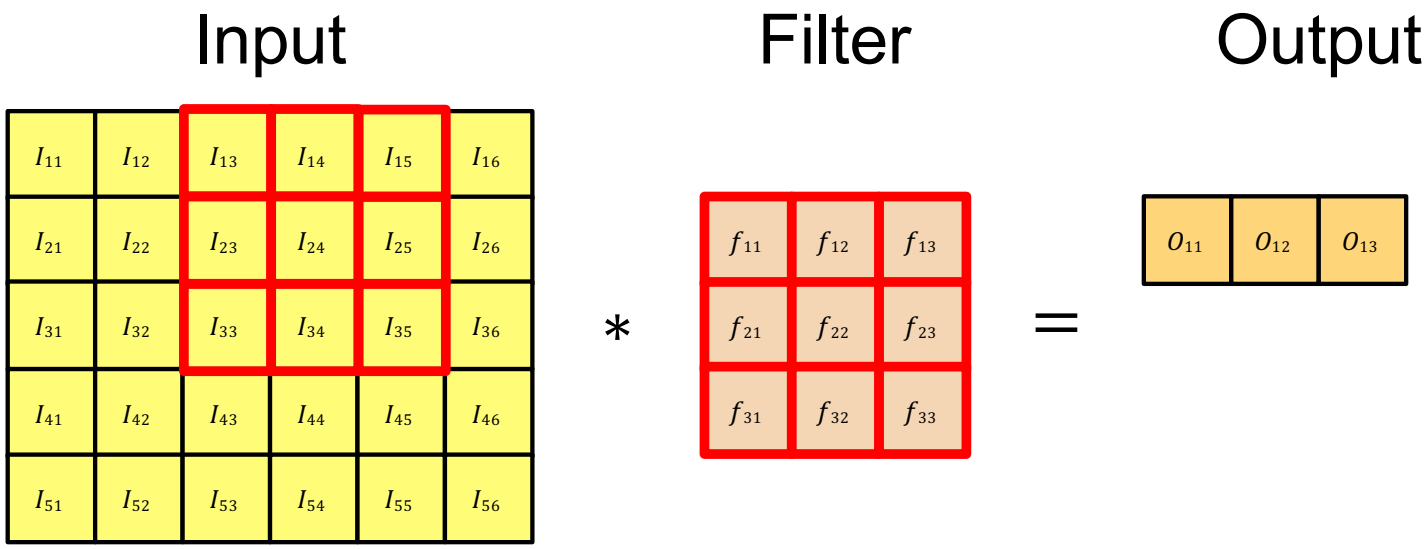
Applying a linear filter



$$O_{12} = I_{12} \cdot f_{11} + I_{13} \cdot f_{12} + I_{14} \cdot f_{13} + \ldots + I_{34} \cdot f_{33}$$

Adapted from D. Fouhey and J. Johnson

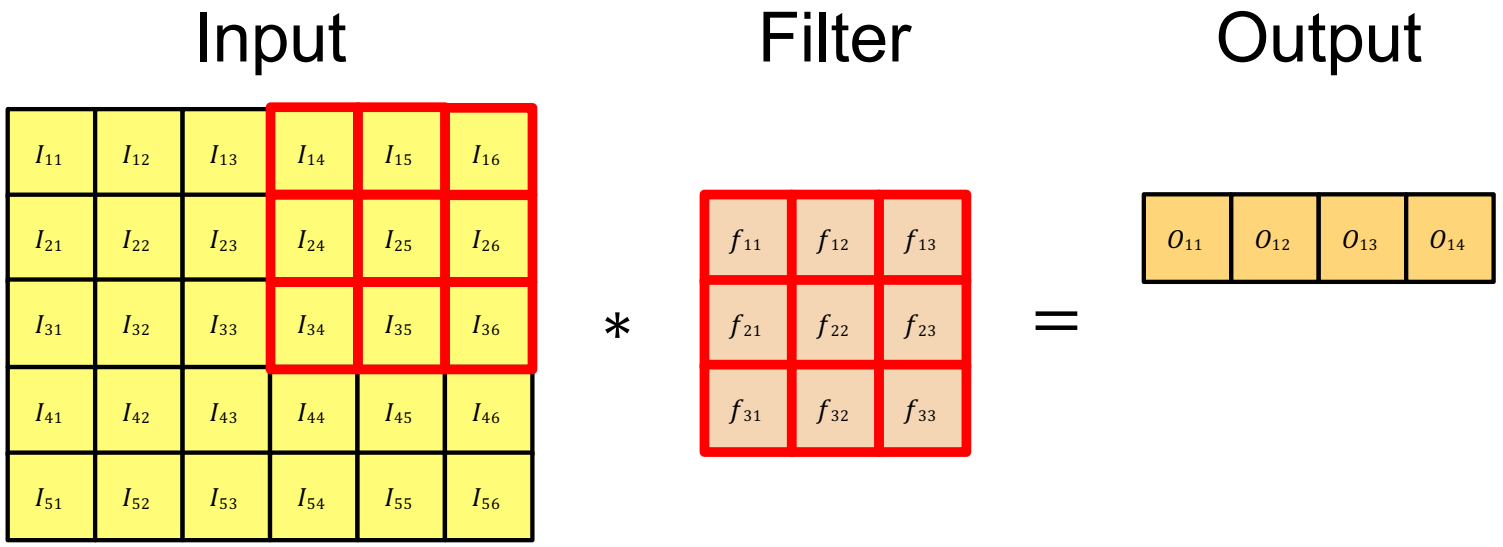
Applying a linear filter



$$O_{13} = I_{13} \cdot f_{11} + I_{14} \cdot f_{12} + I_{15} \cdot f_{13} + \ldots + I_{35} \cdot f_{33}$$

Adapted from D. Fouhey and J. Johnson

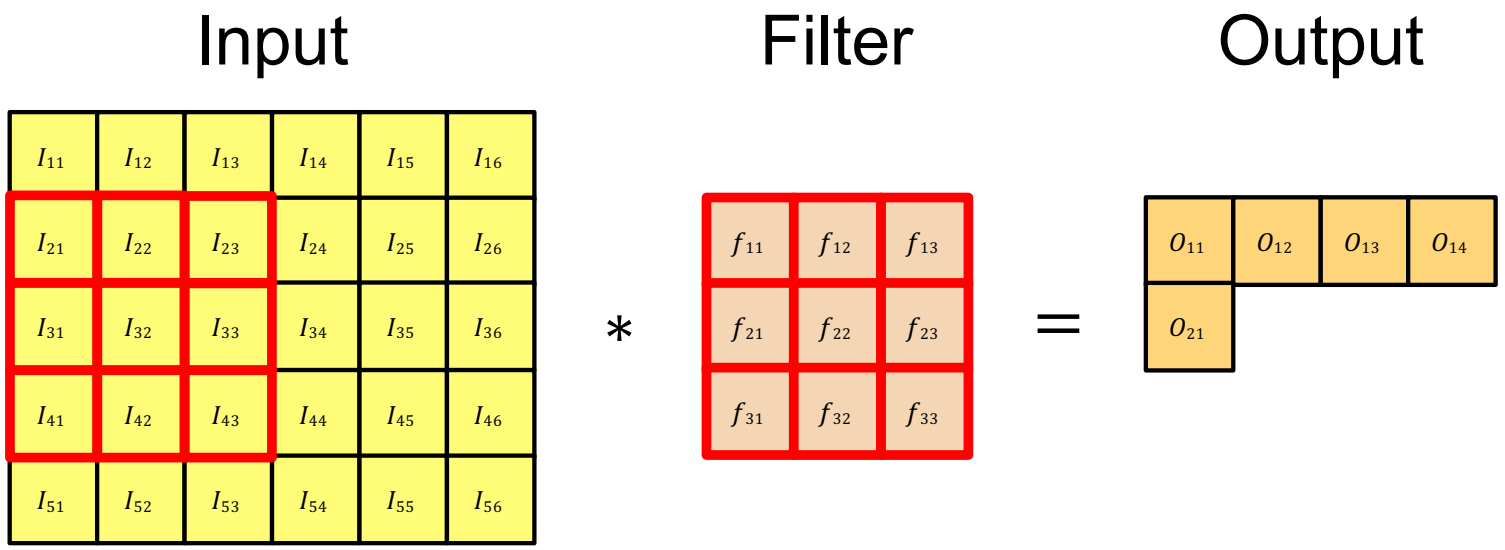
Applying a linear filter



$$O_{14} = I_{14} \cdot f_{11} + I_{15} \cdot f_{12} + I_{16} \cdot f_{13} + \ldots + I_{36} \cdot f_{33}$$

