Perspective projection

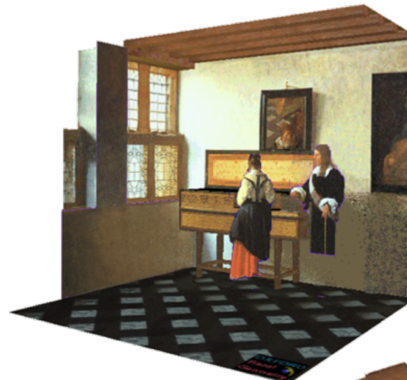
A. Mantegna, *Martyrdom of St. Christopher*, c. 1450

Overview

- Motivation: recovery of 3D structure
- Pinhole projection model
- Properties of projection
- Perspective projection matrix
- Orthographic projection

Given an image, can we recover 3D structure?

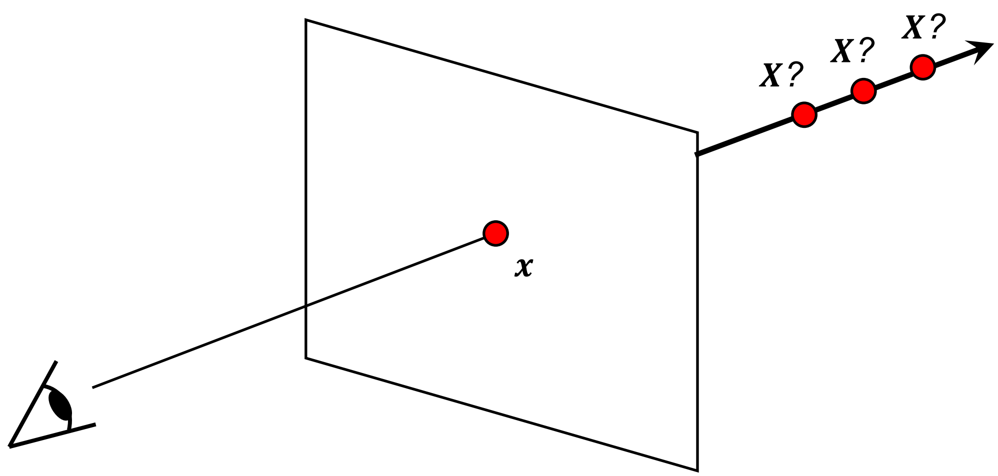
J. Vermeer, *Music Lesson*, 1662



A. Criminisi, M. Kemp, and A. Zisserman, Bringing Pictorial Space to Life: computer techniques for the analysis of paintings, *Proc. Computers and the History of Art*, 2002

