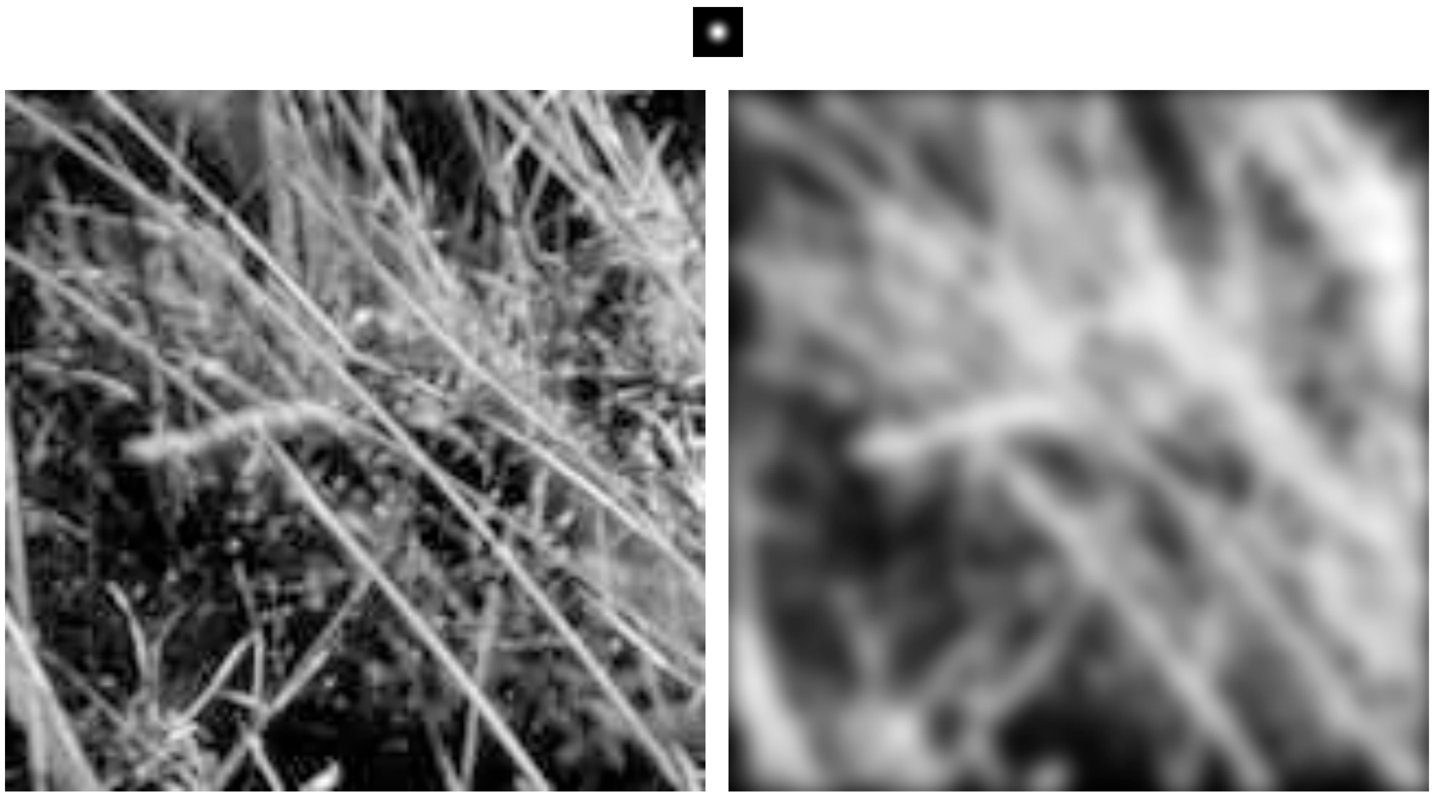
Image filtering



Recall: Image transformations

- What are different kinds of image transformations?
  - Range transformations or point processing
  - Image warping
  - Image filtering

Image filtering: Outline

- Linear filtering and its properties
- Gaussian filters and their properties
- Nonlinear filtering: Median filtering
- Fun filtering application: Hybrid images

Sliding window operations

- Let's slide a fixed-size window over the image and perform the same simple computation at each window location
- Example use case: how do we reduce image noise?
  - Let's take the *average* of pixel values in each window
  - More generally, we can take a *weighted sum* where the weights are given by a *filter kernel*

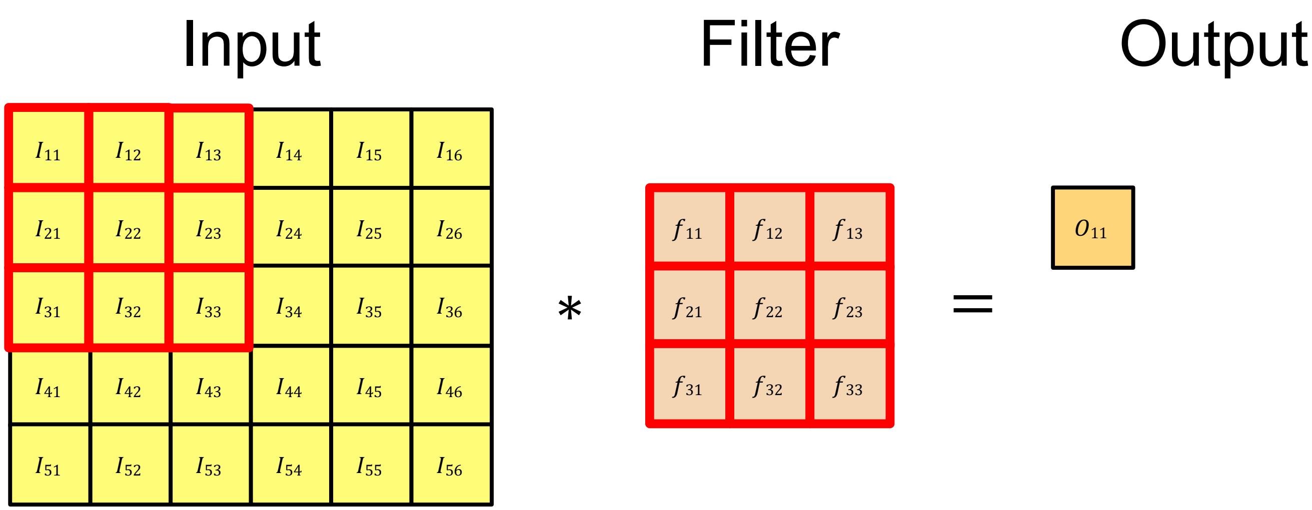
Applying a linear filter

Input | | Filter | | Output

$$\begin{bmatrix} I_{11} & I_{12} & I_{13} & I_{14} & I_{15} & I_{16} \\ I_{21} & I_{22} & I_{23} & I_{24} & I_{25} & I_{26} \\ I_{31} & I_{32} & I_{33} & I_{34} & I_{35} & I_{36} \\ I_{41} & I_{42} & I_{43} & I_{44} & I_{45} & I_{46} \\ I_{51} & I_{52} & I_{53} & I_{54} & I_{55} & I_{56} \end{bmatrix} * \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} =$$

Adapted from [D. Fouhey and J. Johnson]

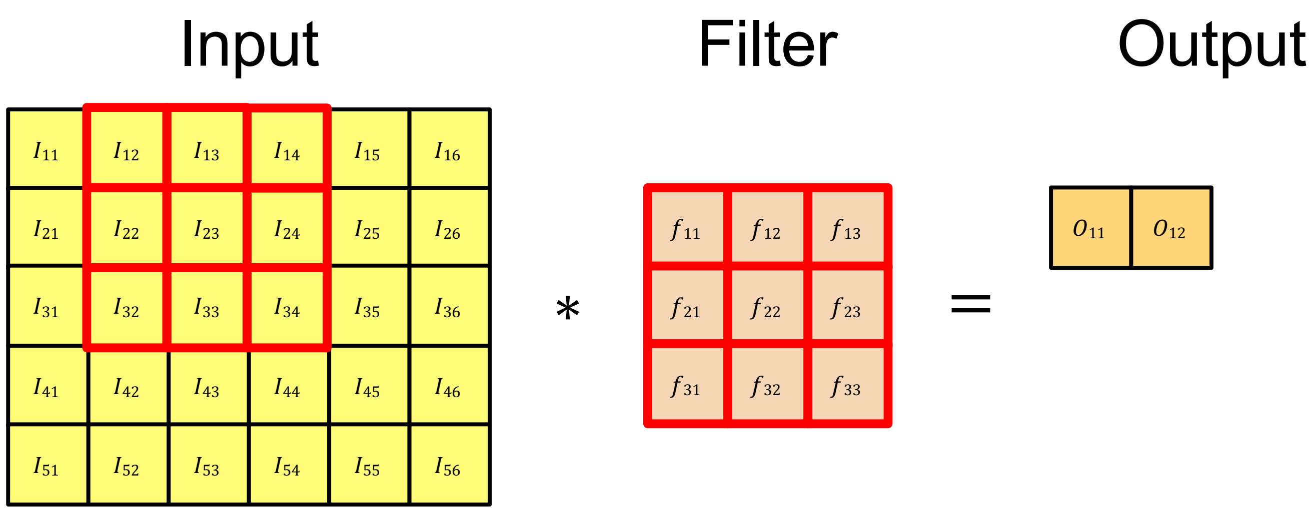
Applying a linear filter



$$O_{11} = I_{11} \cdot f_{11} + I_{12} \cdot f_{12} + I_{13} \cdot f_{13} + \ldots + I_{33} \cdot f_{33}$$

Adapted from D. Fouhey and J. Johnson

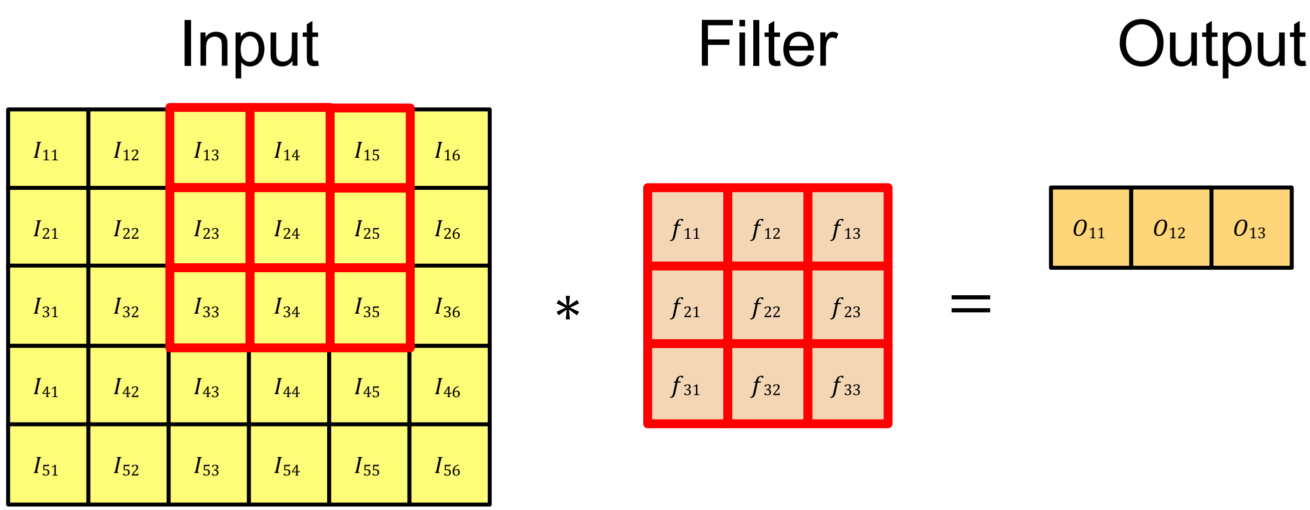
Applying a linear filter



$$O_{12} = I_{12} \cdot f_{11} + I_{13} \cdot f_{12} + I_{14} \cdot f_{13} + \ldots + I_{34} \cdot f_{33}$$