Adapted from D. Fouhey and J. Johnson

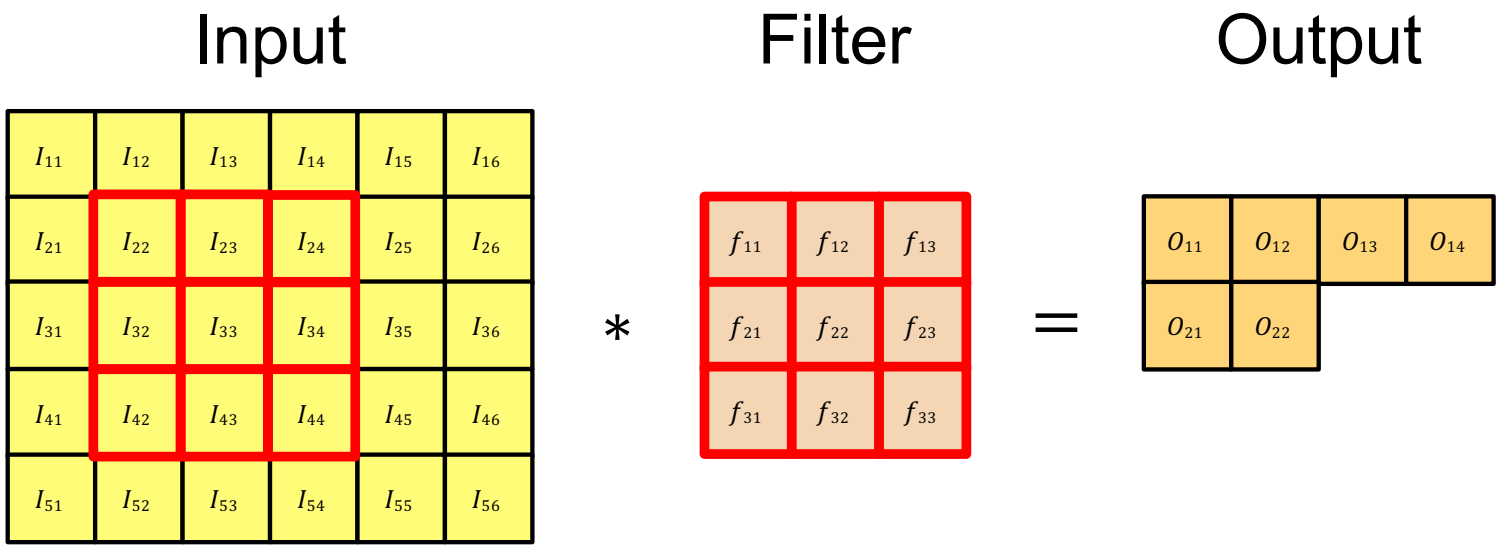
Applying a linear filter



$$O_{21} = I_{21} \cdot f_{11} + I_{22} \cdot f_{12} + I_{23} \cdot f_{13} + \ldots + I_{43} \cdot f_{33}$$

Adapted from D. Fouhey and J. Johnson

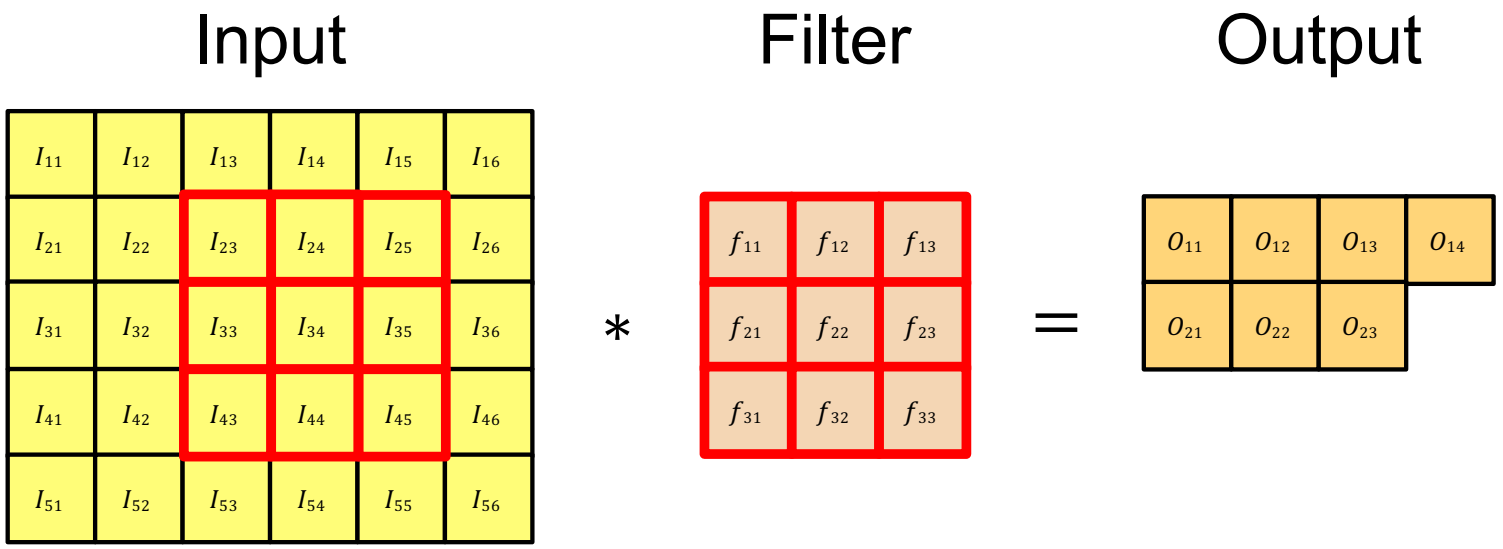
Applying a linear filter



$$O_{22} = I_{22} \cdot f_{11} + I_{23} \cdot f_{12} + I_{24} \cdot f_{13} + \ldots + I_{44} \cdot f_{33}$$

Adapted from D. Fouhey and J. Johnson

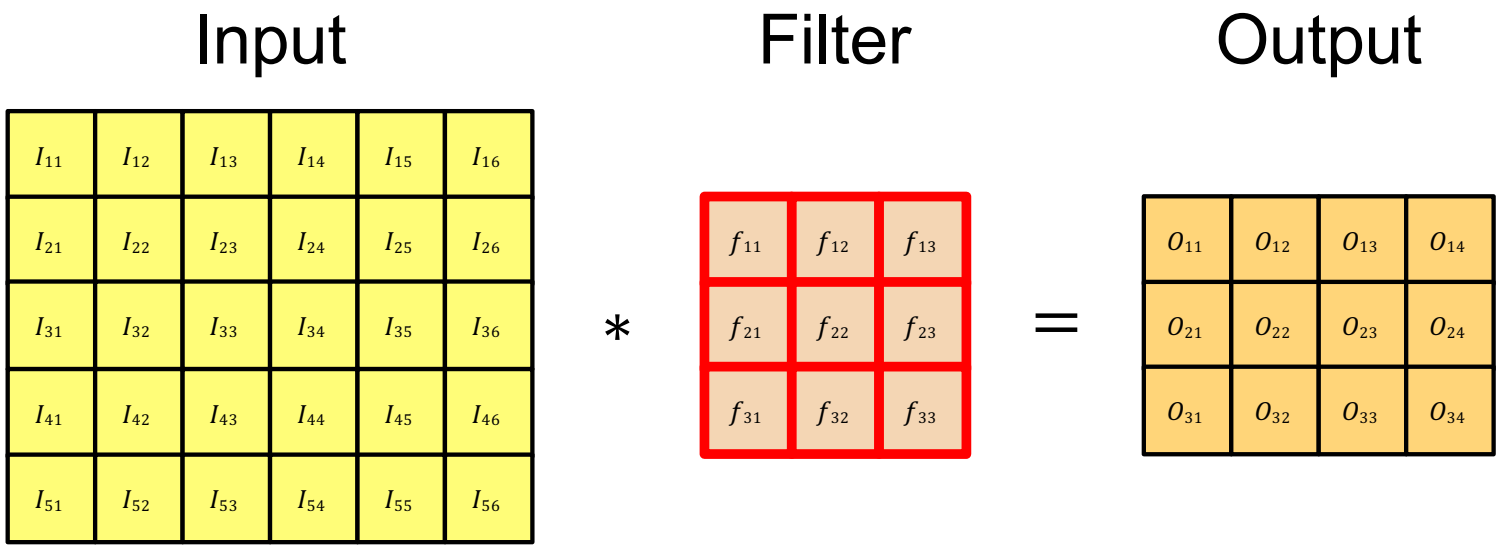
Applying a linear filter



$$O_{23} = I_{23} \cdot f_{11} + I_{24} \cdot f_{12} + I_{25} \cdot f_{13} + \ldots + I_{45} \cdot f_{33}$$

Adapted from D. Fouhey and J. Johnson

Applying a linear filter



What filter values should we use to find the average in a 3×3 window?

Adapted from D. Fouhey and J. Johnson

Convolution

For the moment, think of an image as a two dimensional array of intensities. Write $\mathcal{I}_{ij}$ for the pixel at position i, j. We will construct a small array (a *mask* or *kernel*) $\mathcal{W}$, and compute a new image $\mathcal{N}$ from the image and the mask, using the rule

$$\mathcal{N}_{ij} = \sum_{uv} \mathcal{I}_{i-u,j-v} \mathcal{W}_{uv}$$

which we will write

$$\mathcal{N} = \mathcal{W} * \mathcal{I}.$$

In some sources, you might see $\mathcal{W} ** \mathcal{I}$ (to emphasize the fact that the image is 2D). We sum over all u and v that apply to $\mathcal{W}$; for the moment, do not worry about what happens when an index goes out of the range of $\mathcal{I}$. This operation is known as *convolution*, and $\mathcal{W}$ is often called the *kernel* of the convolution. You should