Things aren't always as they appear…

Single-view ambiguity



Single-view ambiguity

Rashad Alakbarov shadow sculptures

Anamorphic perspective

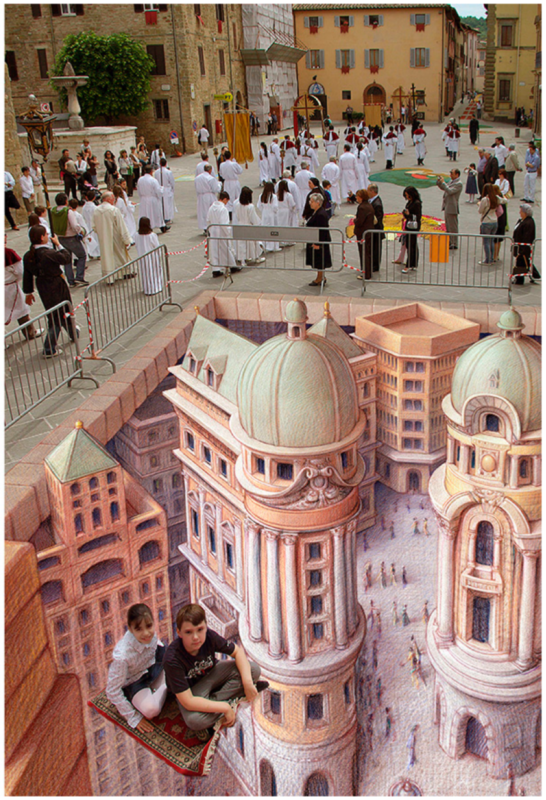


Image source

Anamorphic perspective

H. Holbein The Younger, *The Ambassadors*, 1533

https://en.wikipedia.org/wiki/Anamorphosis

Our goal: Recovery of 3D structure

- When certain assumptions hold, we can recover structure from a single view

- In general, we need *multi-view geometry*

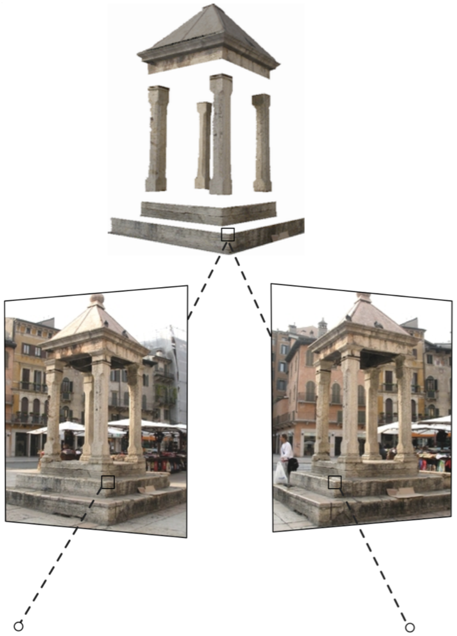


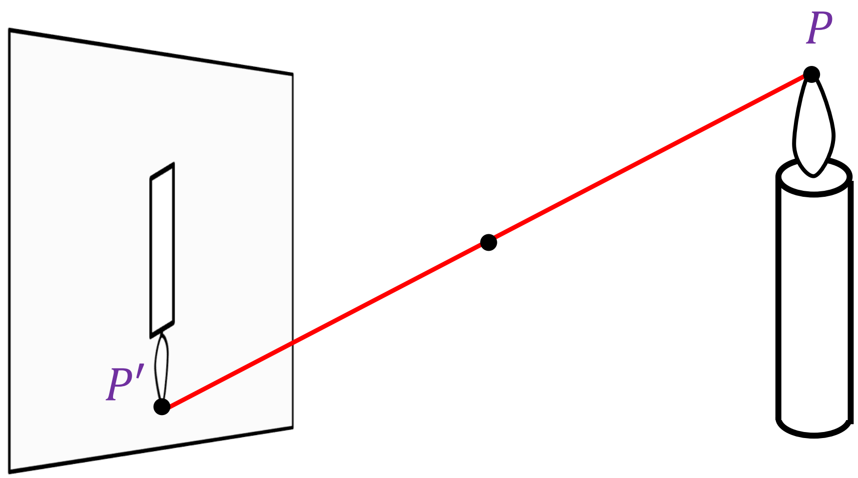
Image source

- But first, we need to understand the geometry of a single camera…

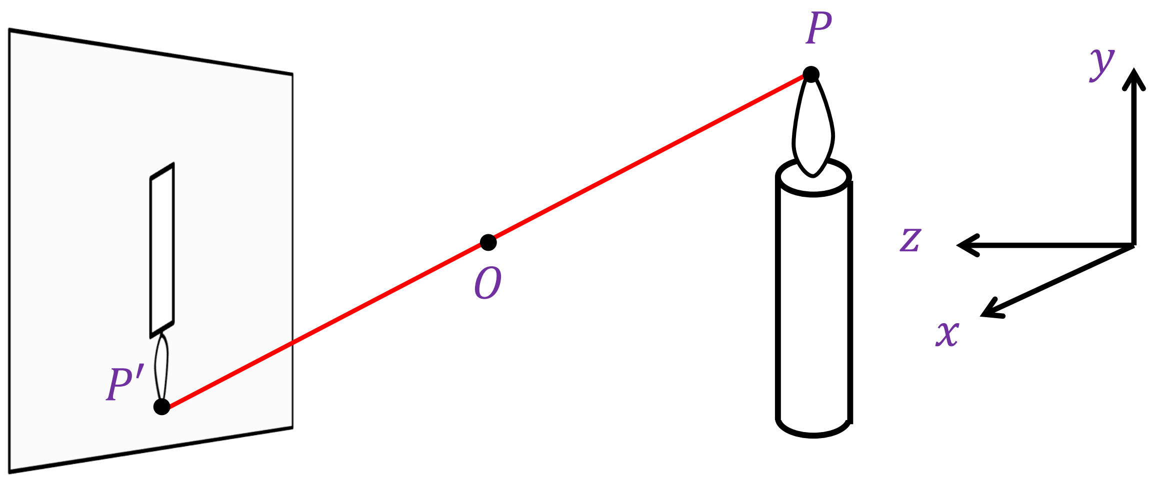
Overview

- Motivation: recovery of 3D structure
- Pinhole projection model

Review: Perspective projection equations



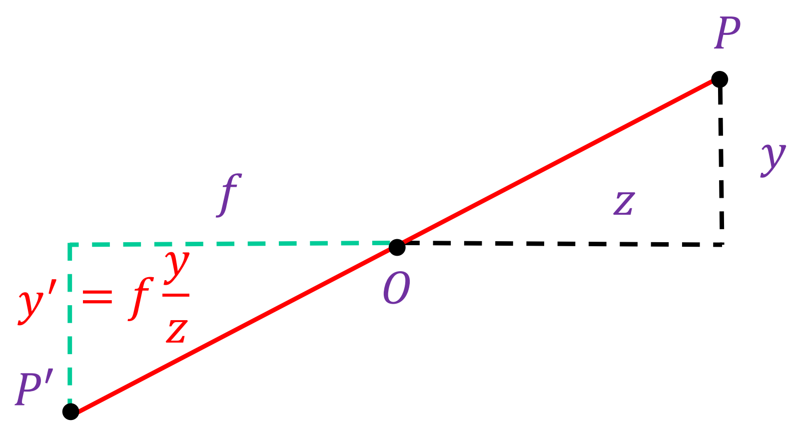
Review: Perspective projection equations



Canonical coordinate system

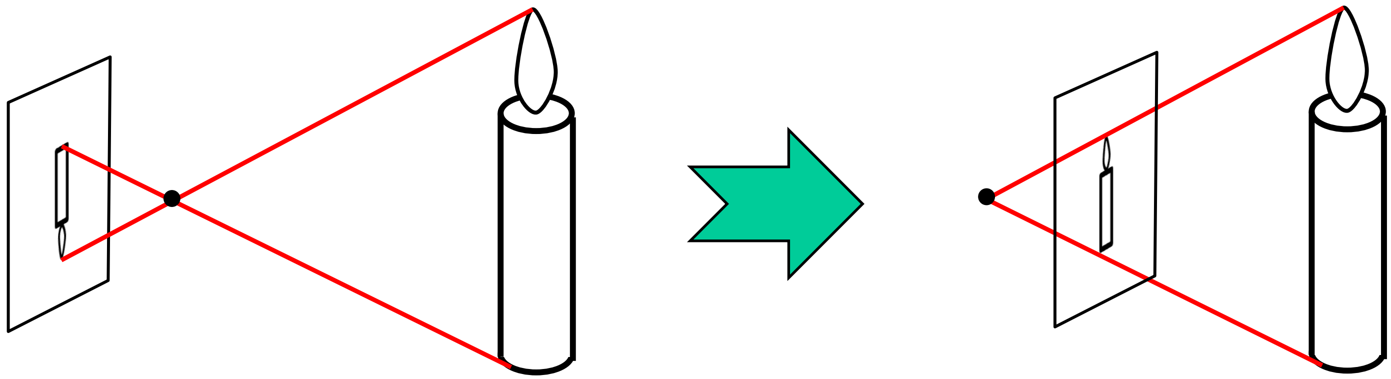
- The optical center (O) is at the origin
- The z axis is the *optical axis* perpendicular to the image plane
- The xy plane is parallel to the image plane, x and y axes are horizontal and vertical directions of the image plane

Review: Perspective projection equations



$$(x, y, z) \rightarrow \left(f\frac{x}{z}, f\frac{y}{z}\right)$$

Review: Perspective projection equations

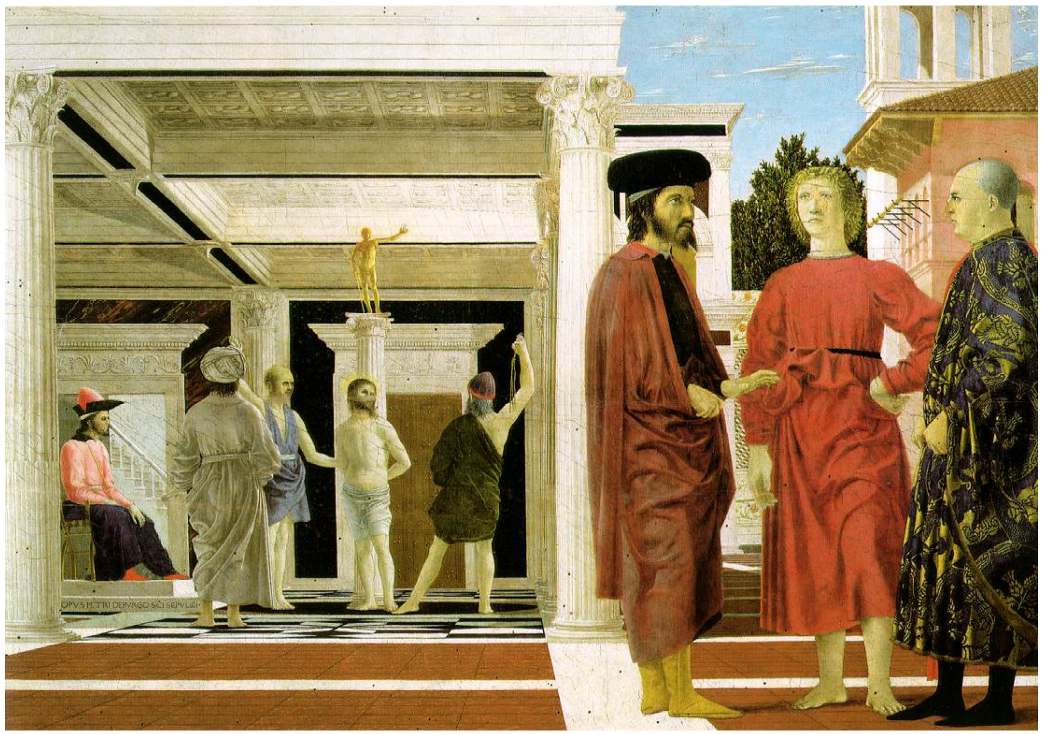


Note: instead of dealing with an image that is upside down, most of the time we will pretend that the image plane is *in front* of the camera center