Adapted from [D. Fouhey and J. Johnson]

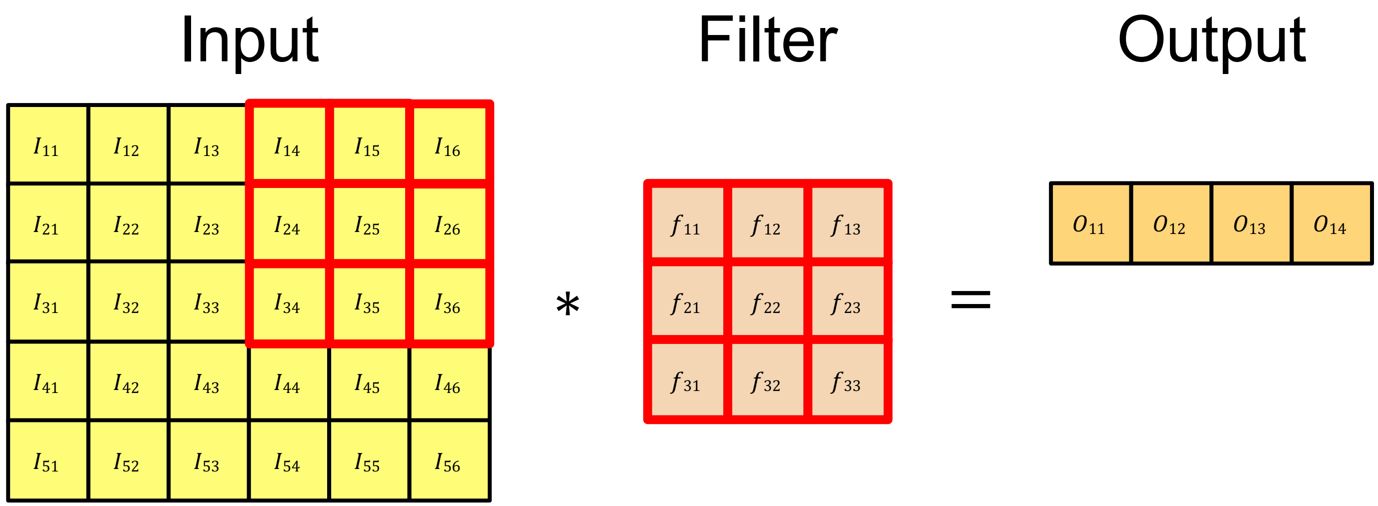
Applying a linear filter



$$O_{13} = I_{13} \cdot f_{11} + I_{14} \cdot f_{12} + I_{15} \cdot f_{13} + \ldots + I_{35} \cdot f_{33}$$

Adapted from D. Fouhey and J. Johnson

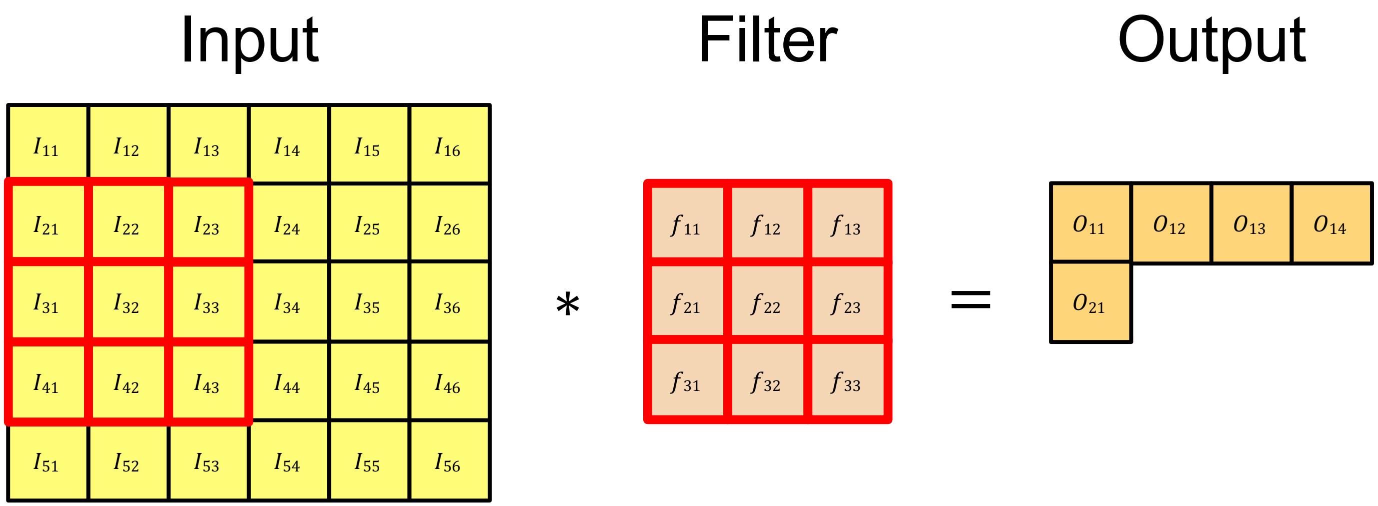
Applying a linear filter



$$O_{14} = I_{14} \cdot f_{11} + I_{15} \cdot f_{12} + I_{16} \cdot f_{13} + \ldots + I_{36} \cdot f_{33}$$

Adapted from [D. Fouhey and J. Johnson]

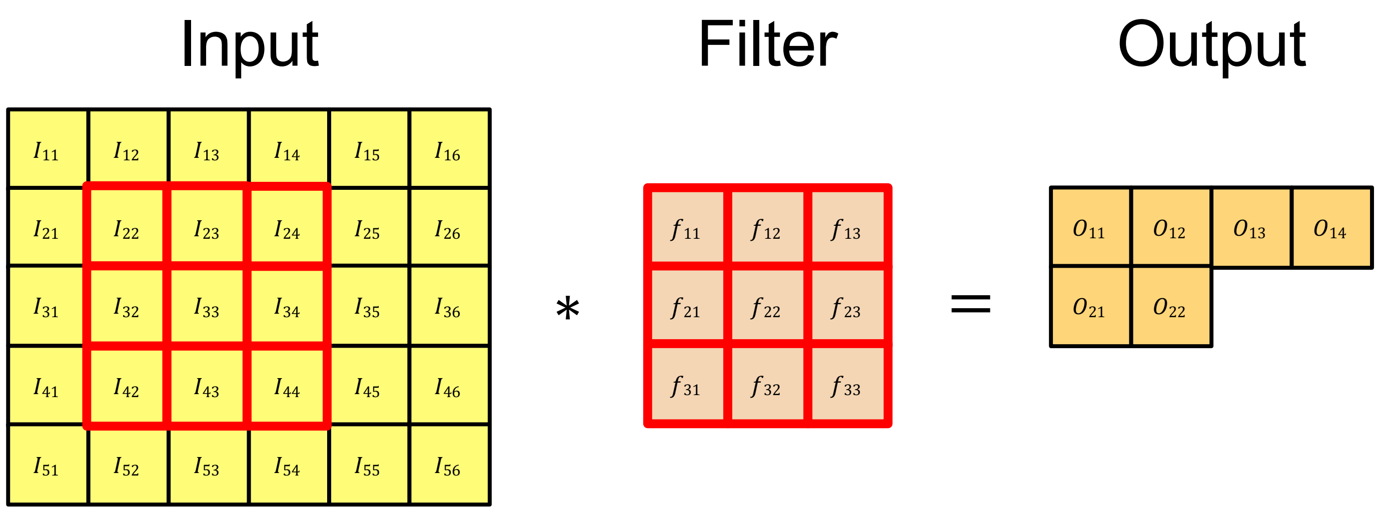
Applying a linear filter



$$O_{21} = I_{21} \cdot f_{11} + I_{22} \cdot f_{12} + I_{23} \cdot f_{13} + \ldots + I_{43} \cdot f_{33}$$

Adapted from [D. Fouhey and J. Johnson]

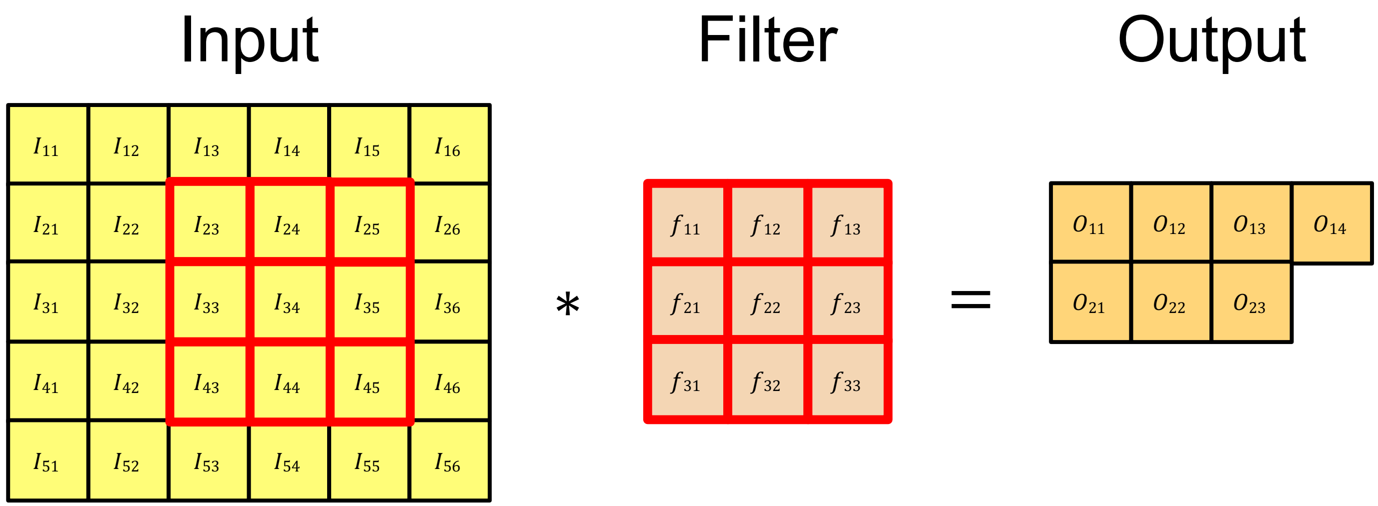
Applying a linear filter



$$O_{22} = I_{22} \cdot f_{11} + I_{23} \cdot f_{12} + I_{24} \cdot f_{13} + \ldots + I_{44} \cdot f_{33}$$