Filtering

look closely at the expression; the "direction" of the dummy variable u (resp. v) has been reversed compared with what you might expect (unless you have a signal processing background). What you might expect – sometimes called *correlation* or *filtering* – would compute

$$\mathcal{N}_{ij} = \sum_{uv} \mathcal{I}_{i+u,j+v} \mathcal{W}_{uv}$$

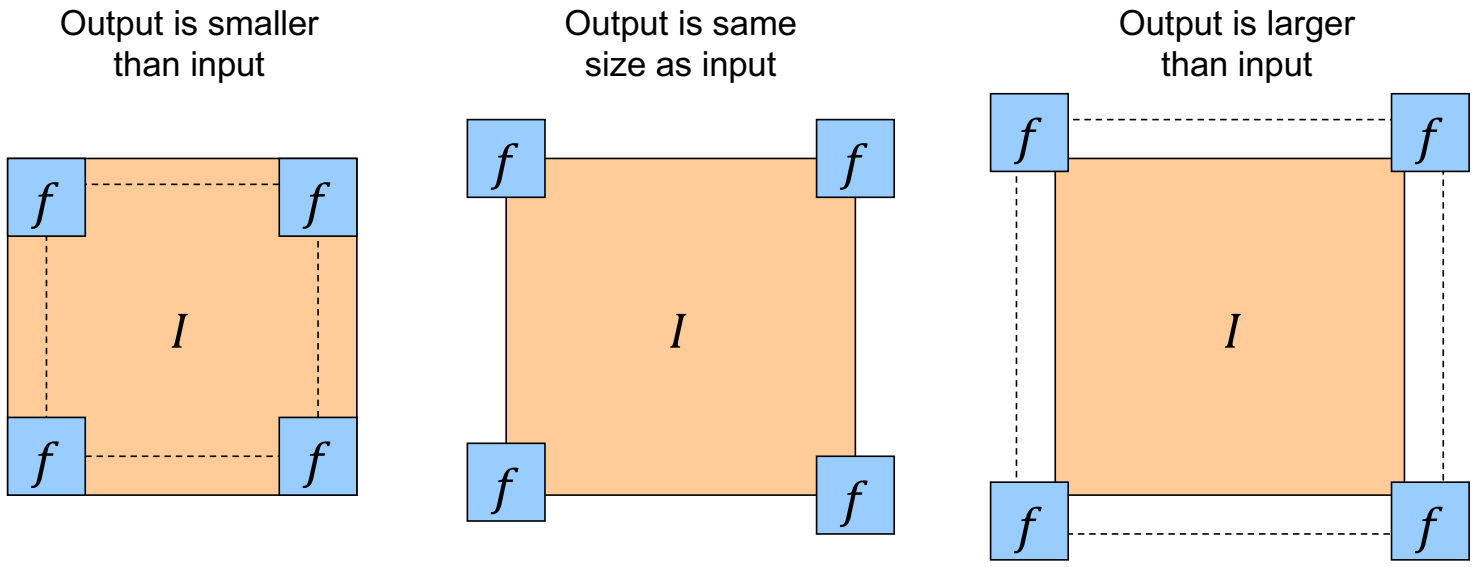
which we will write

$$\mathcal{N} = \texttt{filter}(\mathcal{I}, \mathcal{W}).$$

This difference isn't particularly significant, but if you forget that it is there, you compute the wrong answer.

Practical details: Dealing with edges

- To control the size of the output, we need to use *padding*



Practical details: Dealing with edges

- To control the size of the output, we need to use *padding*
- What values should we pad the image with?

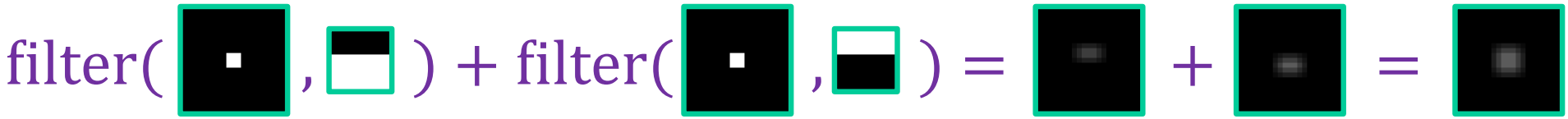
Practical details: Dealing with edges

- To control the size of the output, we need to use *padding*
- What values should we pad the image with?
  - Zero pad (or clip filter)
  - Wrap around
  - Copy edge
  - Reflect across edge

Source: S. Marschner

Properties: Linearity

$$\text{filter}(I, f_1 + f_2) = \text{filter}(I, f_1) + \text{filter}(I, f_2)$$



Adapted from D. Fouhey and J. Johnson

Properties: Linearity

$$\text{filter}(I, f_1 + f_2) = \text{filter}(I, f_1) + \text{filter}(I, f_2)$$

Also:

$$\text{filter}(I_1 + I_2, f) = \text{filter}(I_1, f) + \text{filter}(I_2, f)$$

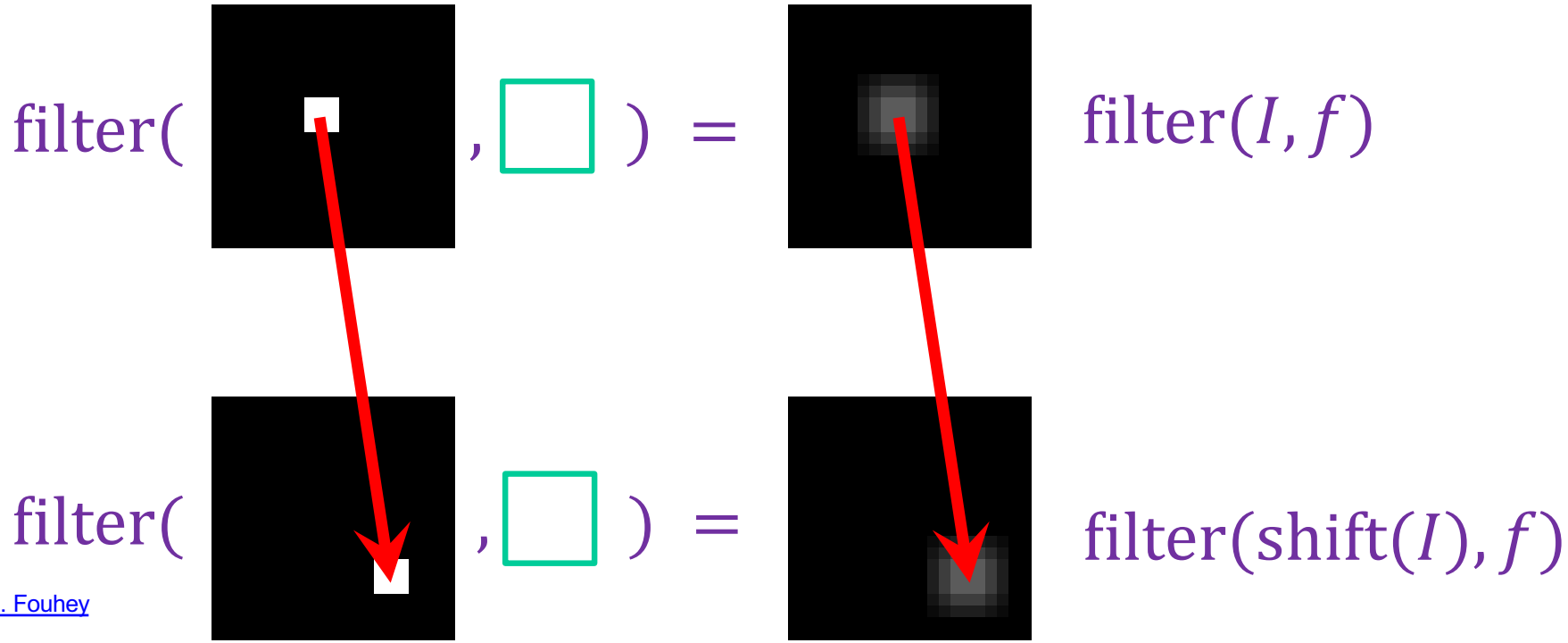
$$\text{filter}(kI, f) = k\ \text{filter}(I, f)$$

$$\text{filter}(I, kf) = k\ \text{filter}(I, f)$$

Adapted from D. Fouhey and J. Johnson

Properties: Shift-invariance

$$\text{filter}(\text{shift}(I), f) = \text{shift}(\text{filter}(I, f))$$



Adapted from D. Fouhey and J. Johnson

More linear filtering properties

- Commutativity: $f * g = g * f$
  - For infinite signals, no difference between filter and signal
- Associativity: $f * (g * h) = (f * g) * h$
  - Convolving several filters one after another is equivalent to convolving with one combined filter:

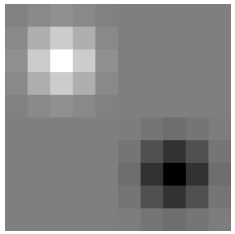
$$\left((g * f_1) * f_2\right) * f_3 = g * (f_1 * f_2 * f_3)$$

- Identity: for *unit impulse* e, $f * e = f$

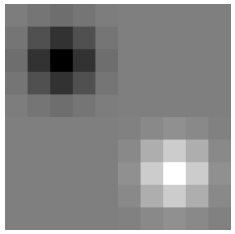
Note: Filtering vs. “convolution”

- In classical signal processing terminology, convolution is filtering with a *flipped* kernel, and filtering with an upright kernel is known as *cross-correlation*
  - Check convention of filtering function you plan to use!

Filtering or “cross-correlation”
(Kernel in original orientation)



“Convolution”
(Kernel flipped in x and y)



Adapted from D. Fouhey and J. Johnson

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

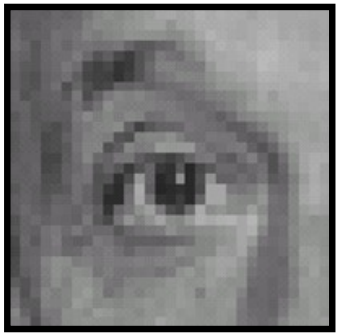
Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

One surrounded by zeros is the *identity filter*



Filtered (no change)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

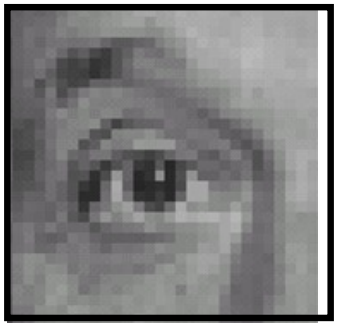
?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0



Shifted *left*
By one pixel

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

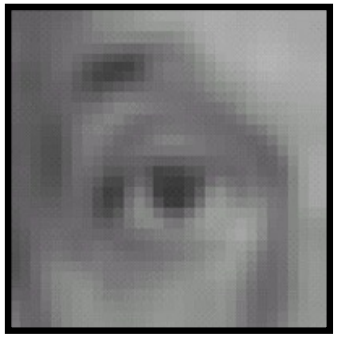
?