Overview

- Motivation: recovery of 3D structure
- Pinhole projection model
- Properties of projection
  - What happens to:
    - Lines
    - Planes
    - General 3D shapes

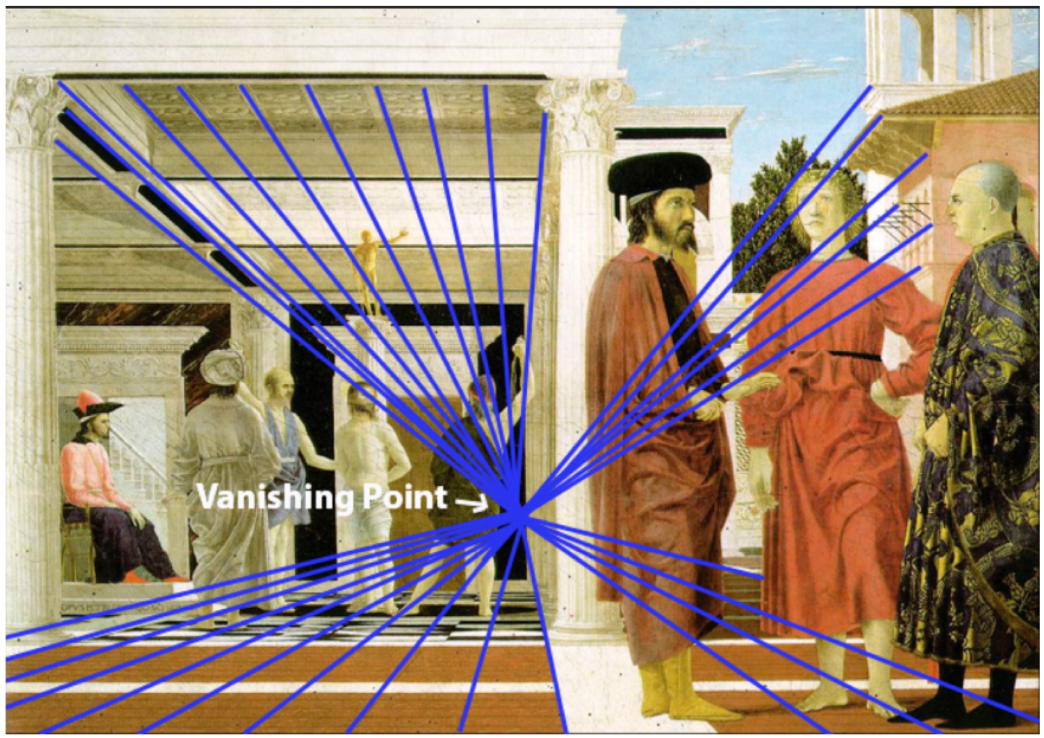
Projection of lines



Piero della Francesca, *Flagellation of Christ*, 1455-1460

Projection of lines

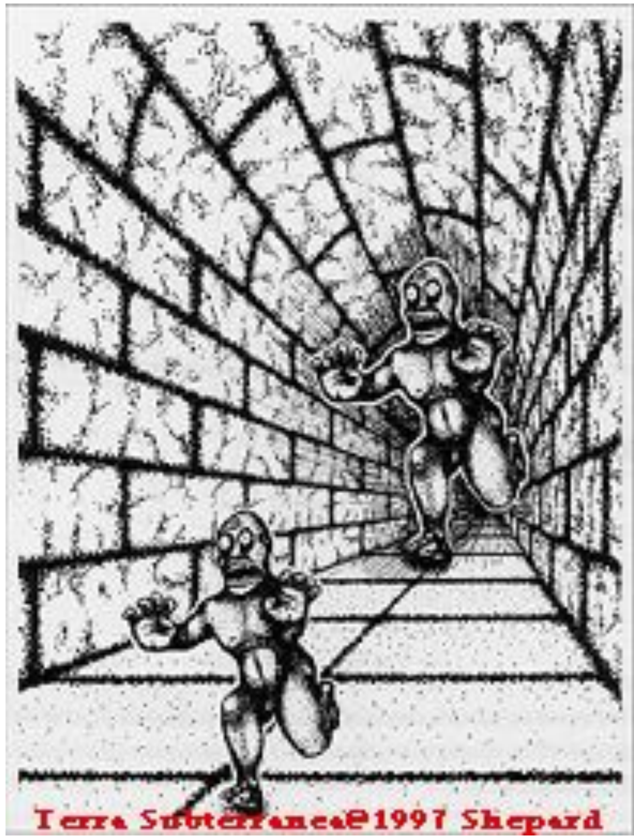
- Parallel lines meet at a *vanishing point*