Adapted from D. Fouhey and J. Johnson

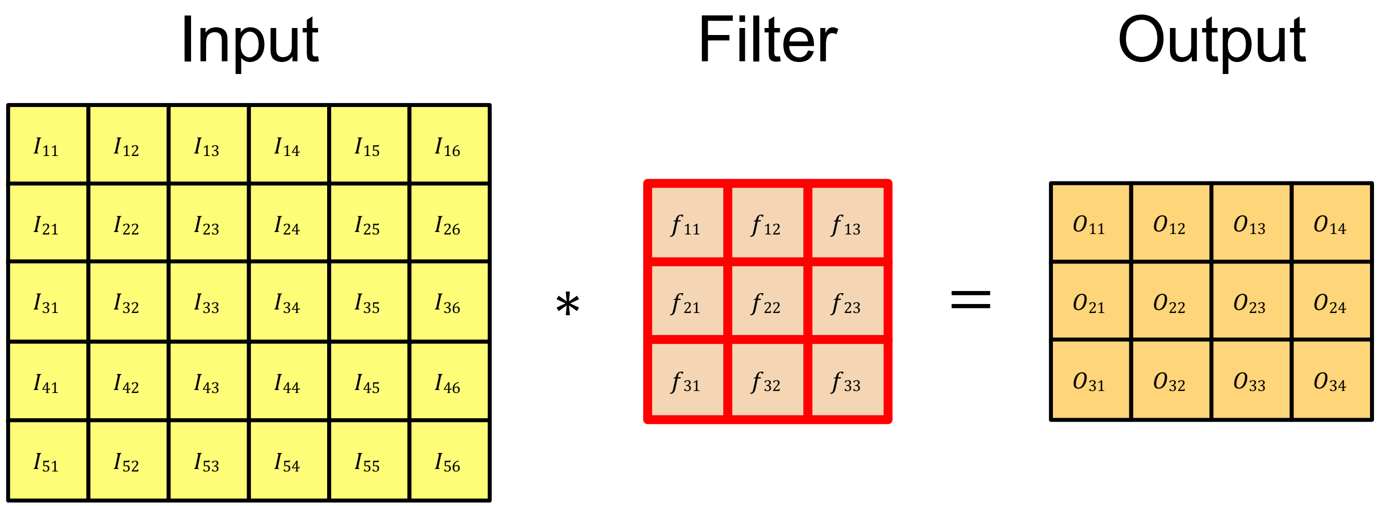
Applying a linear filter



$$O_{23} = I_{23} \cdot f_{11} + I_{24} \cdot f_{12} + I_{25} \cdot f_{13} + \ldots + I_{45} \cdot f_{33}$$

Adapted from D. Fouhey and J. Johnson

Applying a linear filter

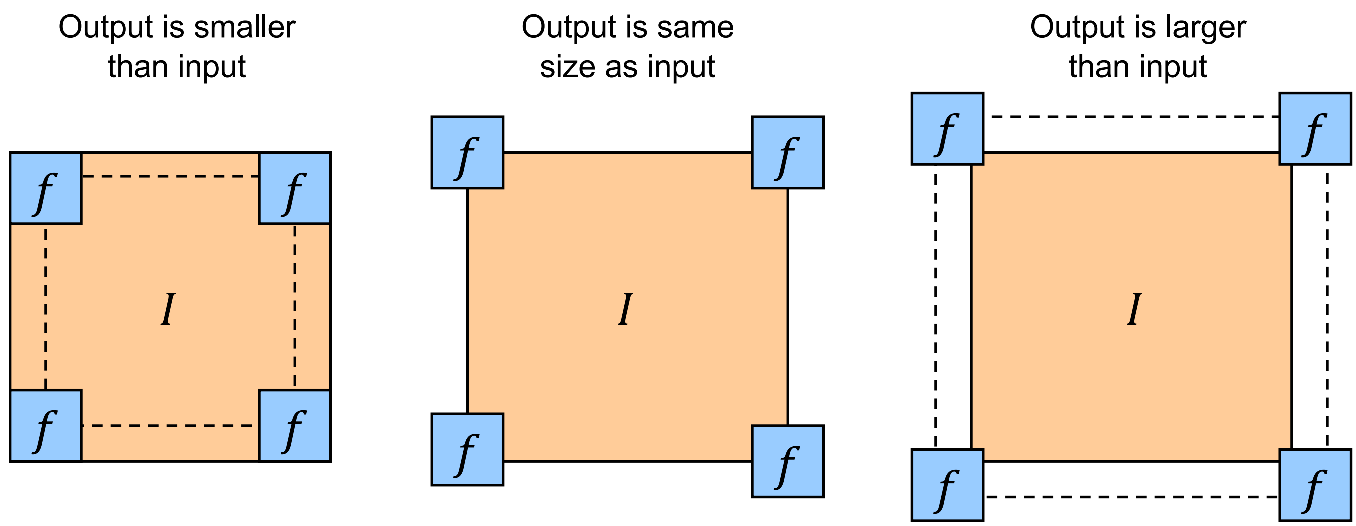


What filter values should we use to find the average in a 3×3 window?

Adapted from [D. Fouhey and J. Johnson]

Practical details: Dealing with edges

- To control the size of the output, we need to use *padding*



Practical details: Dealing with edges

- To control the size of the output, we need to use *padding*
- What values should we pad the image with?

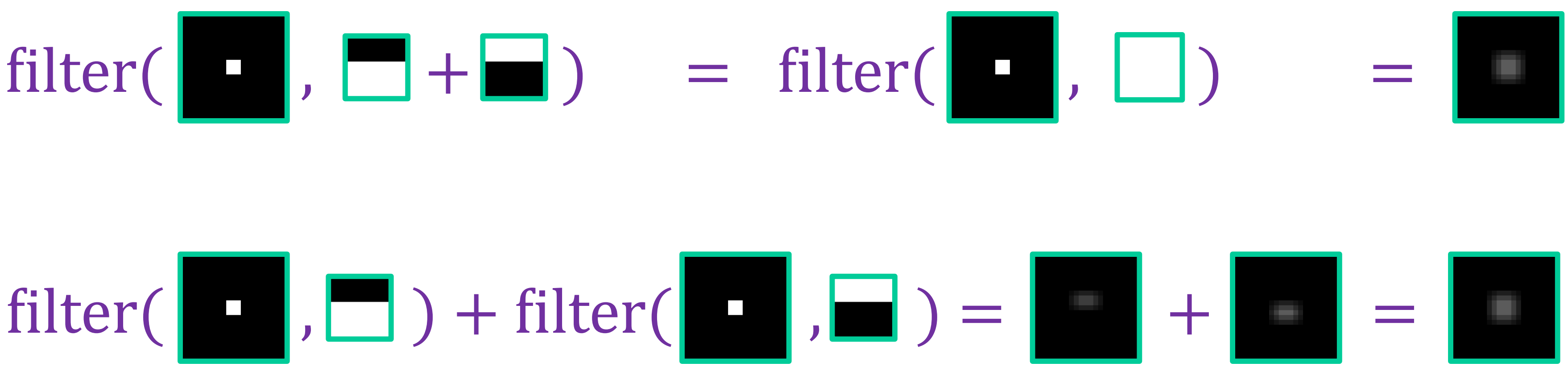
Practical details: Dealing with edges

- To control the size of the output, we need to use *padding*
- What values should we pad the image with?
  - Zero pad (or clip filter)
  - Wrap around
  - Copy edge
  - Reflect across edge

Source: S. Marschner

Properties: Linearity

$$\text{filter}(I, f_1 + f_2) = \text{filter}(I, f_1) + \text{filter}(I, f_2)$$



Adapted from [D. Fouhey and J. Johnson]

Properties: Linearity

$$\text{filter}(I, f_1 + f_2) = \text{filter}(I, f_1) + \text{filter}(I, f_2)$$

Also:

$$\text{filter}(I_1 + I_2, f) = \text{filter}(I_1, f) + \text{filter}(I_2, f)$$

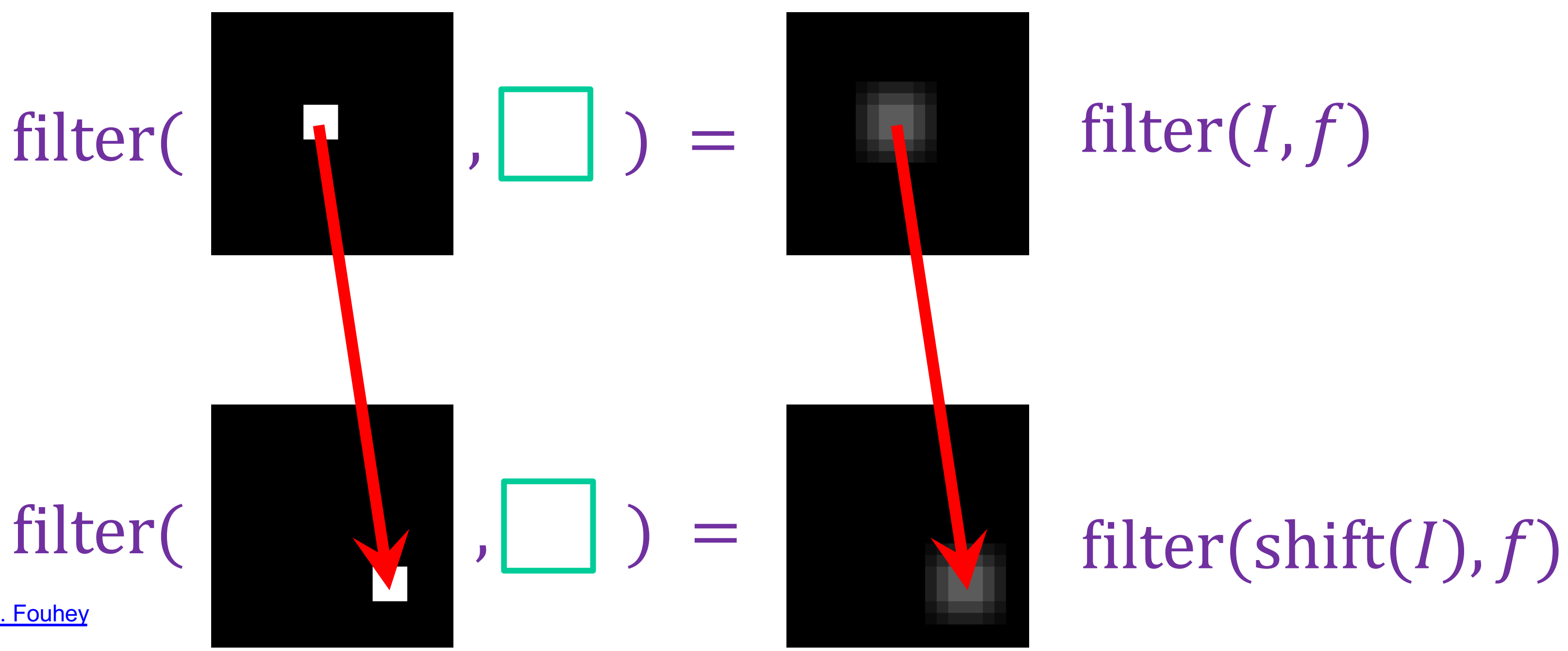
$$\text{filter}(kI, f) = k\ \text{filter}(I, f)$$