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



Blur (with a box filter)

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad ?$$

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

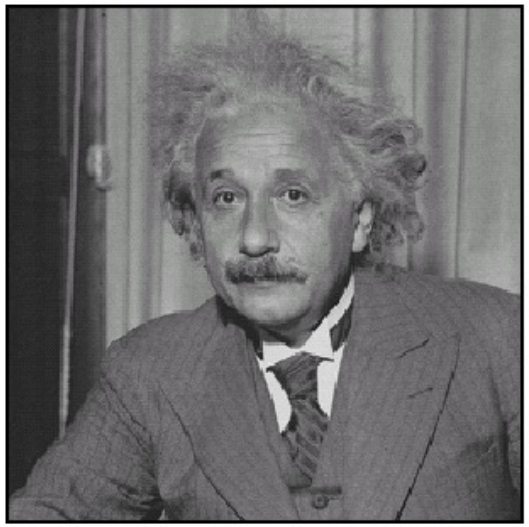
Sharpening filter:
Accentuates differences with local average

(Note that filter sums to 1)

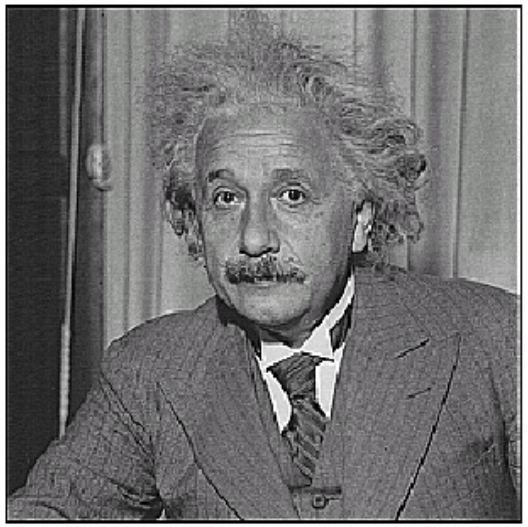
Sharpened

Source: D. Lowe

Sharpening



before



after

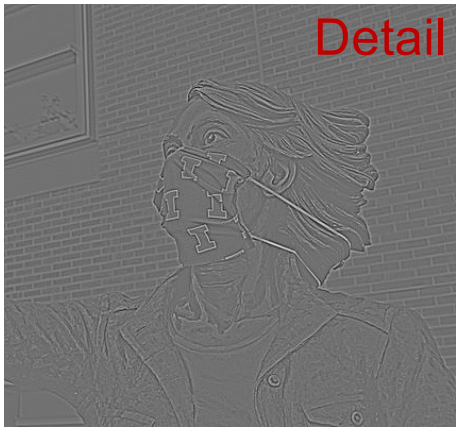
Source: D. Lowe

Sharpening

−

=

+



=

Source:
S. Gupta

Filters are dot products

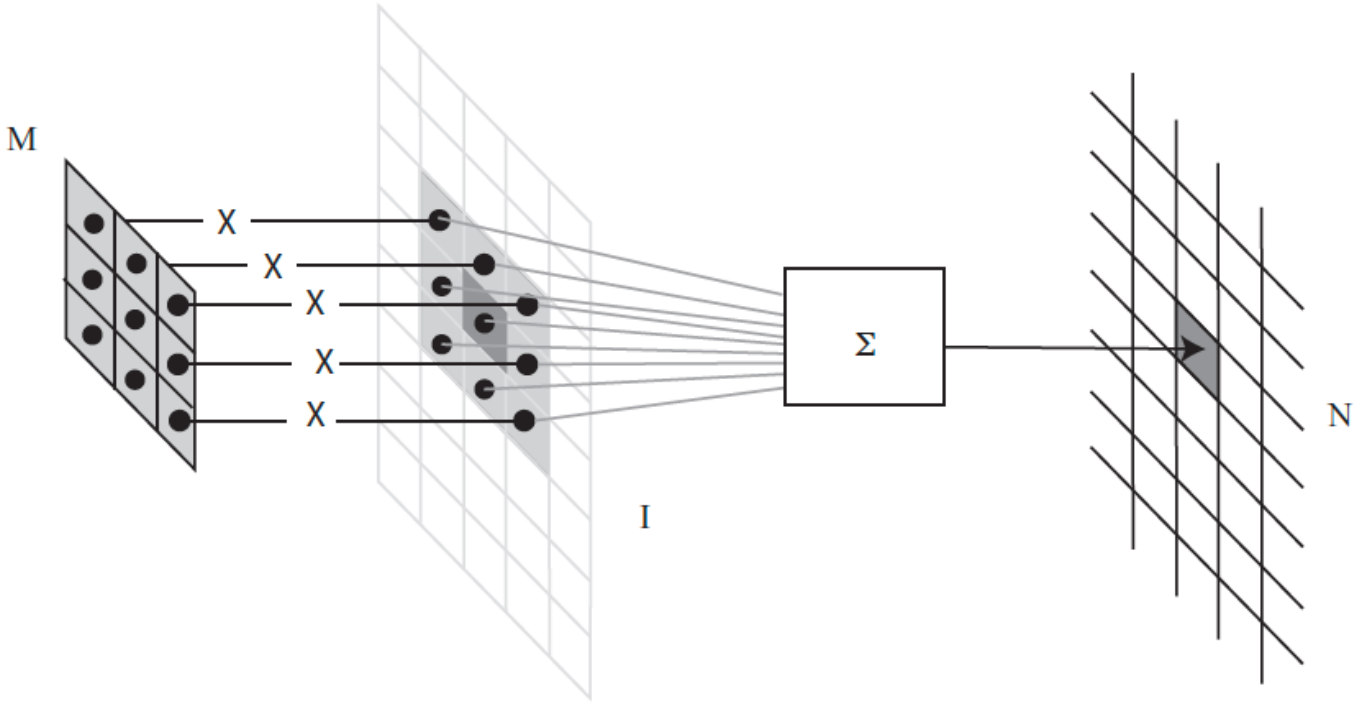


FIGURE 3.1: *To compute the value of $\mathcal{N}$ at some location, you shift a copy of $\mathcal{M}$ (the flipped version of $\mathcal{W}$) to lie over that location in $\mathcal{I}$; you multiply together the non-zero elements of $\mathcal{M}$ and $\mathcal{I}$ that lie on top of one another; and you sum the results.*

Filters are dot products

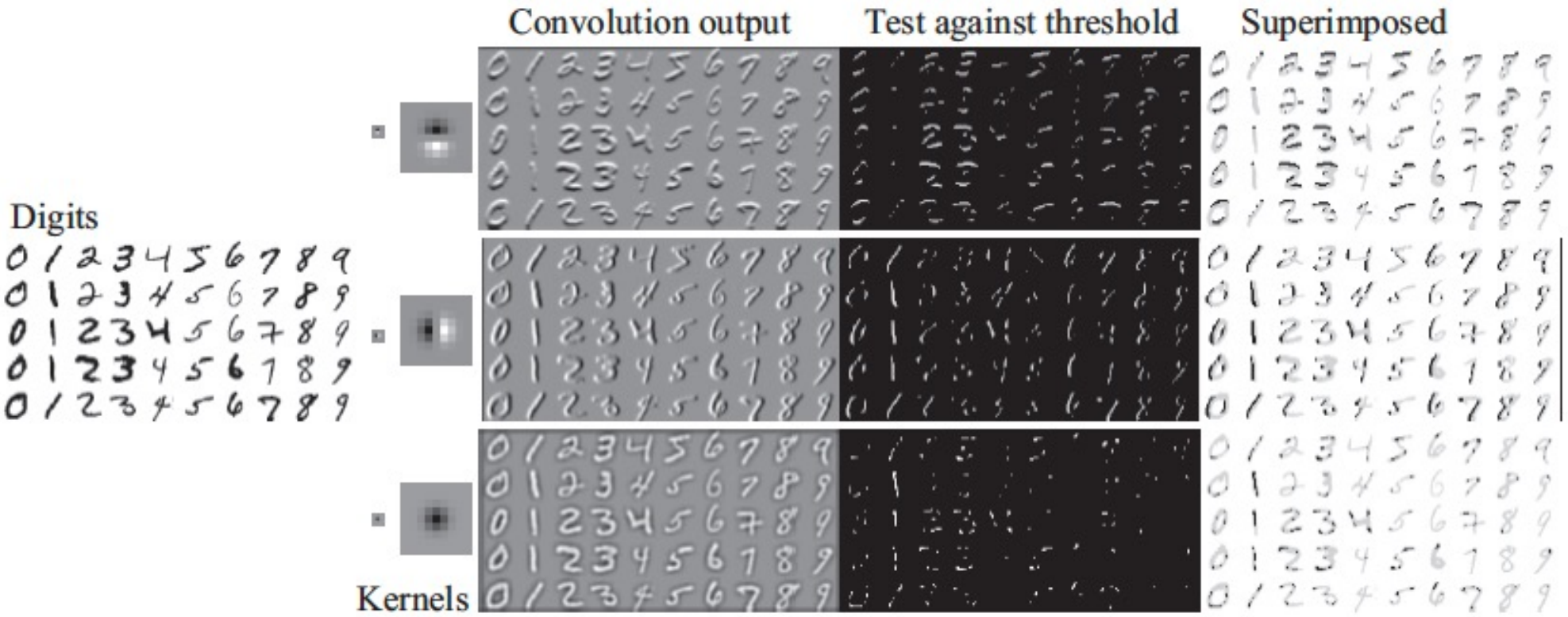
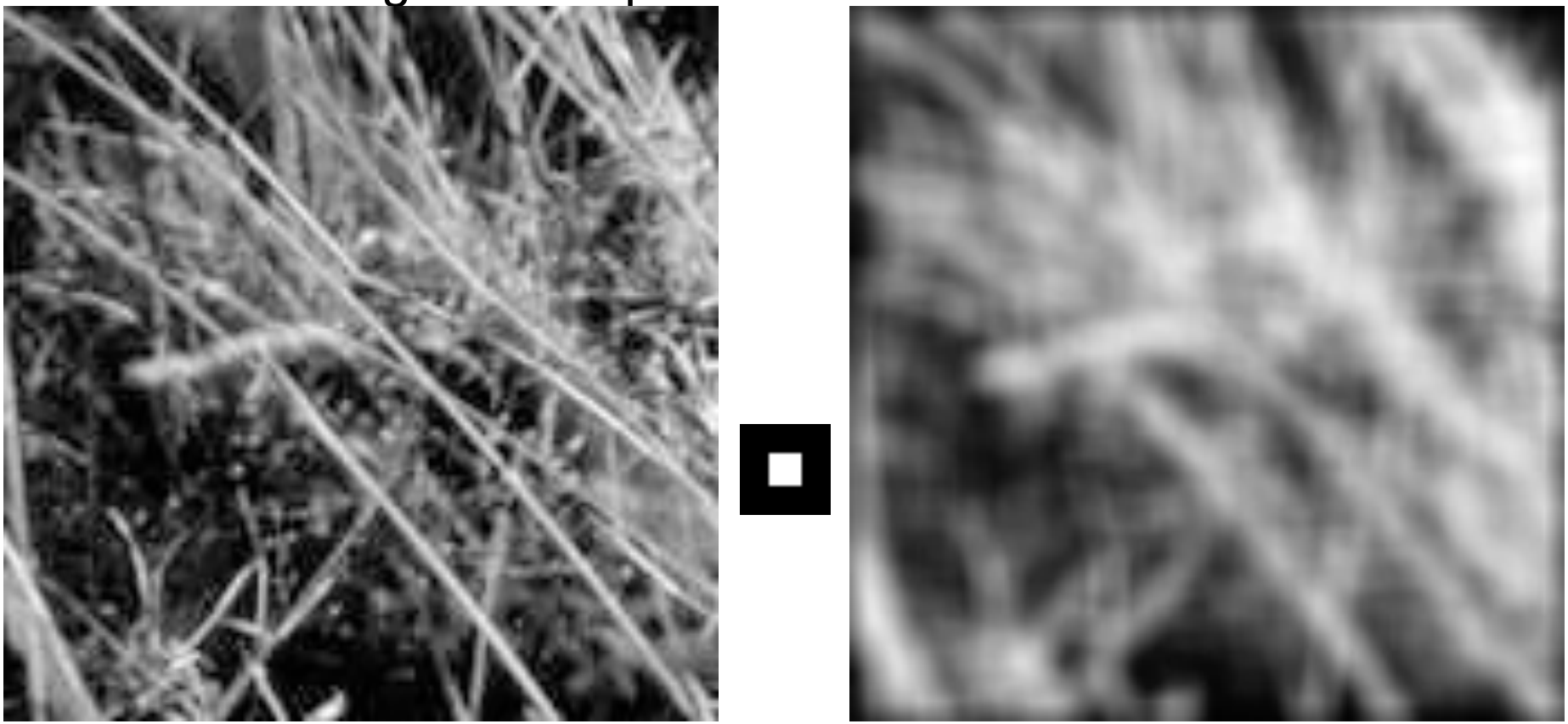