Piero della Francesca, *Flagellation of Christ*, 1455-1460

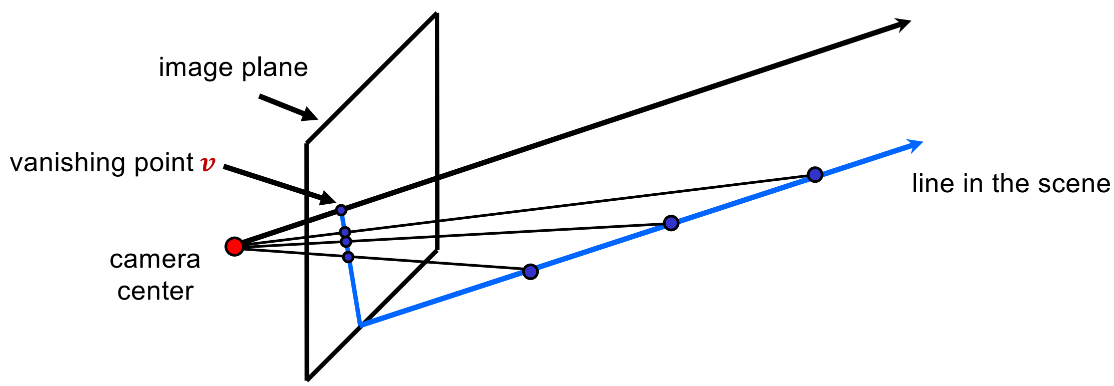
Image source

Converging lines are a powerful perspective cue



Slide by Steve Seitz

Constructing the vanishing point of a line

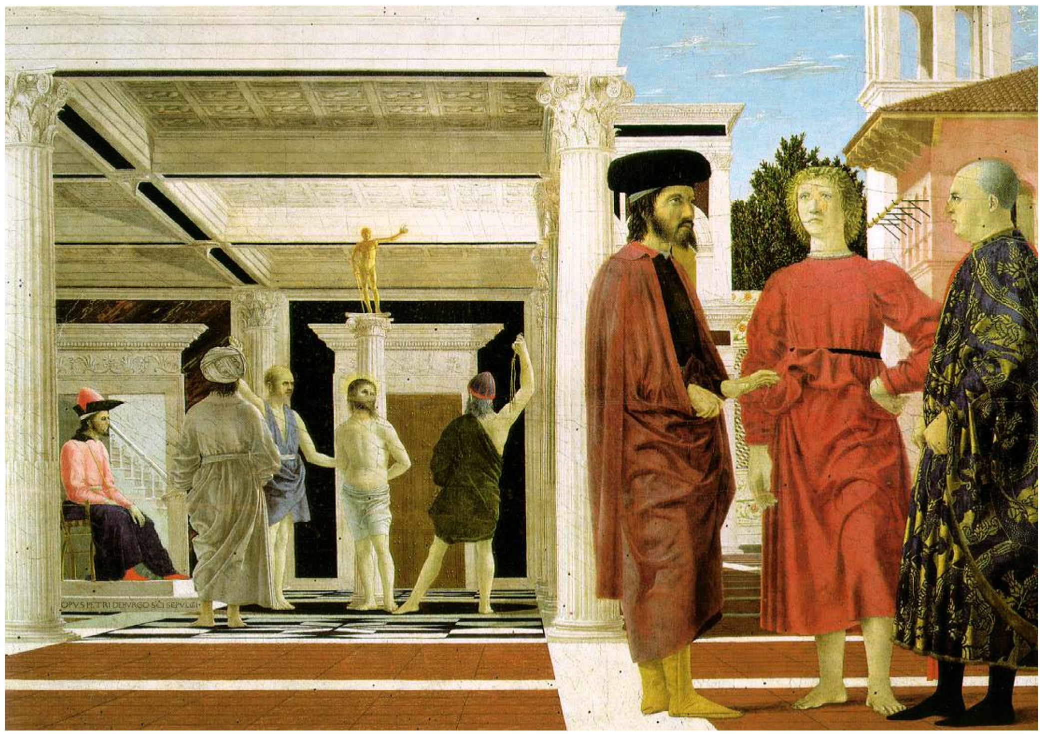


- What about another line going in the same direction?

Adopted from S. Seitz

Projection of lines

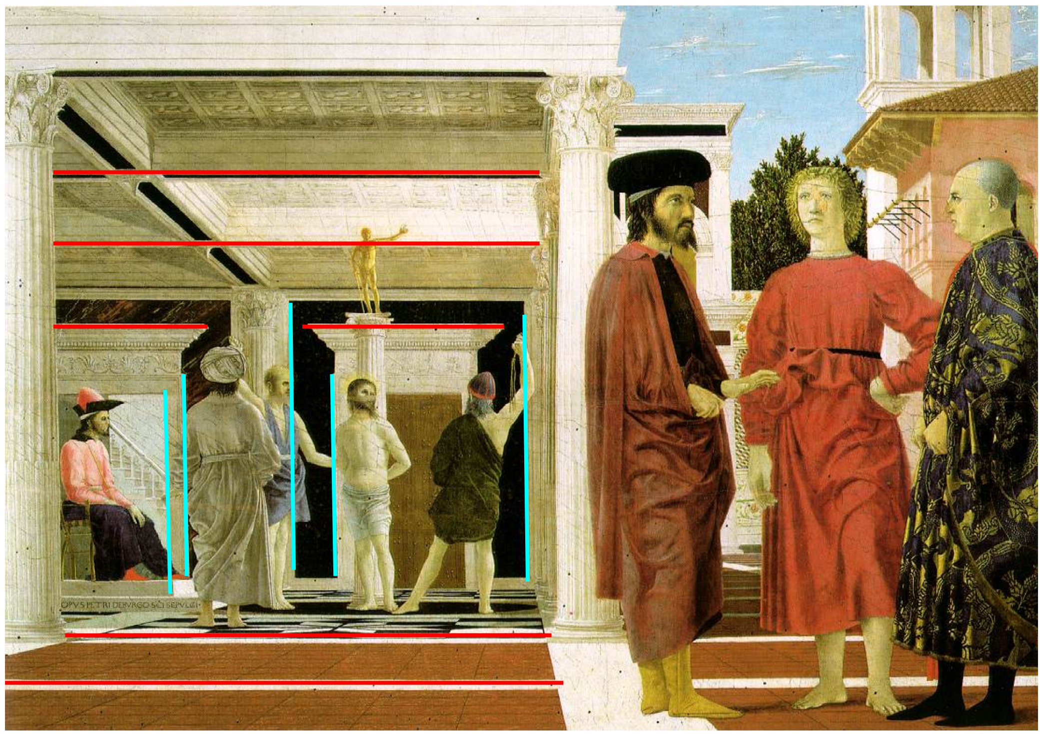
- Do *all* parallel lines converge to a vanishing point?



Piero della Francesca, *Flagellation of Christ*, 1455-1460

Projection of lines

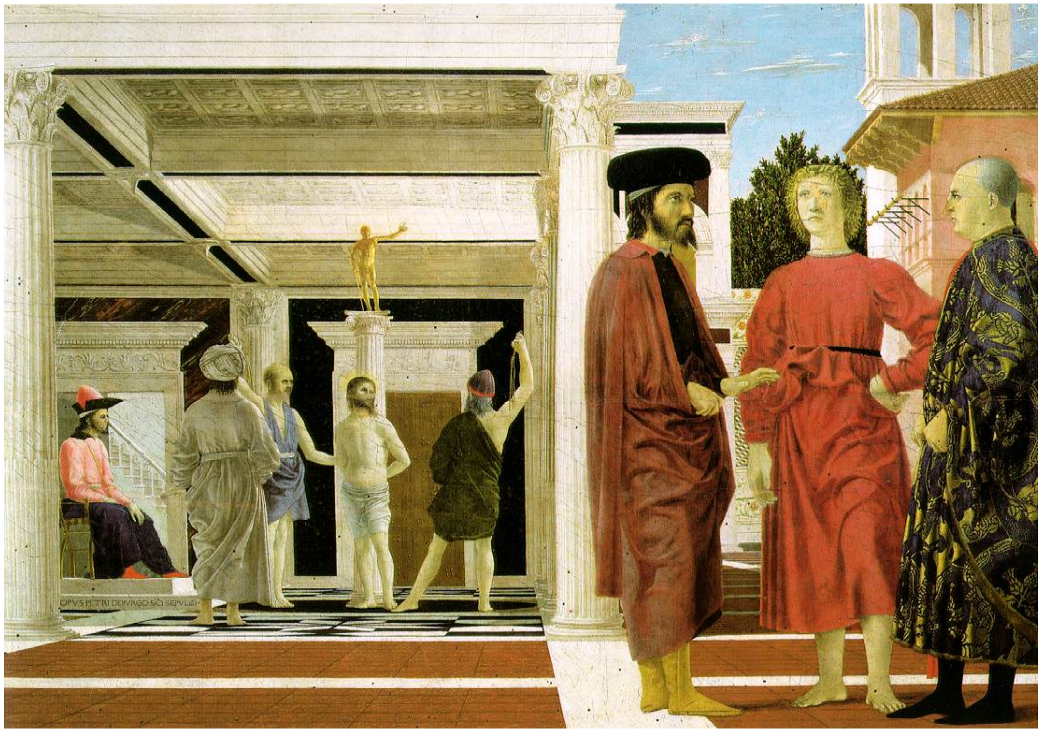
- Do *all* parallel lines converge to a vanishing point?
  - Not parallel lines that are also parallel to the image plane!