$$\text{filter}(I, kf) = k\ \text{filter}(I, f)$$

Adapted from D. Fouhey and J. Johnson

Properties: Shift-invariance

$$\text{filter}(\text{shift}(I), f) \;=\; \text{shift}(\text{filter}(I, f))$$



Adapted from D. Fouhey and J. Johnson

More linear filtering properties

- Commutativity: $f * g = g * f$
  - For infinite signals, no difference between filter and signal
- Associativity: $f * (g * h) = (f * g) * h$
  - Convolving several filters one after another is equivalent to convolving with one combined filter:

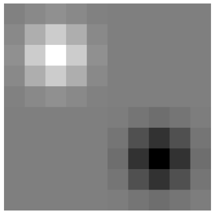
$$\left(\left((g * f_1) * f_2\right) * f_3\right) = g * (f_1 * f_2 * f_3)$$

- Identity: for *unit impulse* e, $f * e = f$

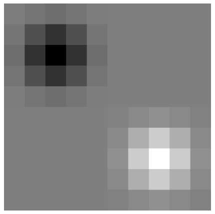
Note: Filtering vs. “convolution”

- In classical signal processing terminology, convolution is filtering with a *flipped* kernel, and filtering with an upright kernel is known as *cross-correlation*
  - Check convention of filtering function you plan to use!

Filtering or “cross-correlation”
(Kernel in original orientation)

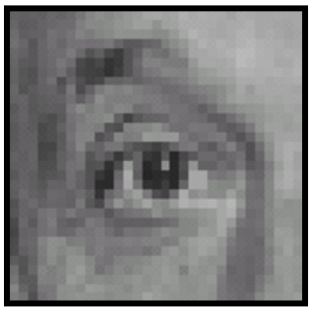


“Convolution”
(Kernel flipped in x and y)



Adapted from D. Fouhey and J. Johnson

Practice with linear filters



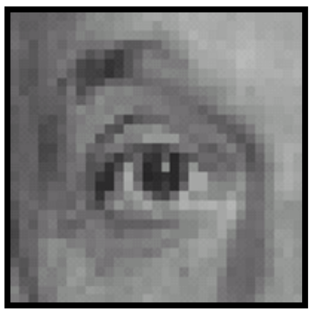
Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

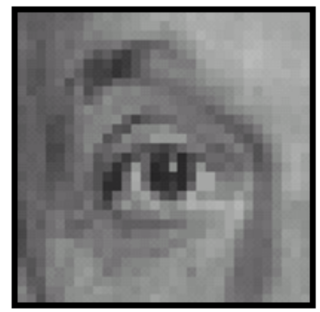
Practice with linear filters



Original

0	0	0
0	1	0
0	0	0

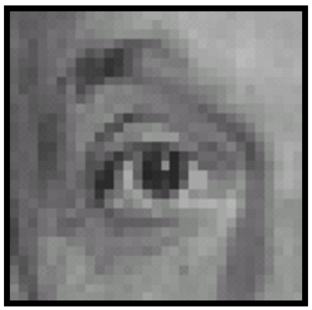
One surrounded by zeros is the *identity filter*



Filtered (no change)

Source: D. Lowe

Practice with linear filters



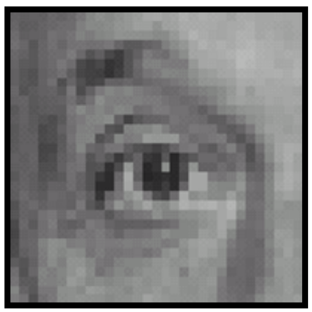
Original

0	0	0
0	0	1
0	0	0

?

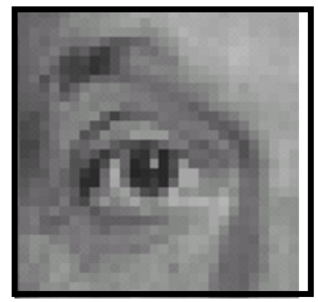
Source: D. Lowe

Practice with linear filters



Original

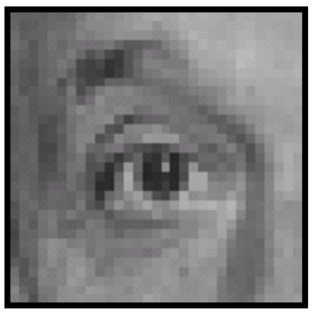
0	0	0
0	0	1
0	0	0



Shifted *left*
By one pixel

Source: D. Lowe

Practice with linear filters



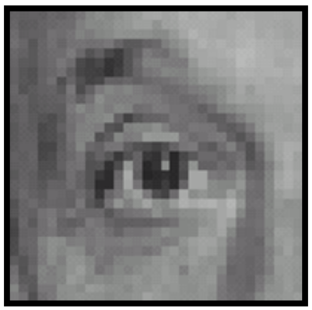
Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

?

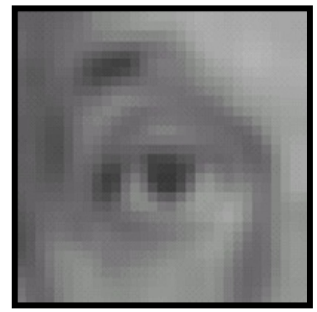
Source: D. Lowe

Practice with linear filters



Original

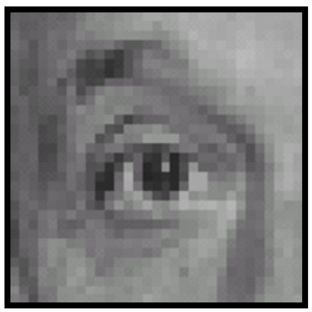
$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$



Blur (with a box filter)

Source: D. Lowe

Practice with linear filters



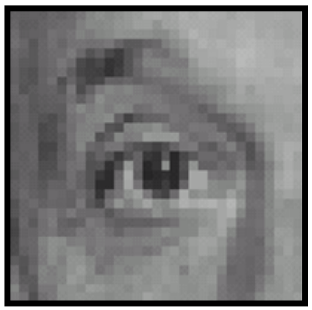
Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

?

Source: D. Lowe

Practice with linear filters



Original

$$\begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

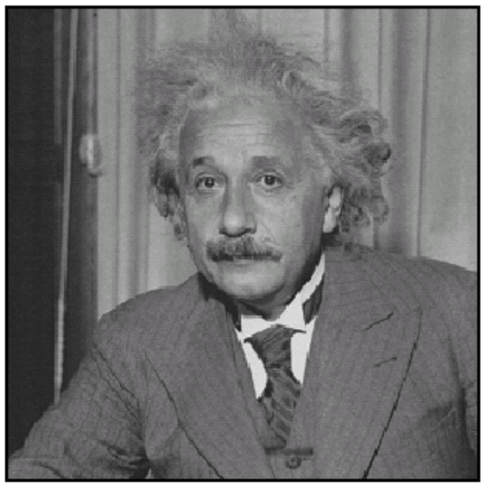
Sharpening filter:
Accentuates differences with local average

(Note that filter sums to 1)