Image filtering: Outline

- Linear filtering and its properties
- Gaussian filters and their properties

Smoothing with box filter revisited

- What's wrong with this picture?



Source: D. Forsyth

Smoothing with box filter revisited

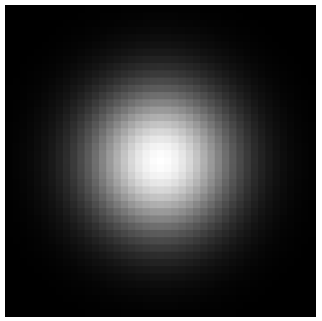
- What's wrong with this picture?
- What's the solution?
  - To eliminate edge effects, weight contribution of neighborhood pixels according to their closeness to the center

"proportional to"
(renormalize values to sum to 1)

$$G(x, y) \propto \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

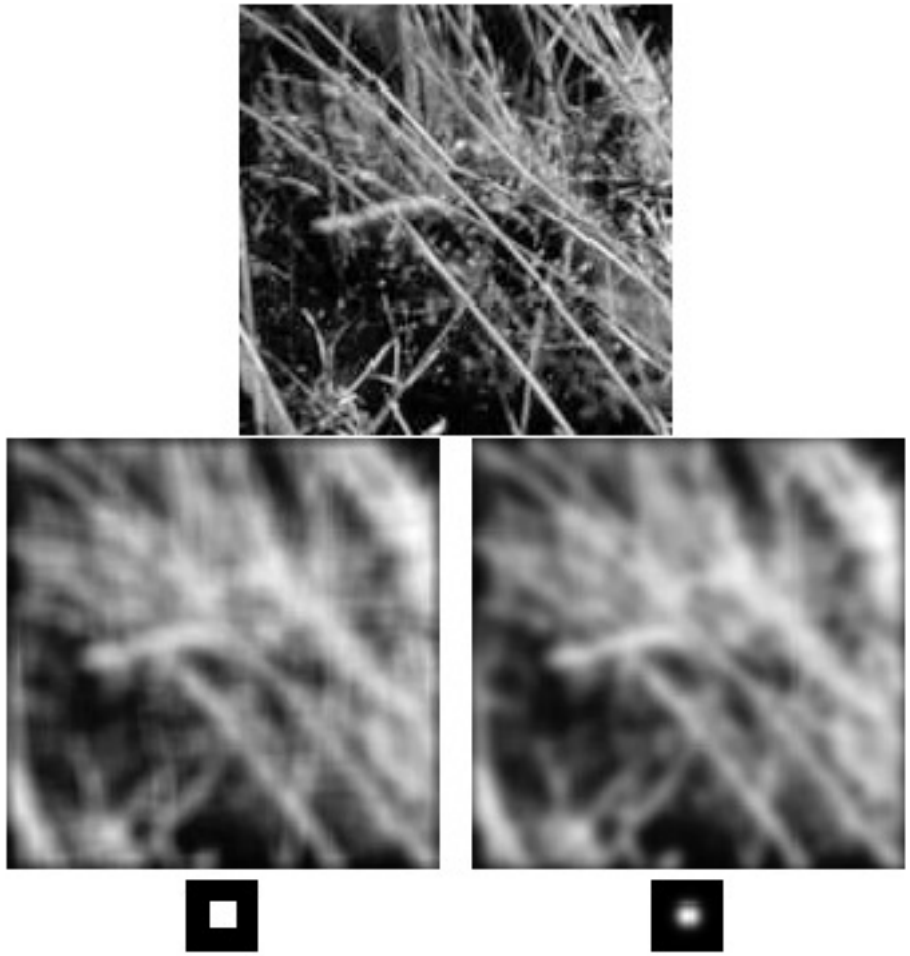
standard deviation
(determines size of "blob")

Gaussian filter



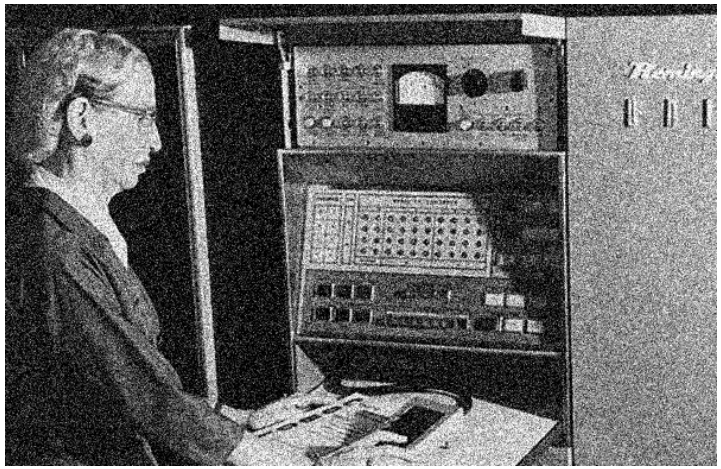
Adapted from D. Fouhey and J. Johnson

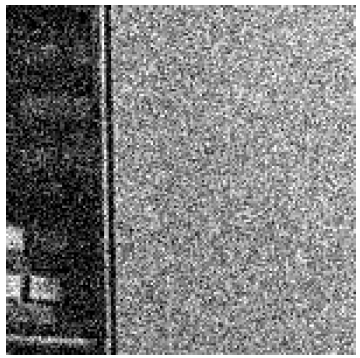
Gaussian vs. box filtering



Applying Gaussian filters

Input image
(no filter)