Piero della Francesca, *Flagellation of Christ*, 1455-1460

Projection of planes

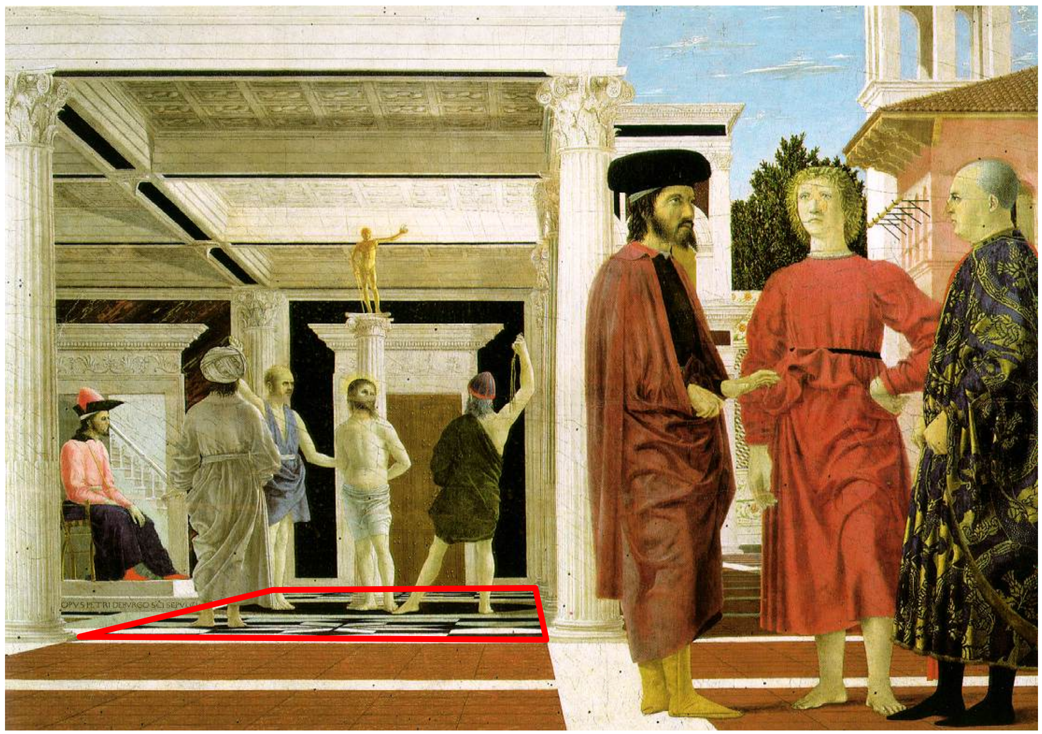
- What happens to planar patterns?



Piero della Francesca, *Flagellation of Christ*, 1455-1460

Projection of planes

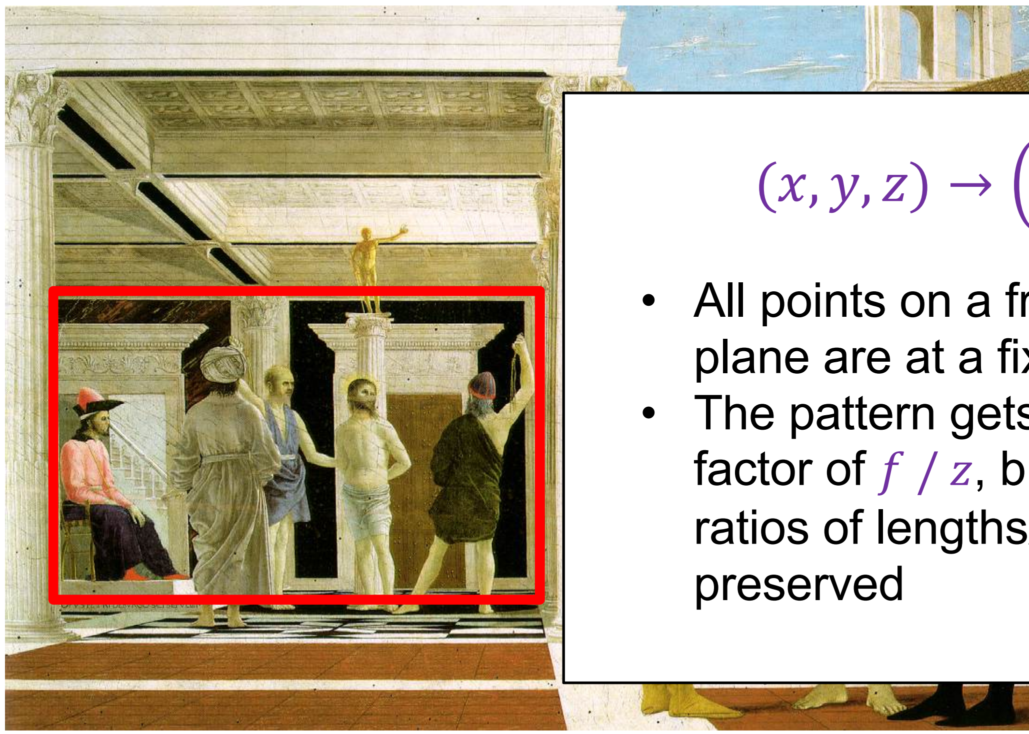
- Patterns on *non-fronto-parallel* planes are distorted by a 2D homography



Piero della Francesca, *Flagellation of Christ*, 1455-1460

Projection of planes

- What about patterns on *fronto-parallel planes*?



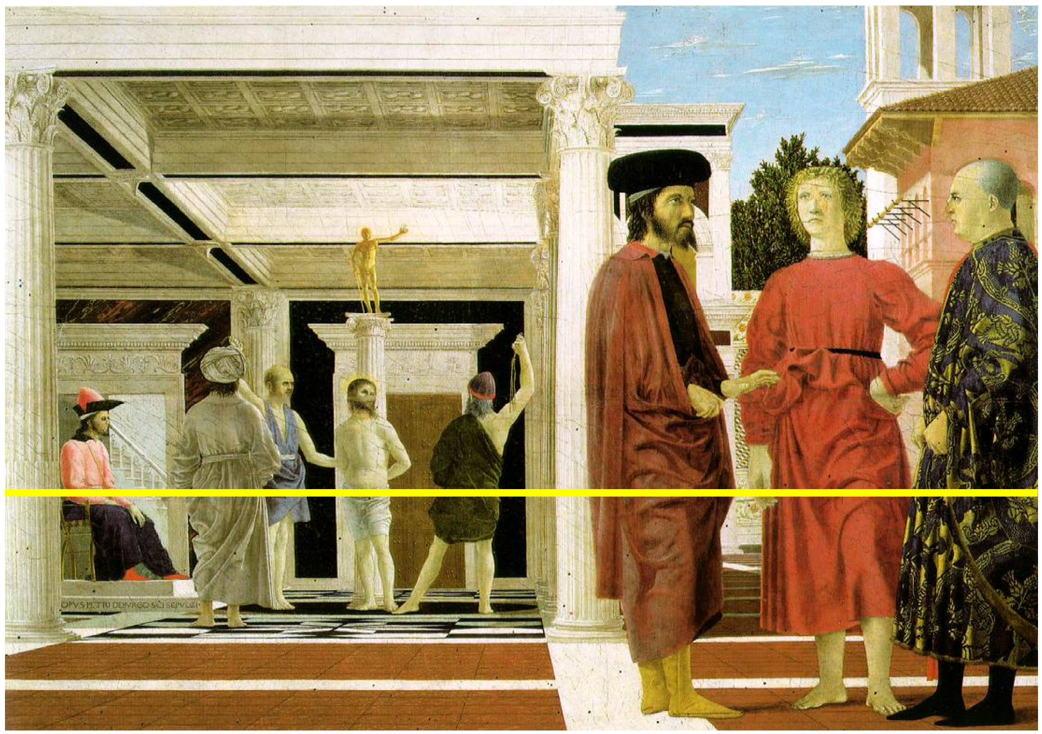
Piero della Francesca, *Flagellation of Christ*, 1455-1460

$$(x, y, z) \rightarrow \left(f\frac{x}{z}, f\frac{y}{z}\right)$$

- All points on a fronto-parallel plane are at a fixed depth z
- The pattern gets scaled by a factor of $f\ /\ z$, but angles and ratios of lengths/areas are preserved