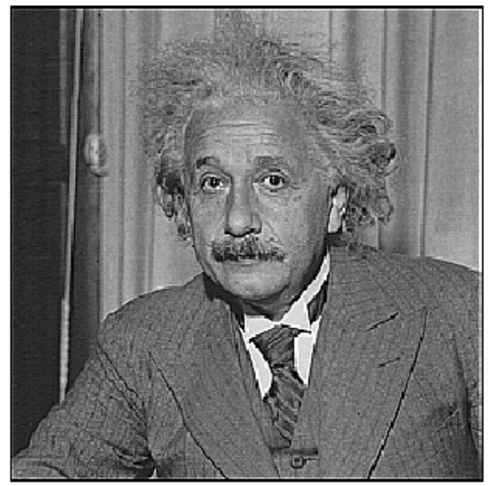
Sharpened

Source: D. Lowe

Sharpening



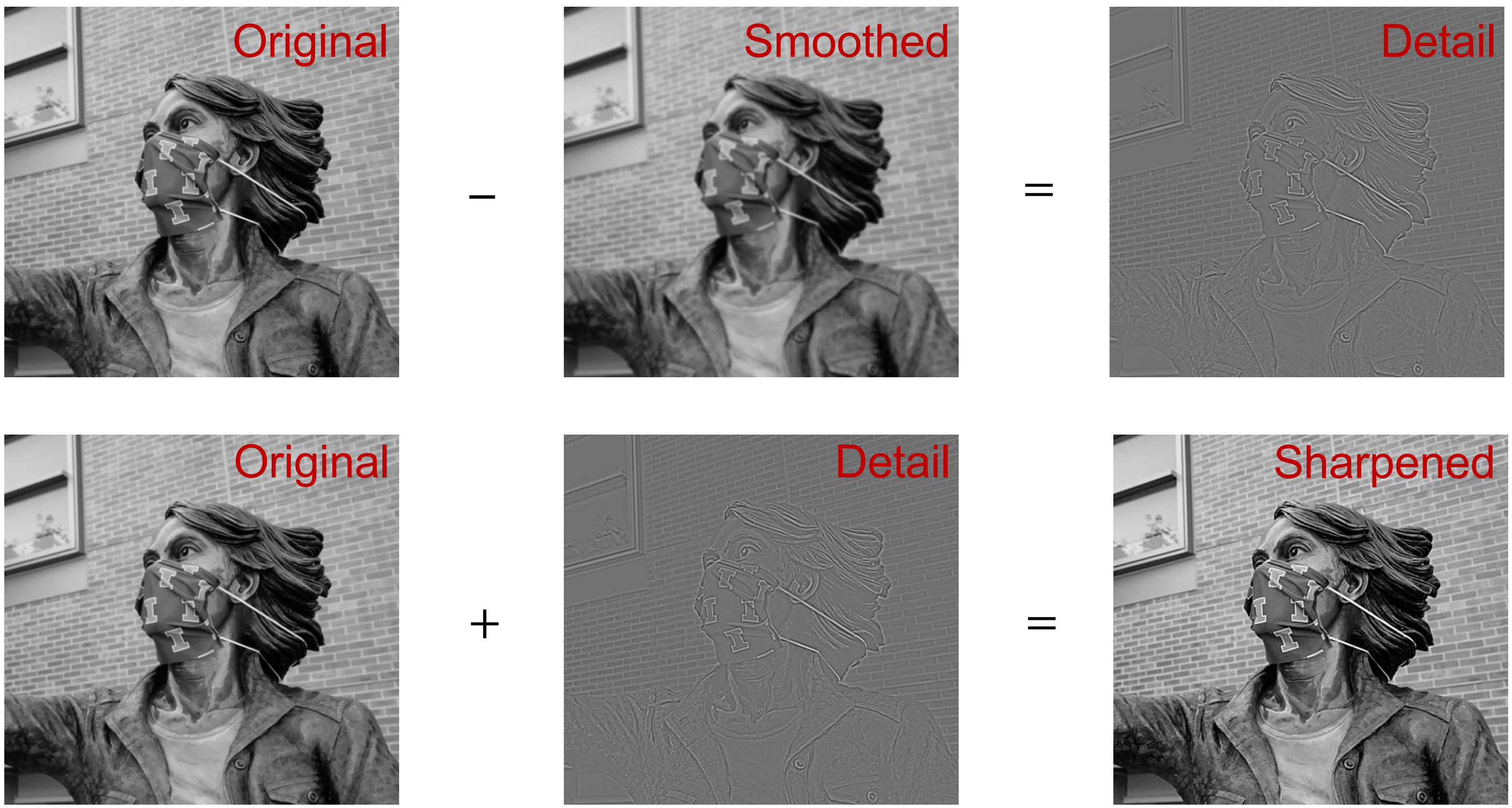
before



after

Source: D. Lowe

Sharpening



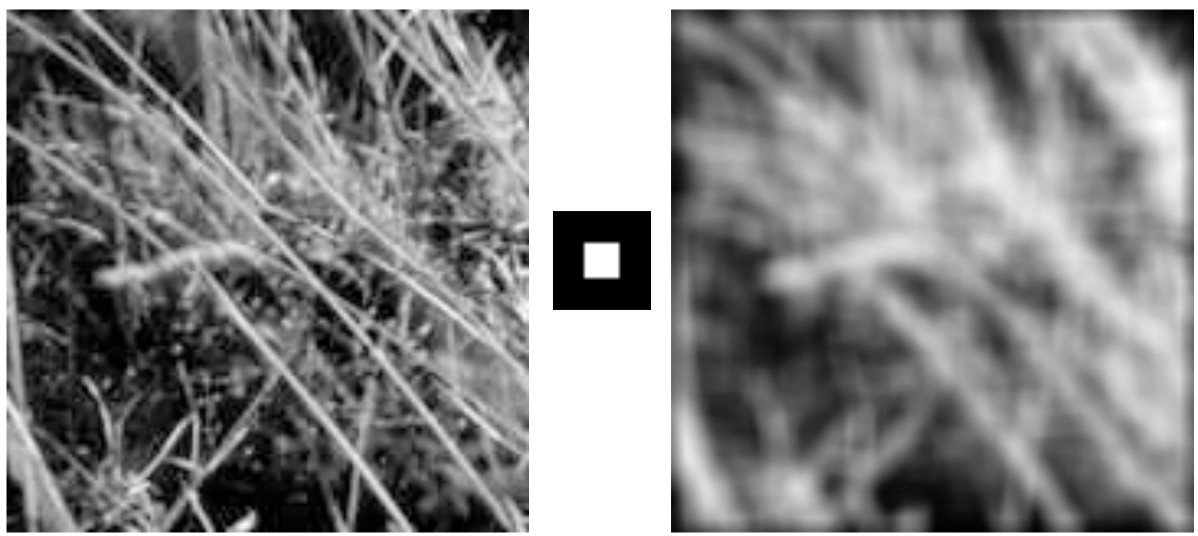
Source:
S. Gupta

Image filtering: Outline

- Linear filtering and its properties
- Gaussian filters and their properties

Smoothing with box filter revisited

- What’s wrong with this picture?
- What’s the solution?



Source: D. Forsyth

Smoothing with box filter revisited

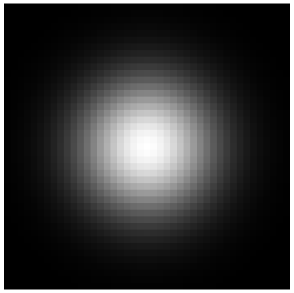
- What's wrong with this picture?
- What's the solution?
  - To eliminate edge effects, weight contribution of neighborhood pixels according to their closeness to the center

"proportional to"
(renormalize values to sum to 1)

$$G(x,y) \propto \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

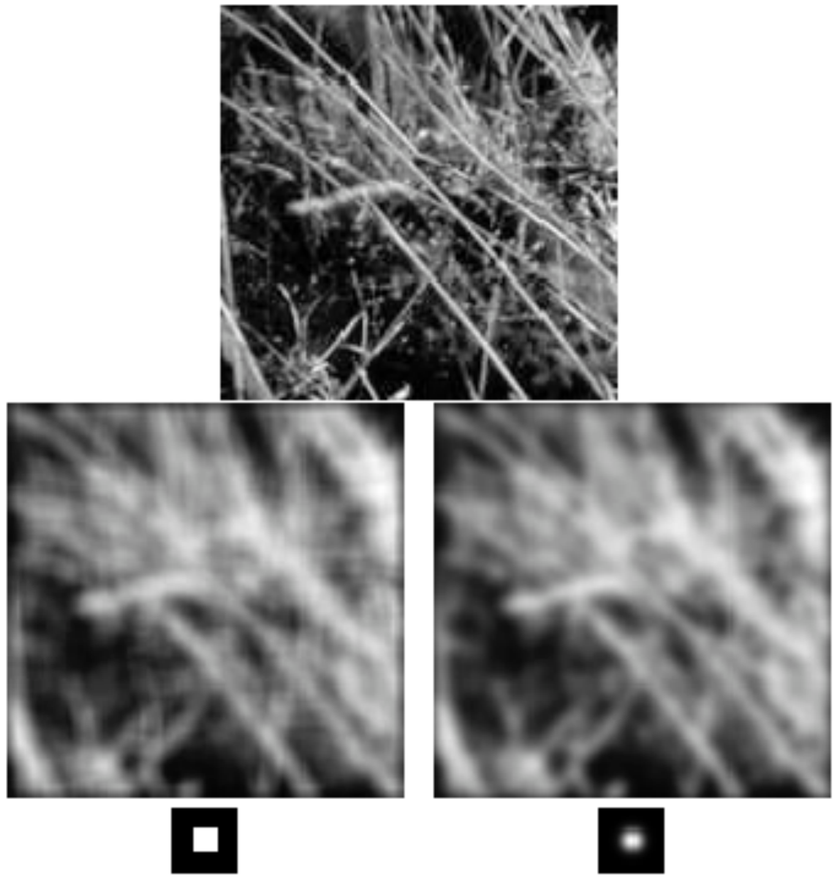
standard deviation
(determines size of "blob")

Gaussian filter



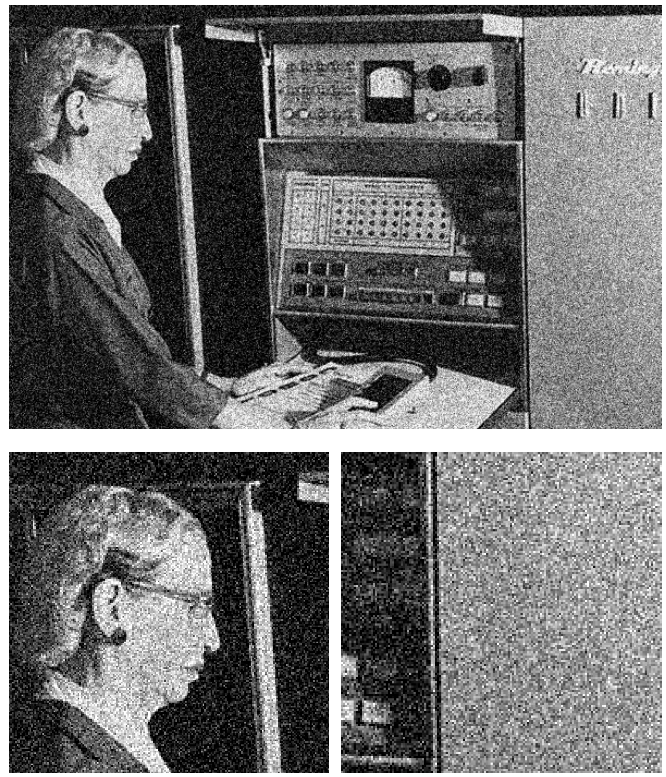
Adapted from D. Fouhey and J. Johnson

Gaussian vs. box filtering



Applying Gaussian filters

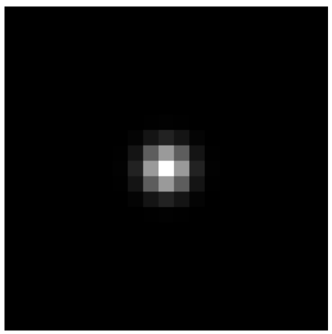
Input image
(no filter)

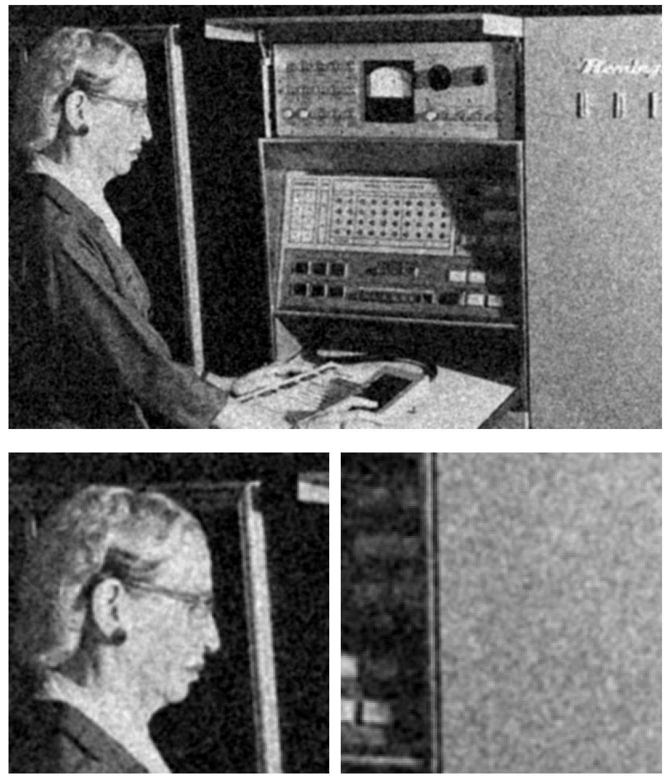


Source: [D. Fouhey and J. Johnson]