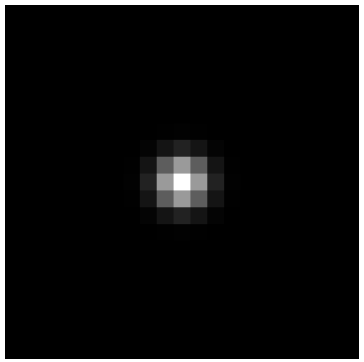


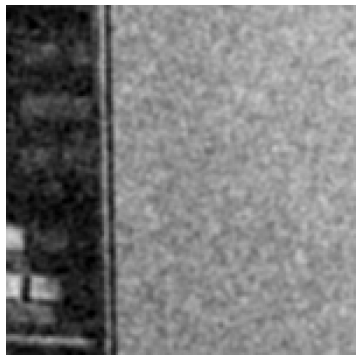


Source: D. Fouhey and J. Johnson

Applying Gaussian filters

$\sigma = 1$

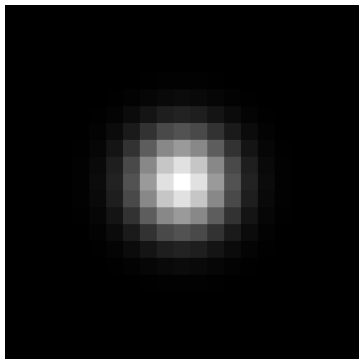


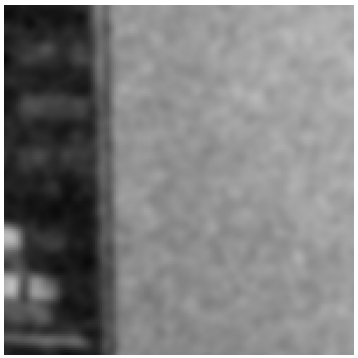


Source: D. Fouhey and J. Johnson

Applying Gaussian filters

$\sigma = 2$

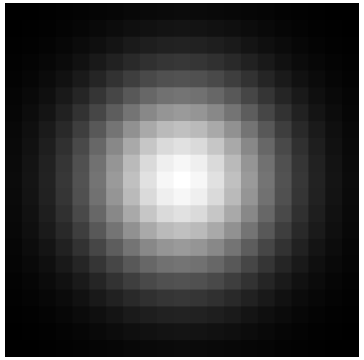




Source: D. Fouhey and J. Johnson

Applying Gaussian filters

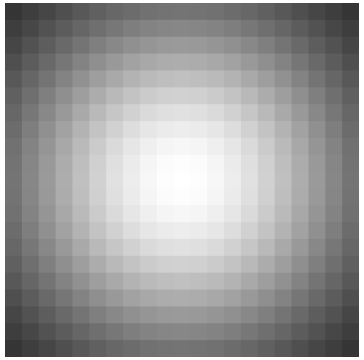
$\sigma = 4$



Source: D. Fouhey and J. Johnson

Applying Gaussian filters

$\sigma = 8$

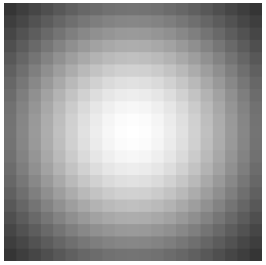


Source: D. Fouhey and J. Johnson

Choosing filter size

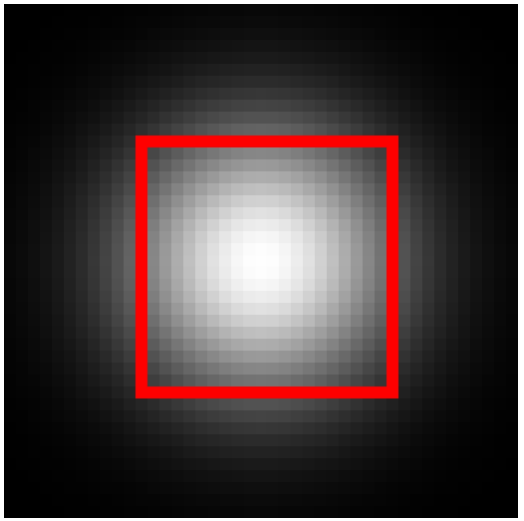
- Rule of thumb: set filter width to about 6σ (captures 99.7% of the energy)

$\sigma = 8$
$\text{Width} = 21$



Too small!

$\sigma = 8$
$\text{Width} = 43$



A bit small (might be OK)

Adapted from D. Fouhey and J. Johnson

Gaussian filters: Properties

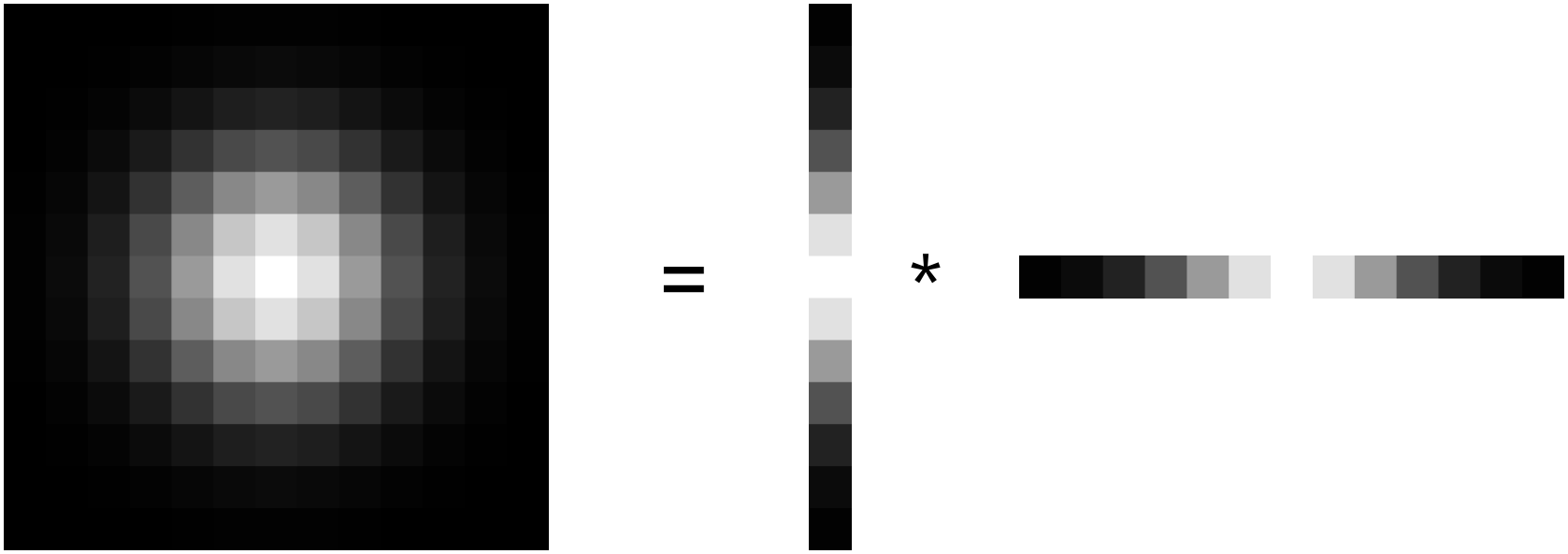
- Gaussian is a *low-pass filter*: it removes high-frequency components from the image (more on this soon)
- Convolution with self is another Gaussian
  - So we can smooth with small-σ kernel, repeat, and get same result as larger-σ kernel would have
  - Convolving *two times* with Gaussian kernel with std. dev. σ is the same as convolving *once* with kernel with std. dev. $\sigma\sqrt{2}$
- Gaussian kernel is *separable*: it factors into product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) =$$

$$\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right) \quad \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{y^2}{2\sigma^2}\right)$$



Adapted from D. Fouhey and J. Johnson

Why is separability useful?

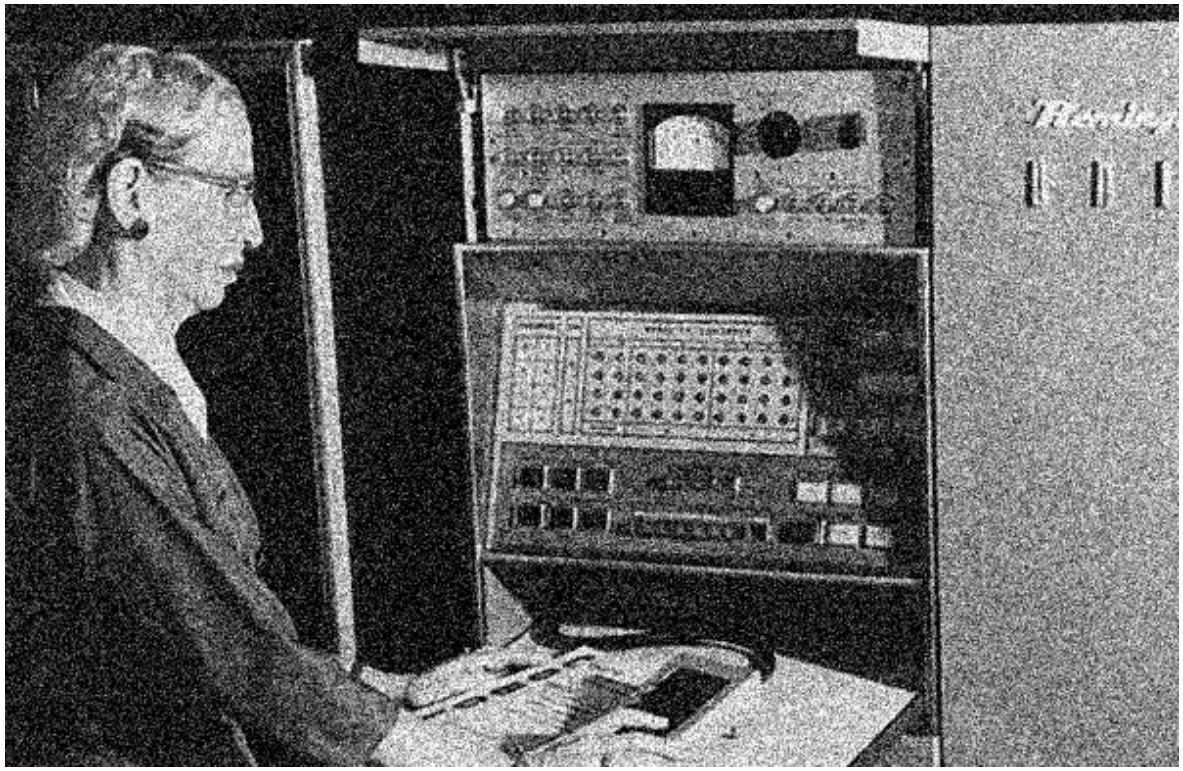
- Separability means that a 2D convolution can be reduced to two 1D convolutions (one along rows and one along columns)
- What is the complexity of filtering an $n \times n$ image with an $m \times m$ kernel?
  - $O(n^2 m^2)$
- What if the kernel is separable?
  - $O(n^2 m)$

Image filtering: Outline

- Linear filtering and its properties
- Gaussian filters and their properties
- Nonlinear filtering: Median filtering