Projection of planes

- Non-fronto-parallel planes have *vanishing lines*

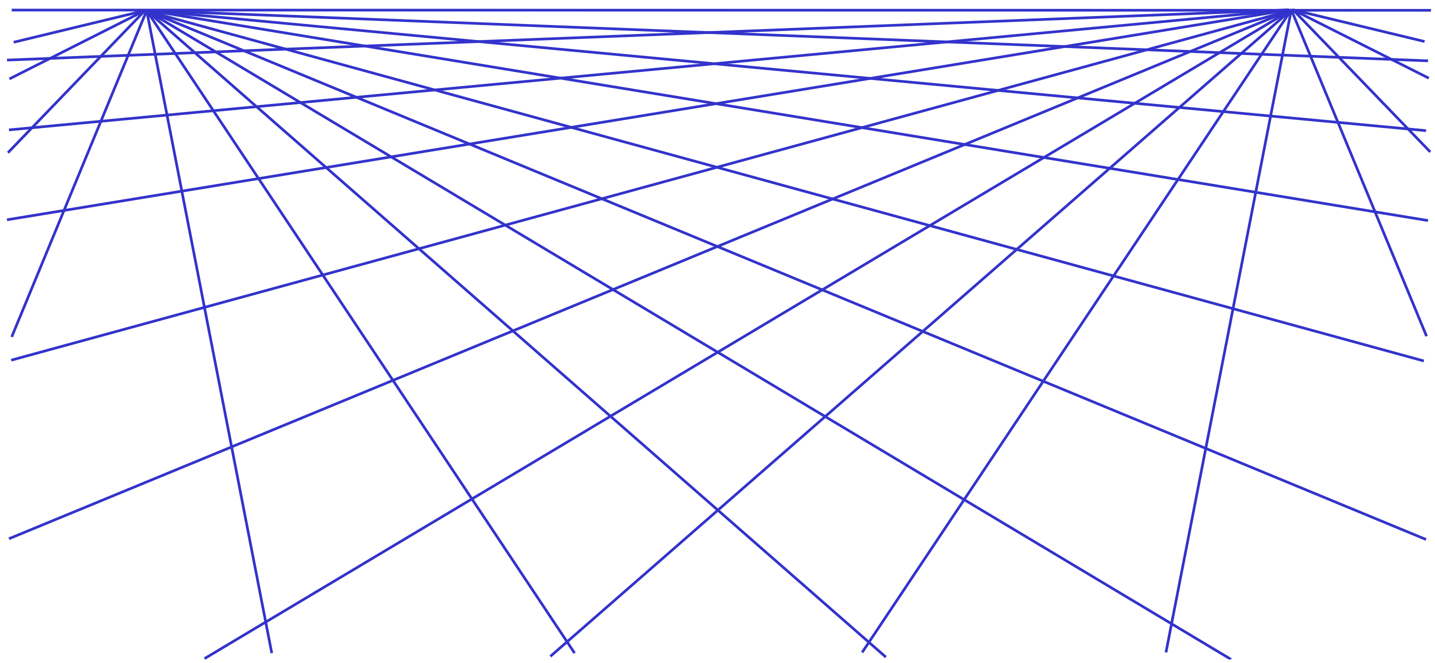


Vanishing line of
the ground plane

Piero della Francesca, *Flagellation of Christ*, 1455-1460

Vanishing lines of planes

- Each family of parallel planes is associated with a *vanishing line* in the image
- How can we construct the vanishing line of a plane?



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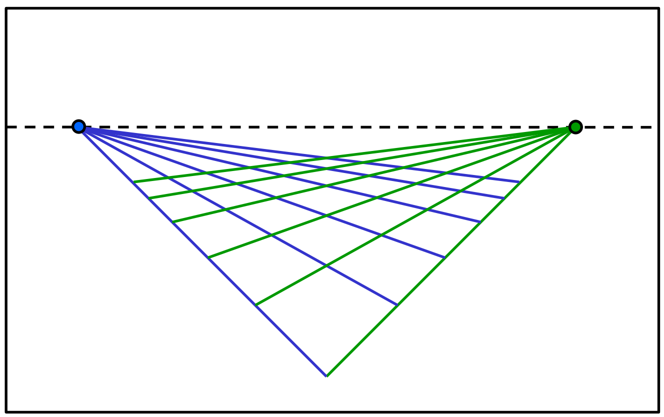


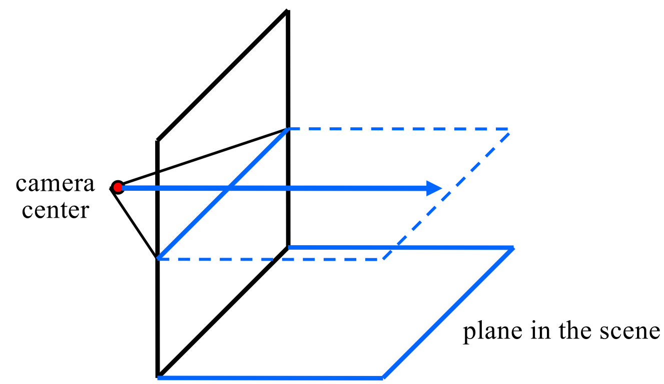


Figure source: S. Seitz

Vanishing lines of planes

- *Horizon*: vanishing line of the ground plane
  - What can the horizon tell us about the relative height of scene points and the camera?

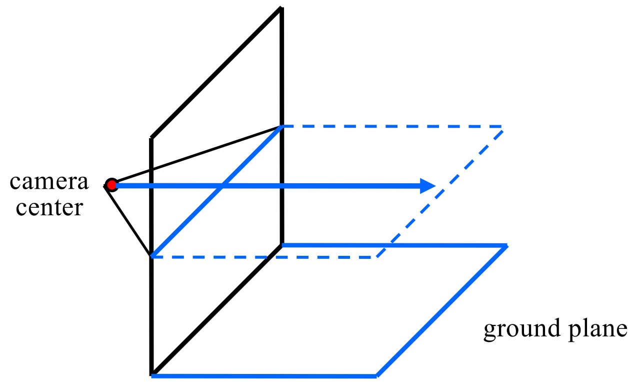


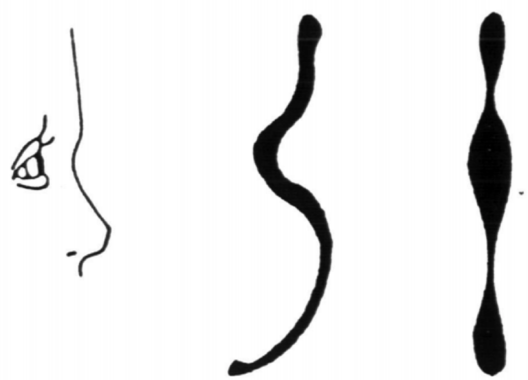
Image source: S. Seitz

Projection of 3D shapes

- What is are the relationships between the geometric properties of general 3D surfaces and their 2D projections?