Applying Gaussian filters

$\sigma = 1$

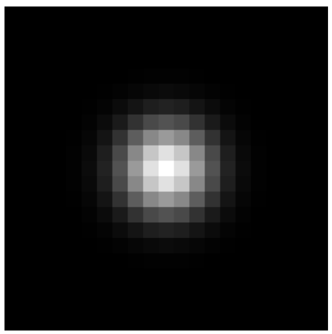


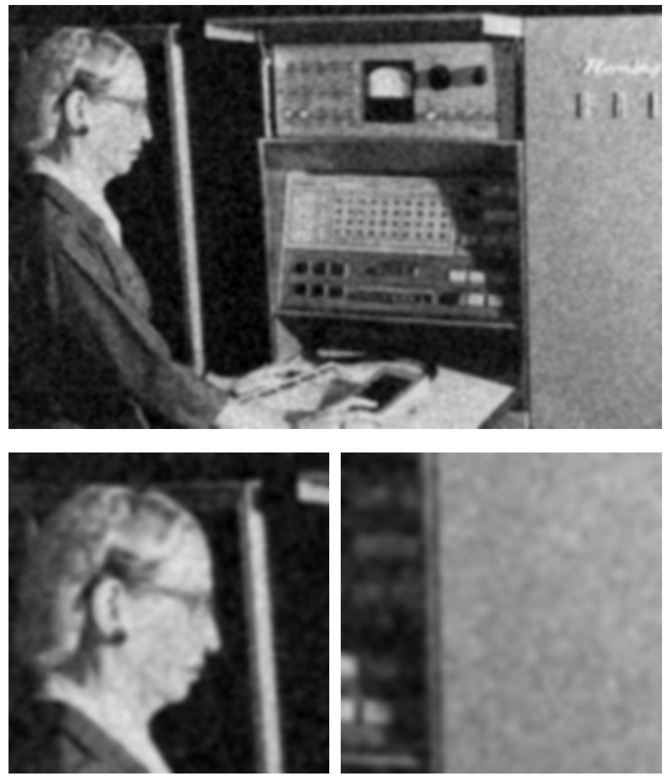


Source: [D. Fouhey and J. Johnson]

Applying Gaussian filters

$\sigma = 2$

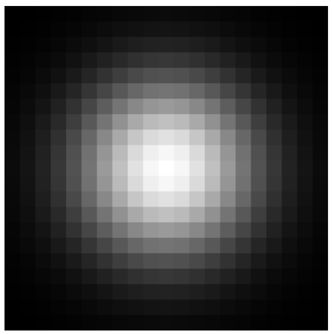




Source: [D. Fouhey and J. Johnson]

Applying Gaussian filters

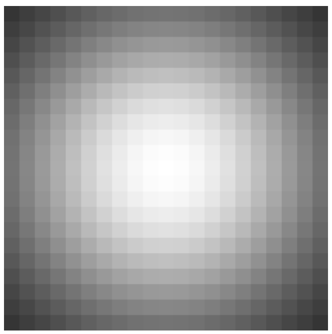
$\sigma = 4$



Source: [D. Fouhey and J. Johnson]

Applying Gaussian filters

$\sigma = 8$

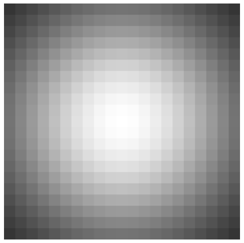


Source: [D. Fouhey and J. Johnson]

Choosing filter size

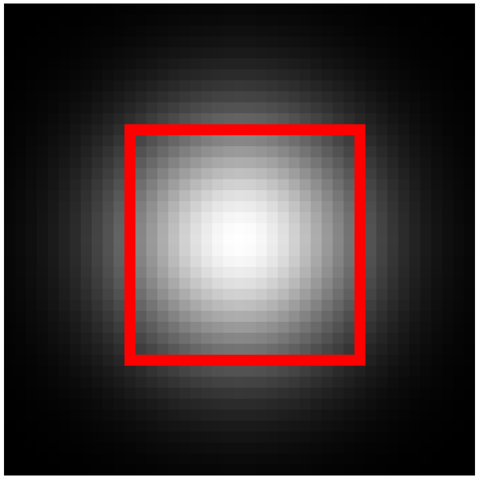
- Rule of thumb: set filter width to about 6σ (captures 99.7% of the energy)

$\sigma = 8$
$\text{Width} = 21$



Too small!

$\sigma = 8$
$\text{Width} = 43$



A bit small (might be OK)

Adapted from D. Fouhey and J. Johnson

Gaussian filters: Properties

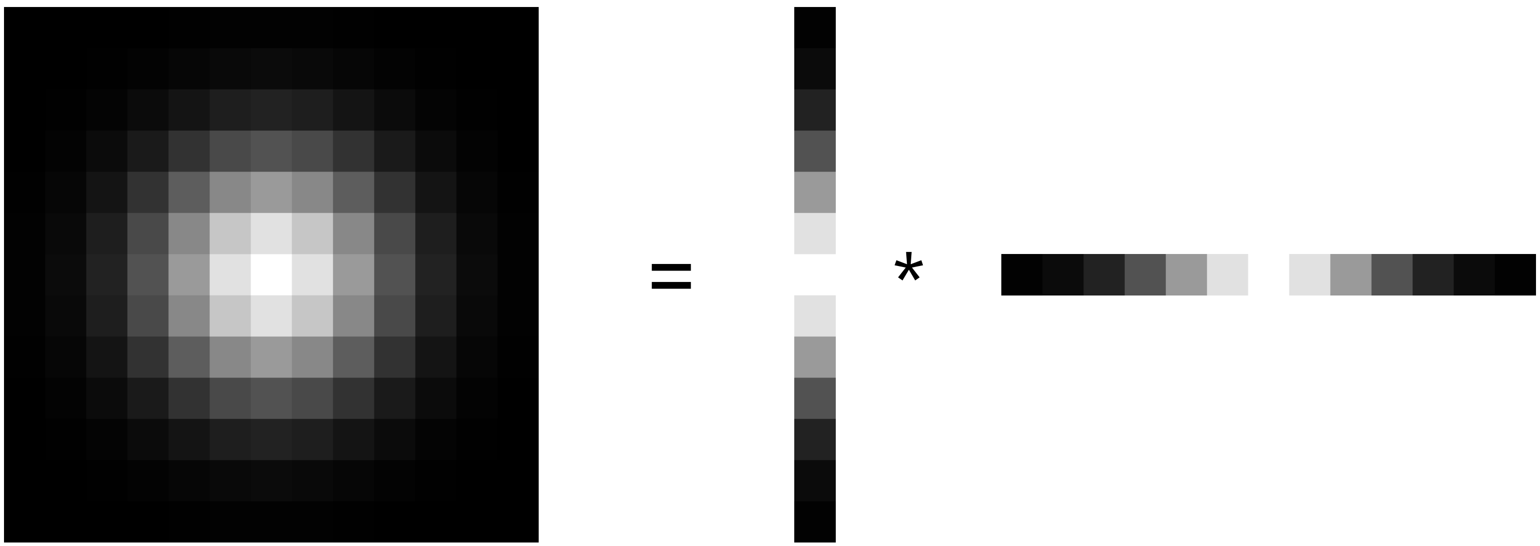
- Gaussian is a *low-pass filter*: it removes high-frequency components from the image (more on this soon)
- Convolution with self is another Gaussian
  - So we can smooth with small-σ kernel, repeat, and get same result as larger-σ kernel would have
  - Convolving *two times* with Gaussian kernel with std. dev. σ is the same as convolving *once* with kernel with std. dev. $\sigma\sqrt{2}$
- Gaussian kernel is *separable*: it factors into product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) =$$

$$\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right) \quad \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{y^2}{2\sigma^2}\right)$$



Adapted from D. Fouhey and J. Johnson

Why is separability useful?

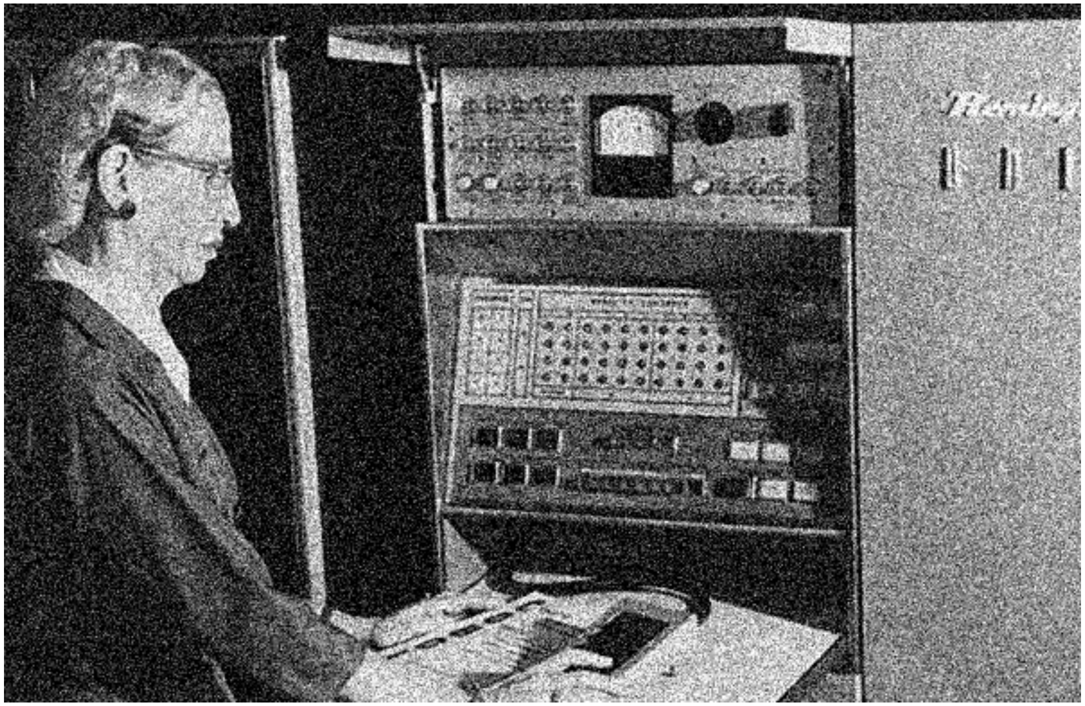
- Separability means that a 2D convolution can be reduced to two 1D convolutions (one along rows and one along columns)
- What is the complexity of filtering an $n \times n$ image with an $m \times m$ kernel?
  - $O(n^2 m^2)$
- What if the kernel is separable?
  - $O(n^2 m)$