Different types of noise

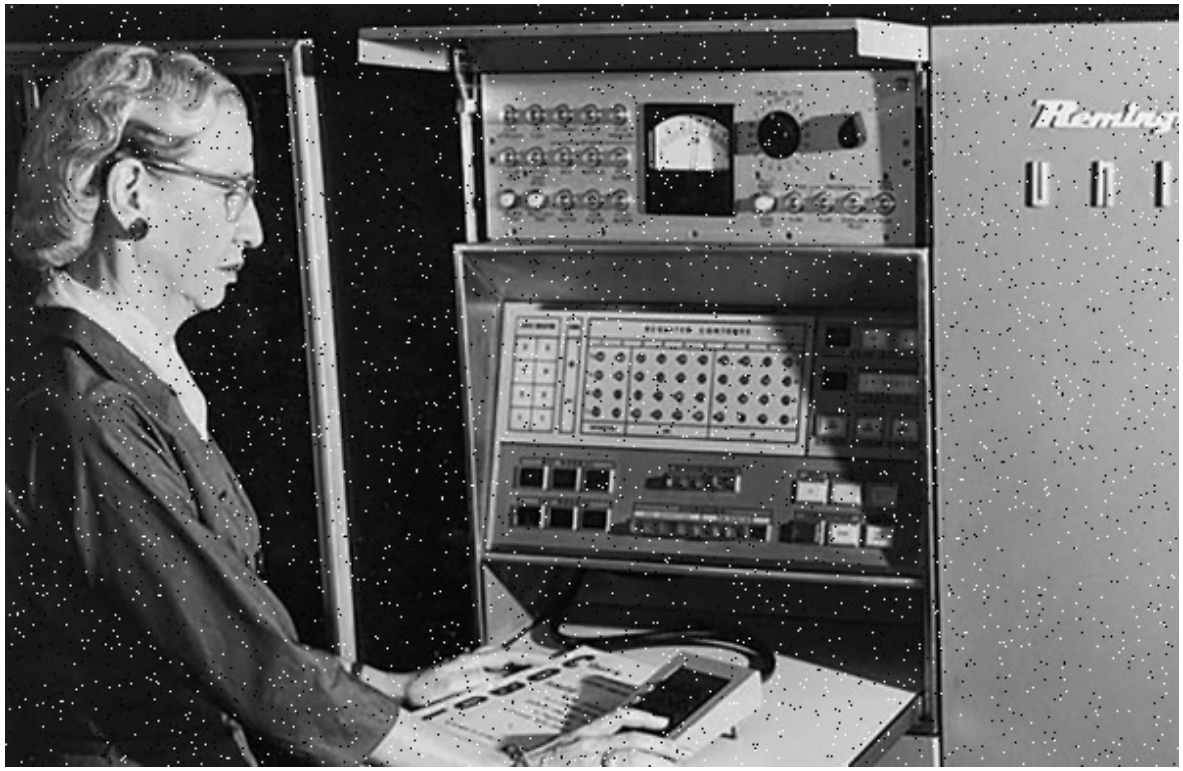
- Gaussian filtering is appropriate for *additive, zero-mean* noise (assuming nearby pixels share the same value)



Adapted from D. Fouhey and J. Johnson

Different types of noise

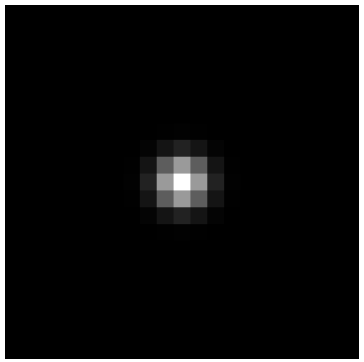
- What about *impulse* or *shot noise,* i.e., when some pixels are arbitrarily replaced by spurious values?

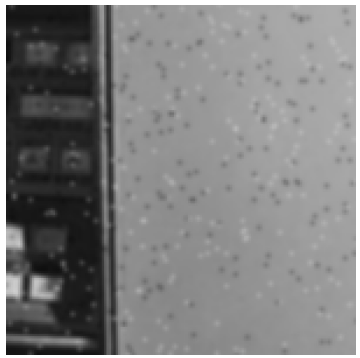


Adapted from D. Fouhey and J. Johnson

Where Gaussian filtering fails

$\sigma = 1$

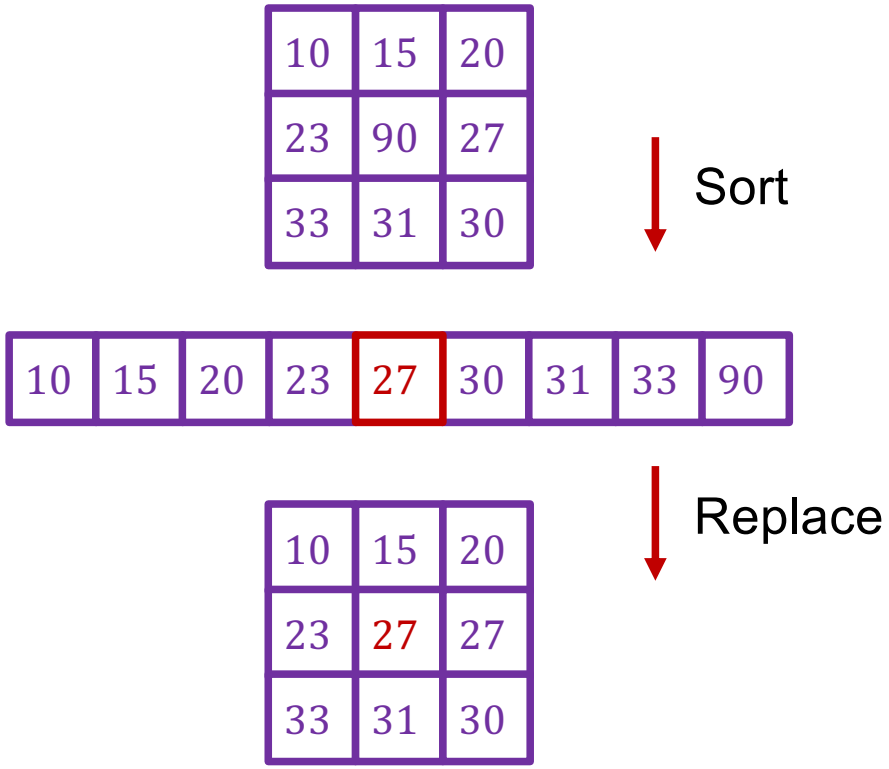




Adapted from D. Fouhey and J. Johnson

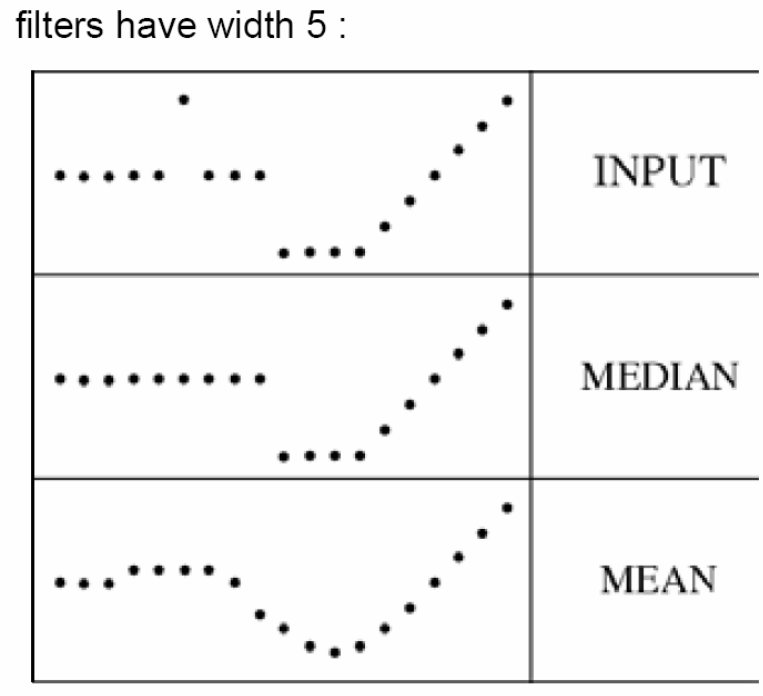
Alternative idea: Median filtering

- A **median filter** operates over a window by selecting the median intensity in the window



Median filter

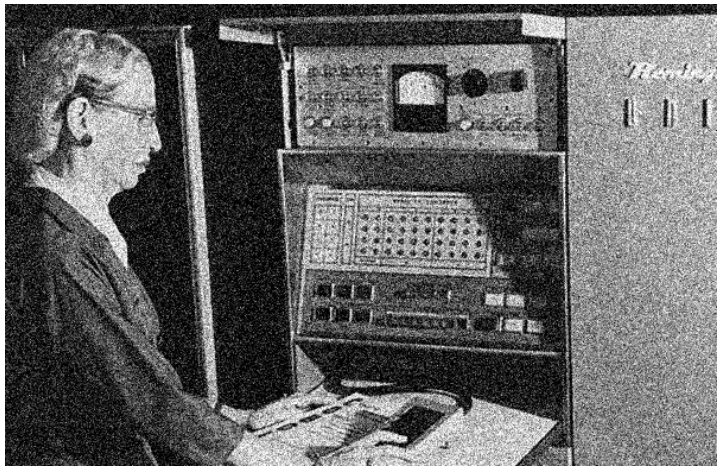
- What advantage does median filtering have over Gaussian filtering?
  - Robustness to outliers

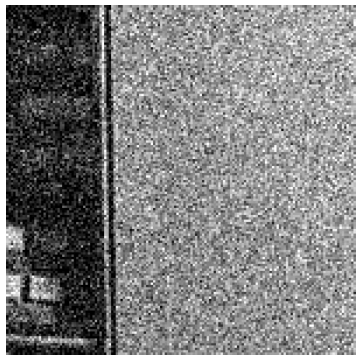


Source: K. Grauman

Applying median filter

Input image
(no filter)