Barbara Hepworth sculpture

Projection of 3D shapes



(a)

Figure 3. (a) A figure taken from Marr (1982).

J. Koenderink. What does the occluding contour tell us about solid shape? Perception 13 (321-330), 1984

Projection of 3D shapes

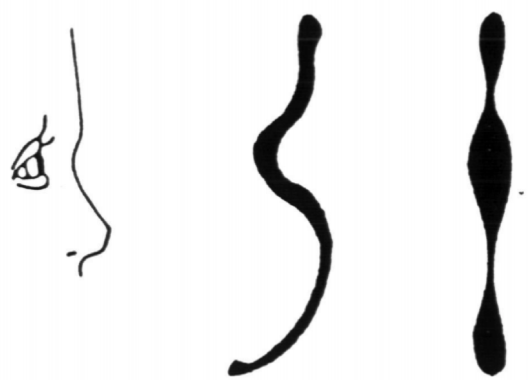
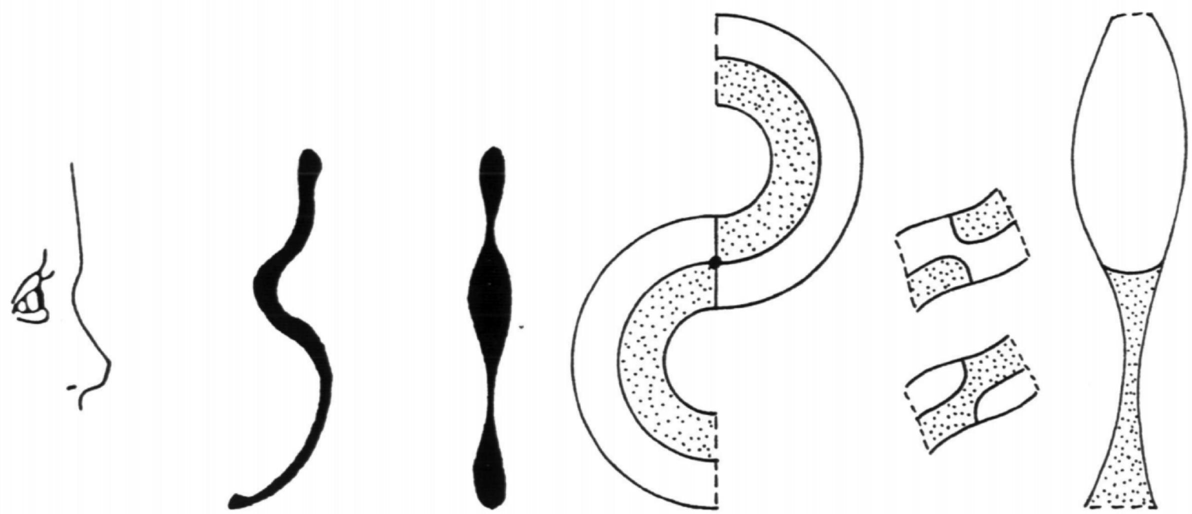

(a)

Figure 3. (a) A figure taken from Marr (1982). The suggestion is that convexities and concavities in the projection of the snake have to do with relative *distances* rather than with local shapes.

J. Koenderink. [What does the occluding contour tell us about solid shape?](#) Perception 13 (321-330), 1984

Projection of 3D shapes



(a) (b)

Figure 3. (a) A figure taken from Marr (1982). The suggestion is that convexities and concavities in the projection of the snake have to do with relative *distances* rather than with local shapes. (b) A torus cut into two and pasted together again. The shaded regions are anticlastic, the other regions synclastic. The small insets show the generic case after a small deformation. In projection (right), this 'snake' has convexities where the body is locally egg-shaped, concavities where the body is locally saddle-shaped, inflexions at flexional curves of the body.

J. Koenderink. What does the occluding contour tell us about solid shape? Perception 13 (321-330), 1984

Projection of 3D shapes

Figure 4. Details from Dürer's "Samson killing the lion". (Bartsch #2; the print dates from 1498.)

J. Koenderink. What does the occluding contour tell us about solid shape? Perception 13 (321-330), 1984

Projection of 3D shapes

- What is the shape of the projection of a sphere?