Image filtering: Outline

- Linear filtering and its properties
- Gaussian filters and their properties
- Nonlinear filtering: Median filtering

Different types of noise

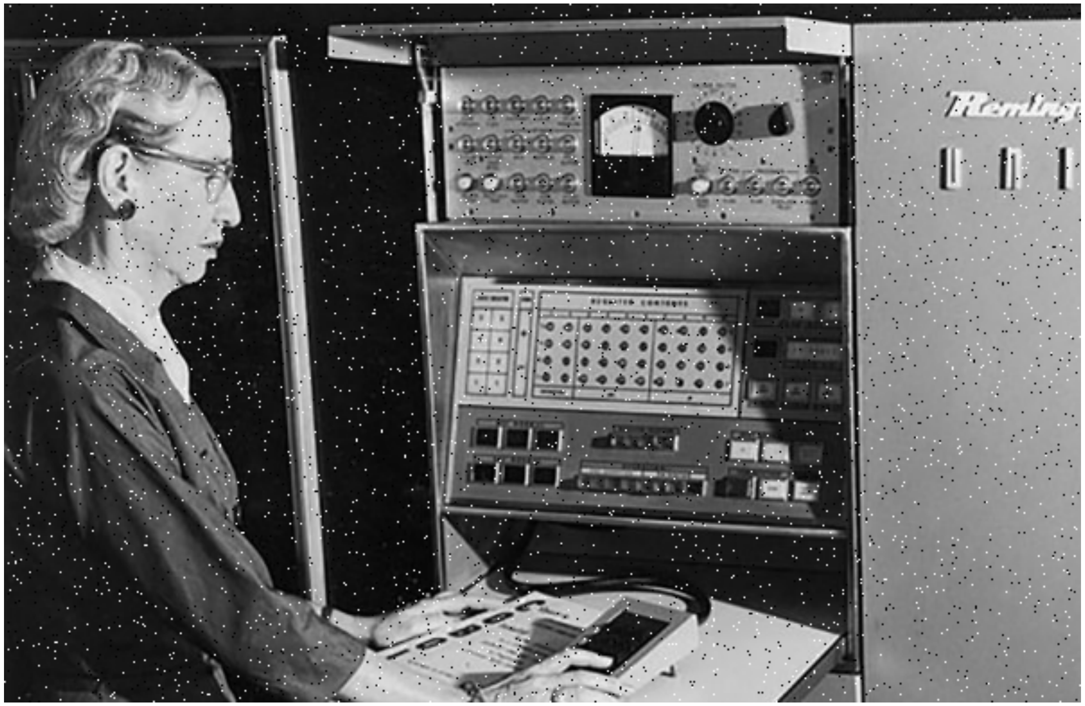
- Gaussian filtering is appropriate for *additive, zero-mean* noise (assuming nearby pixels share the same value)



Adapted from [D. Fouhey and J. Johnson]

Different types of noise

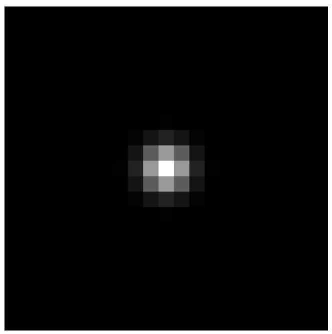
- What about *impulse* or *shot noise,* i.e., when some pixels are arbitrarily replaced by spurious values?

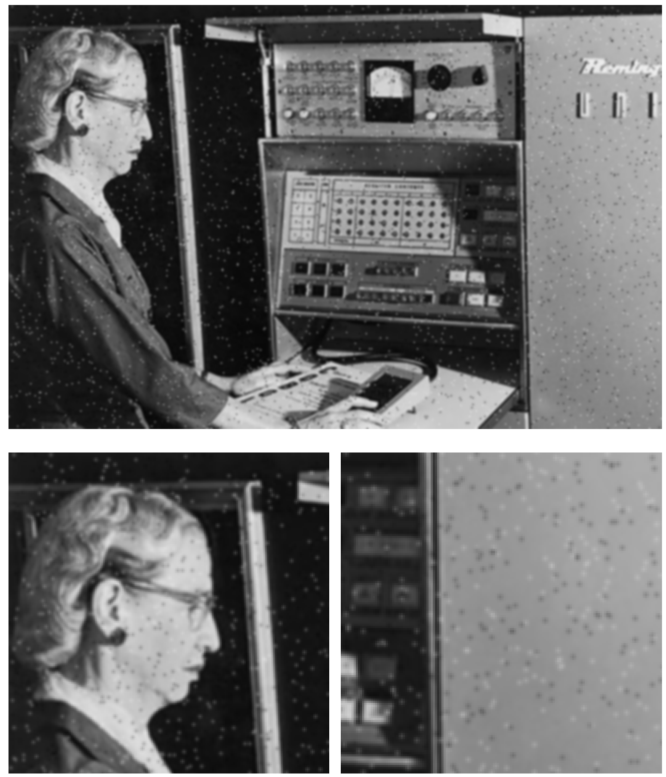


Adapted from D. Fouhey and J. Johnson

Where Gaussian filtering fails

$\sigma = 1$

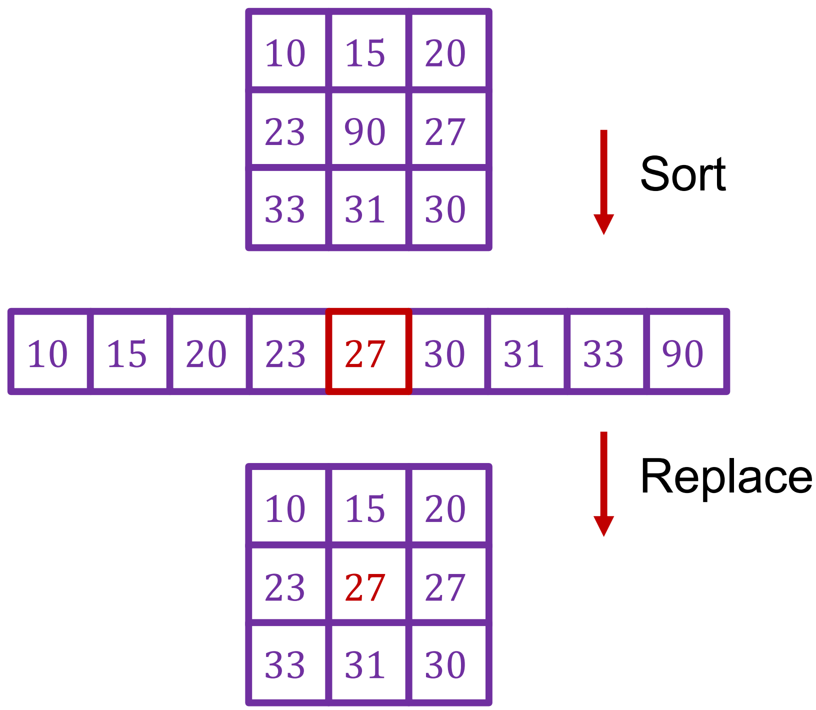




Adapted from [D. Fouhey and J. Johnson]

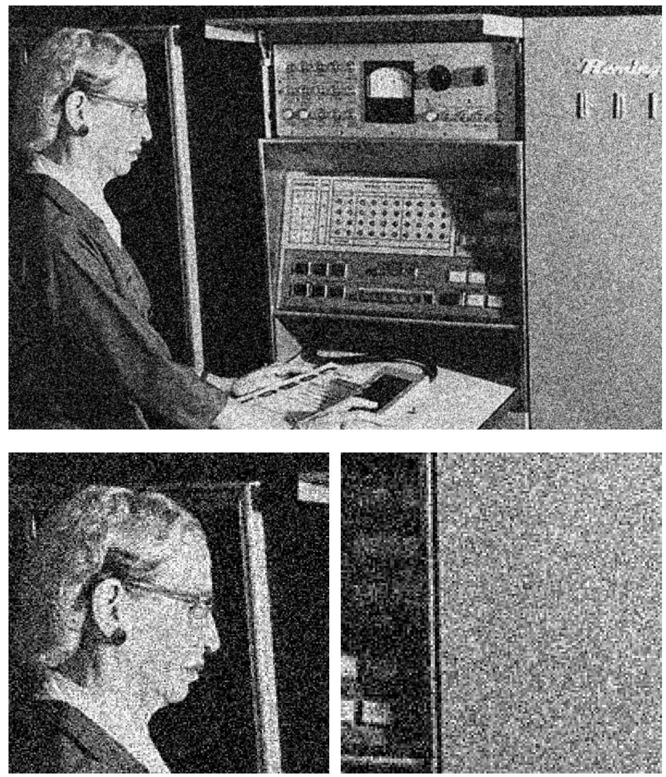
Alternative idea: Median filtering

- A **median filter** operates over a window by selecting the median intensity in the window



Applying median filter

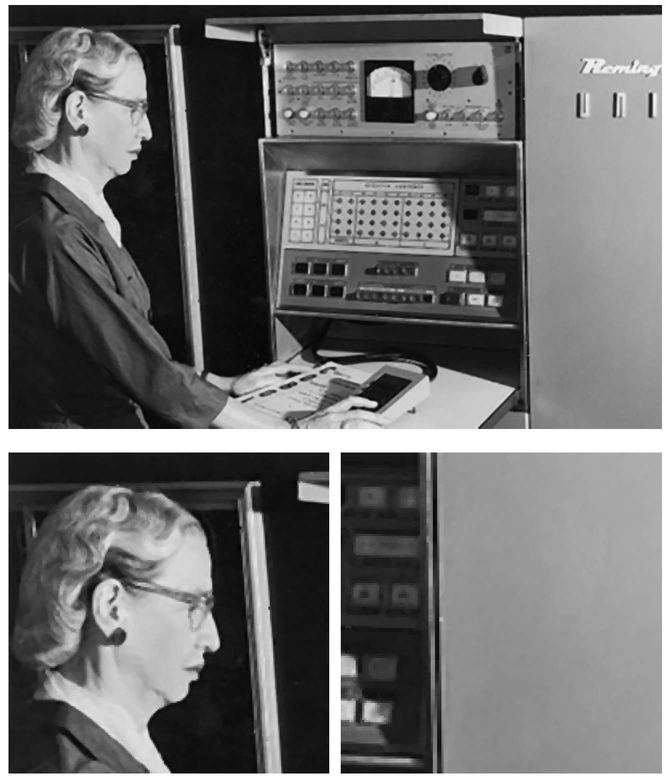
Input image
(no filter)



Adapted from [D. Fouhey and J. Johnson]

Applying median filter

median
filter
$(\text{width} = 3)$



Adapted from [D. Fouhey and J. Johnson]

Applying median filter

median
filter
$(\text{width} = 7)$

Is median filtering linear?