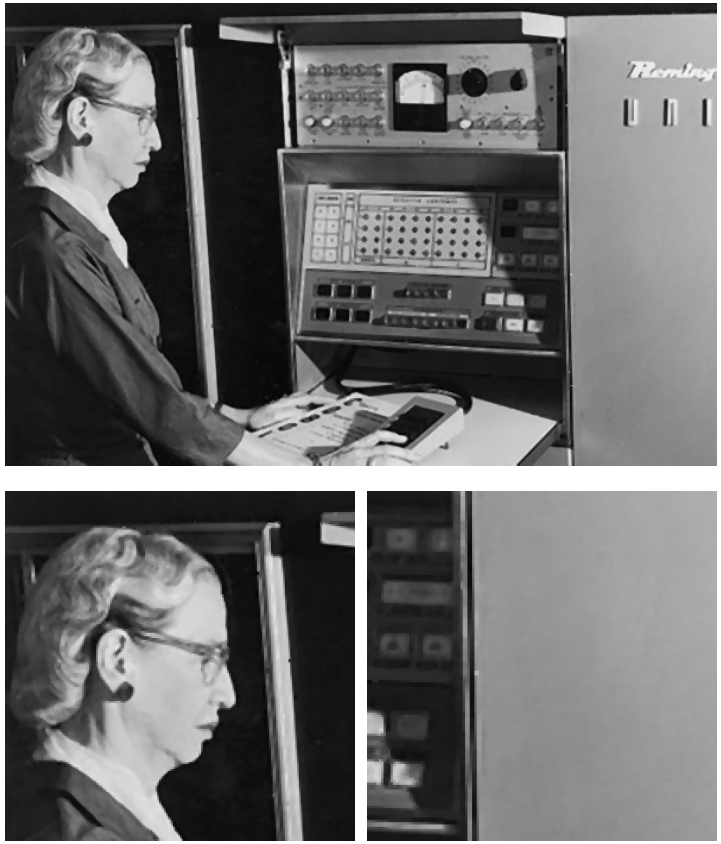




Adapted from D. Fouhey and J. Johnson

Applying median filter

median
filter
$(\text{width} = 3)$



Adapted from D. Fouhey and J. Johnson

Applying median filter

median
filter
$(\text{width} = 7)$

Is median filtering linear?

Is median filtering linear?

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

median filter

$$1 \quad + \quad 0 \quad \neq \quad 2$$

Adapted from D. Fouhey and J. Johnson

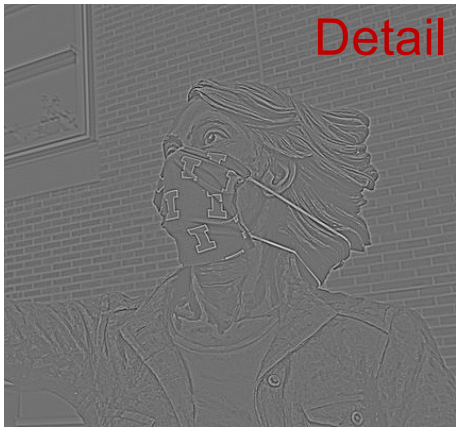
Image filtering: Outline

- Linear filtering and its properties
- Gaussian filters and their properties
- Nonlinear filtering: Median filtering
- Fun filtering application: Hybrid images

Application: Hybrid images

A. Oliva, A. Torralba, P.G. Schyns,
Hybrid Images, SIGGRAPH 2006

Recall: Sharpening

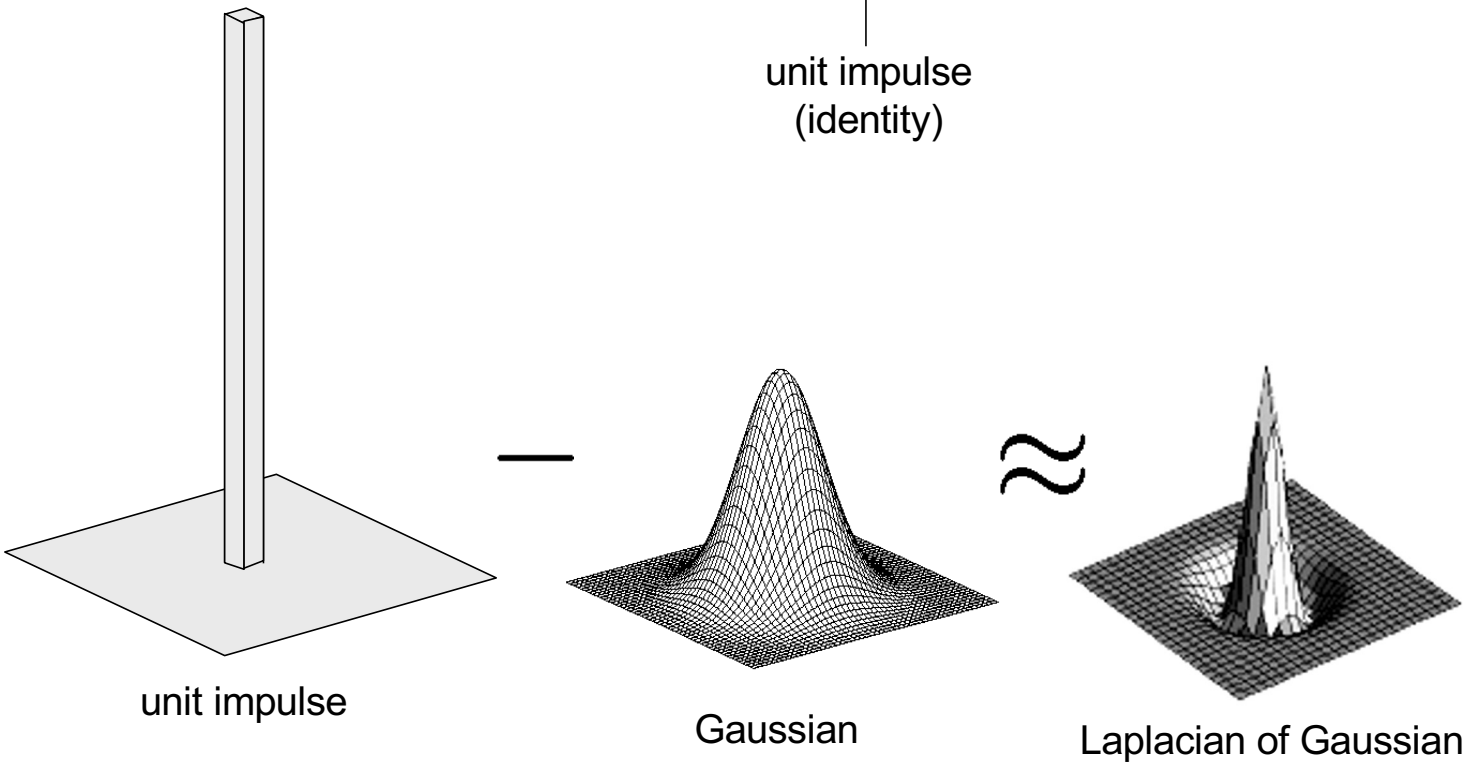


Source:
S. Gupta

“Detail” filter

$$I - I * g = I * (e - g)$$

unit impulse
(identity)



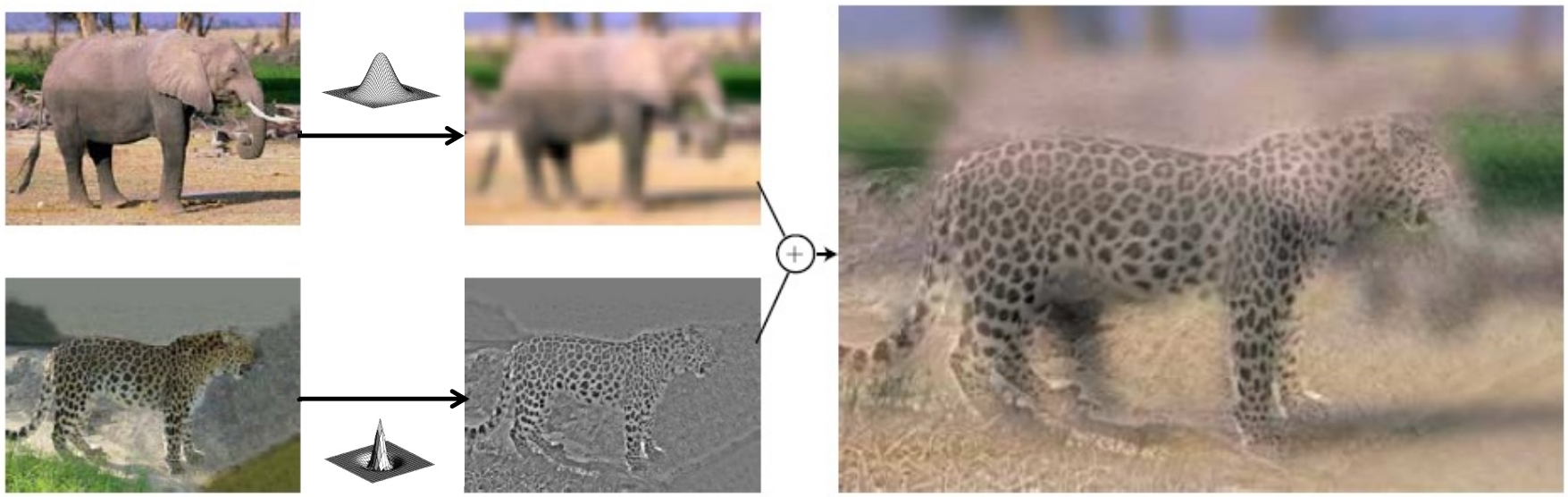
unit impulse

Gaussian

Laplacian of Gaussian

Application: Hybrid images

Gaussian filter



Laplacian filter

Application: Hybrid images