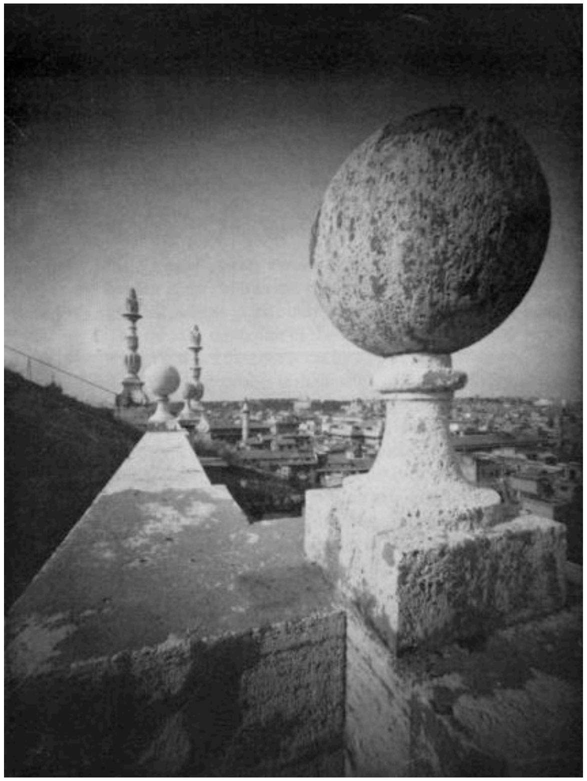


Image source: F. Durand

Projection of 3D shapes

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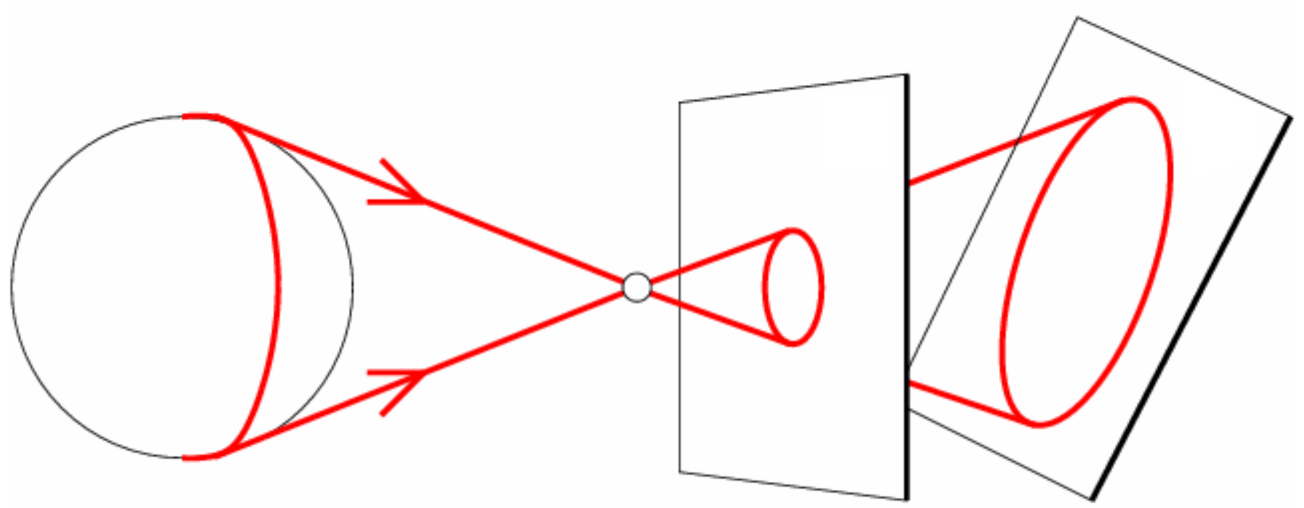
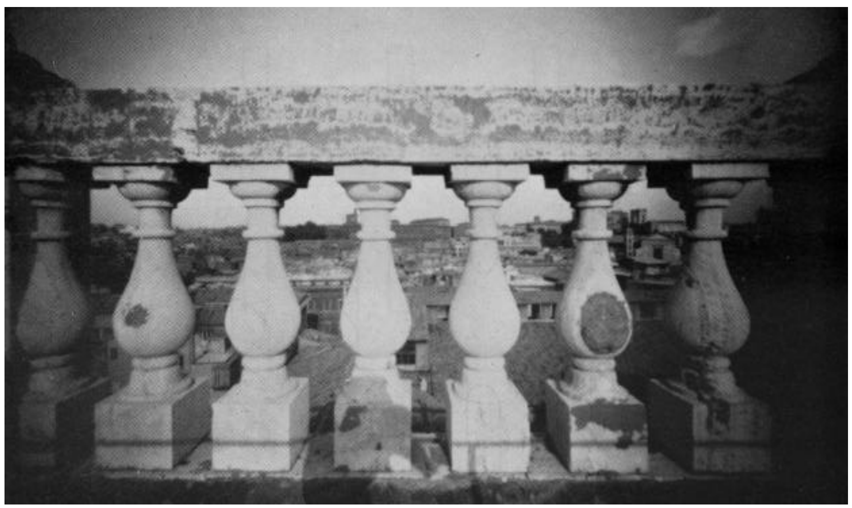
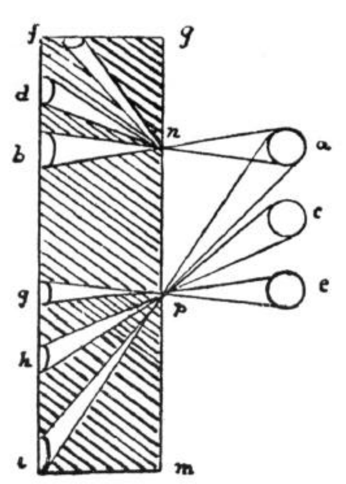


Projection of 3D shapes

- Are the widths of the projected columns equal?
  - The exterior columns are wider
  - This is not an optical illusion, and is not due to lens flaws
  - Phenomenon pointed out by Leonardo Da Vinci





Source: F. Durand

Perspective distortion: People

Overview

- Motivation: recovery of 3D structure
- Pinhole projection model
- Properties of projection
- Perspective projection matrix
- Orthographic projection

Recall: Homogeneous coordinates

- To form homogeneous coordinates from normal Euclidean coordinates, append 1 as the last entry:

$$(x, y) \Longrightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous *image* coordinates

$$(x, y, z) \Longrightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous *scene* coordinates

- To convert *from* homogeneous coordinates, divide by the last entry:

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Longrightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Longrightarrow (x/w, y/w, z/w)$$

In homogeneous coordinates, all scalar multiples represent the same point!

Perspective projection matrix

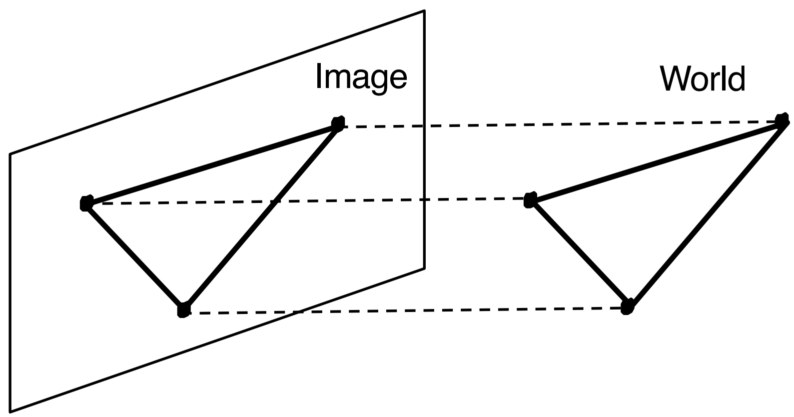
- Projection is a matrix multiplication using homogeneous coordinates:

$$\begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z} \right)$$

divide by the third coordinate

Orthographic projection

- Special case of perspective projection
  - Distance from center of projection to image plane is infinite
  - Also called “parallel projection”



Slide by Steve Seitz

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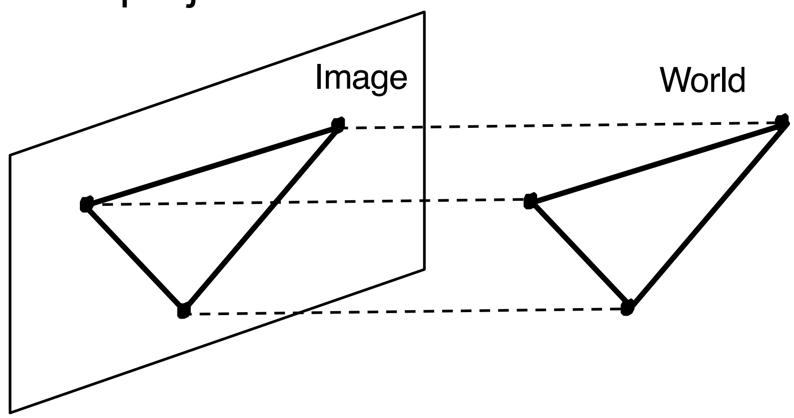
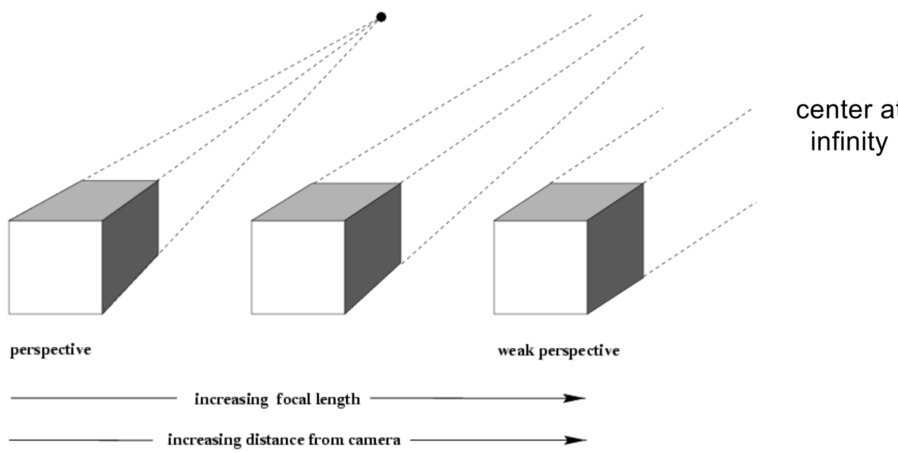


  - Assuming projection along the z axis, what's the matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Slide by Steve Seitz

Approximating an orthographic camera



Source: Hartley & Zisserman