Is median filtering linear?

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

median filter ↓

$$1 + 0 \neq 2$$

Adapted from [D. Fouhey and J. Johnson]

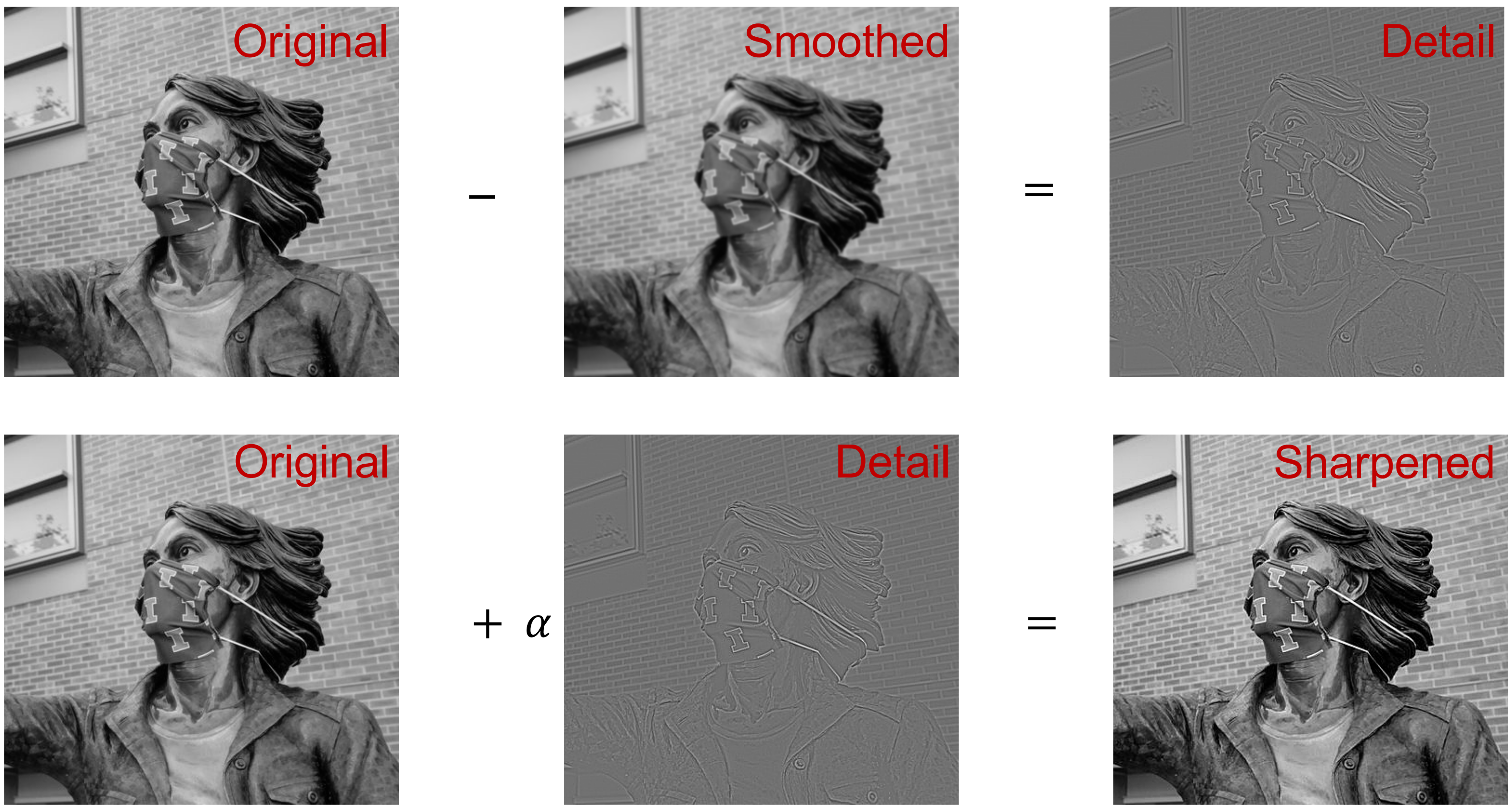
Image filtering: Outline

- Linear filtering and its properties
- Gaussian filters and their properties
- Nonlinear filtering: Median filtering
- Fun filtering application: Hybrid images

Application: Hybrid images

A. Oliva, A. Torralba, P.G. Schyns,
Hybrid Images, SIGGRAPH 2006

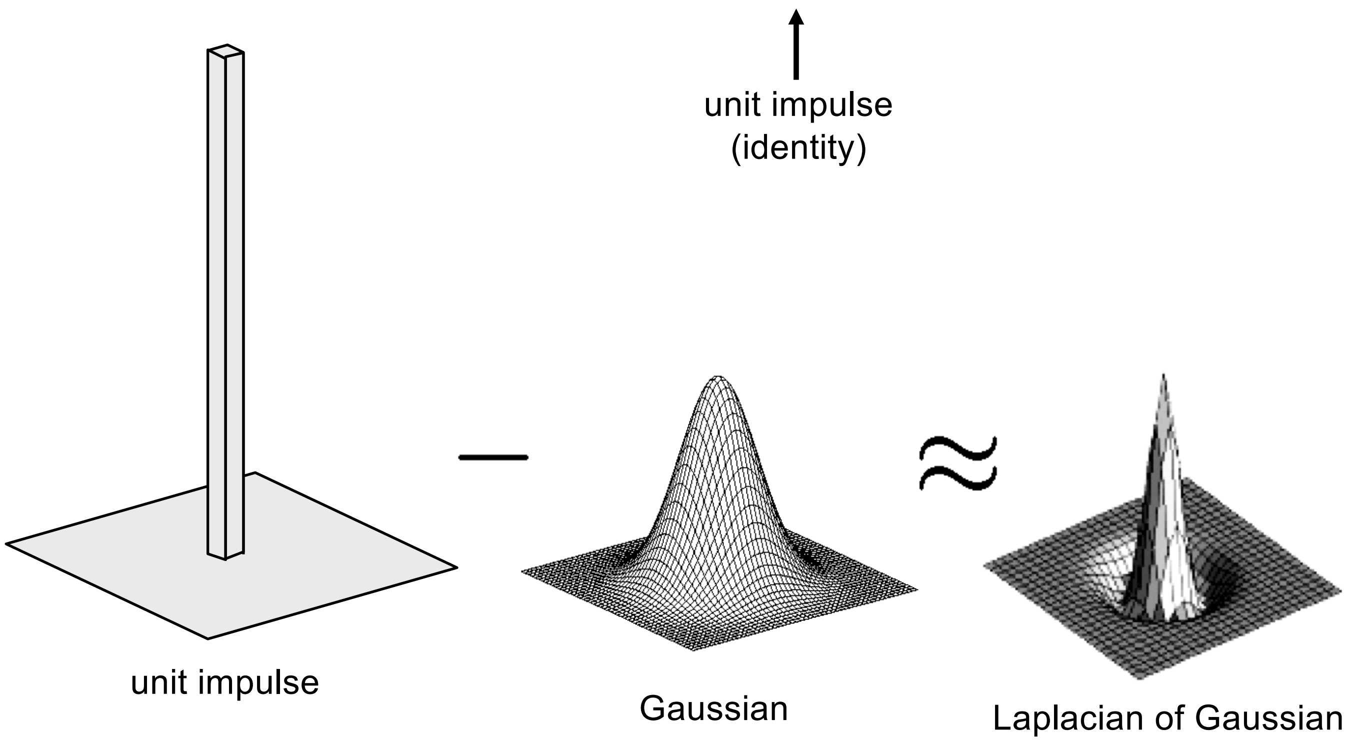
Recall: Sharpening



Source:
S. Gupta

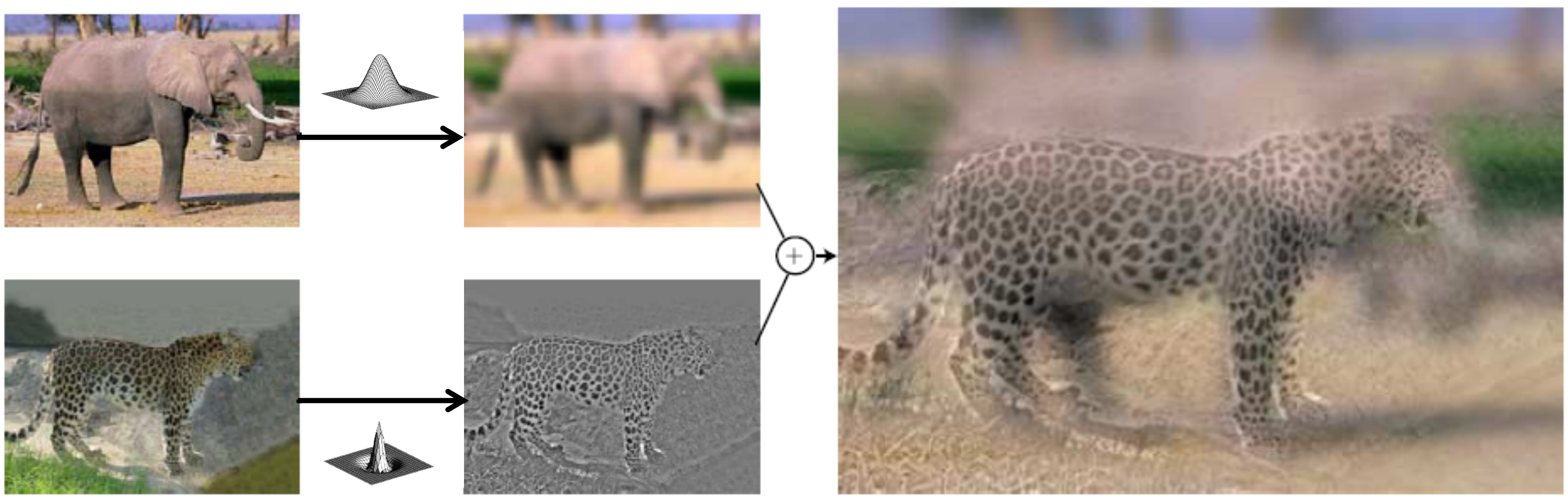
“Detail” filter

$$I - I * g = I * (e - g)$$



Application: Hybrid images

Gaussian filter



Laplacian filter

Application: Hybrid images