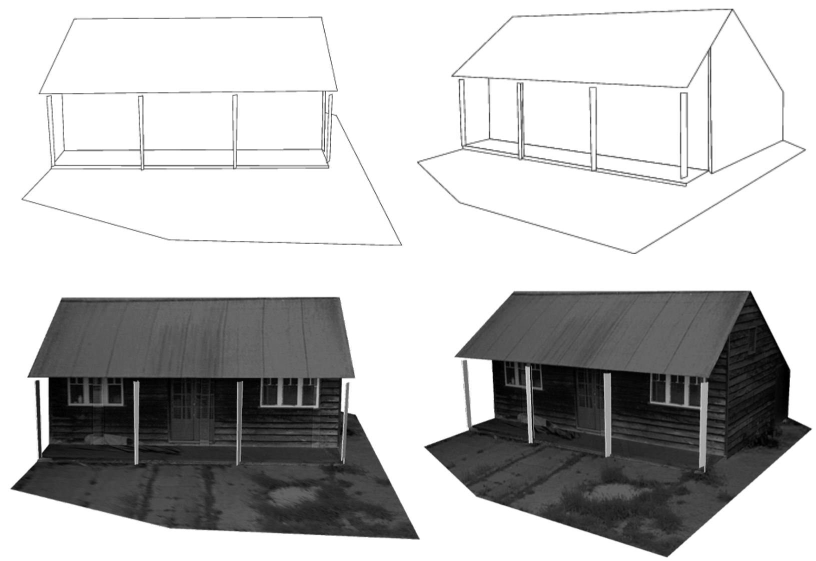
Single-view metrology

Many slides adapted from
S. Seitz, D. Hoiem

R. Magritte, *Personal Values*, 1952

Application: 3D from a single image



A. Criminisi et al. Single View Metrology. IJCV 2000

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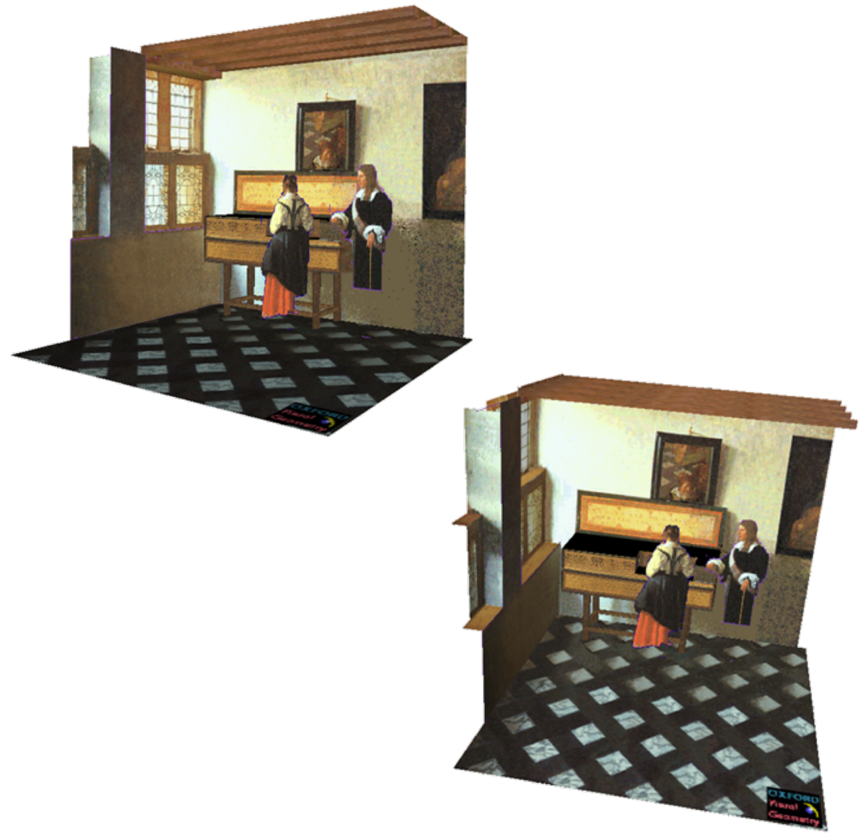
J. Vermeer, Music Lesson, 1662

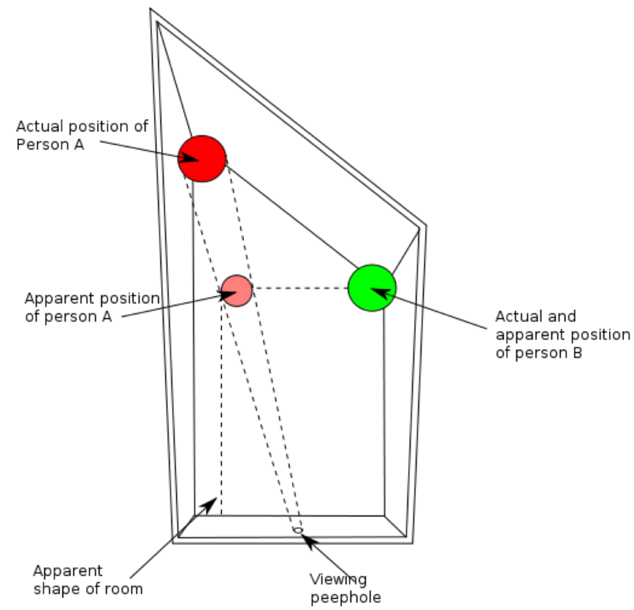


A. Criminisi et al. Bringing Pictorial Space to Life: computer techniques for the analysis of paintings,
Proc. Computers and the History of Art, 2002

Application: Image editing, augmented reality

K. Karsch and V. Hedau and D. Forsyth and D. Hoiem. [Rendering Synthetic Objects into Legacy Photographs]. *SIGGRAPH Asia* 2011

Reminder: Beware!



http://en.wikipedia.org/wiki/Ames_room

Outline

- Camera calibration using vanishing points
- Measurements from a single image
- Applications of single-view metrology

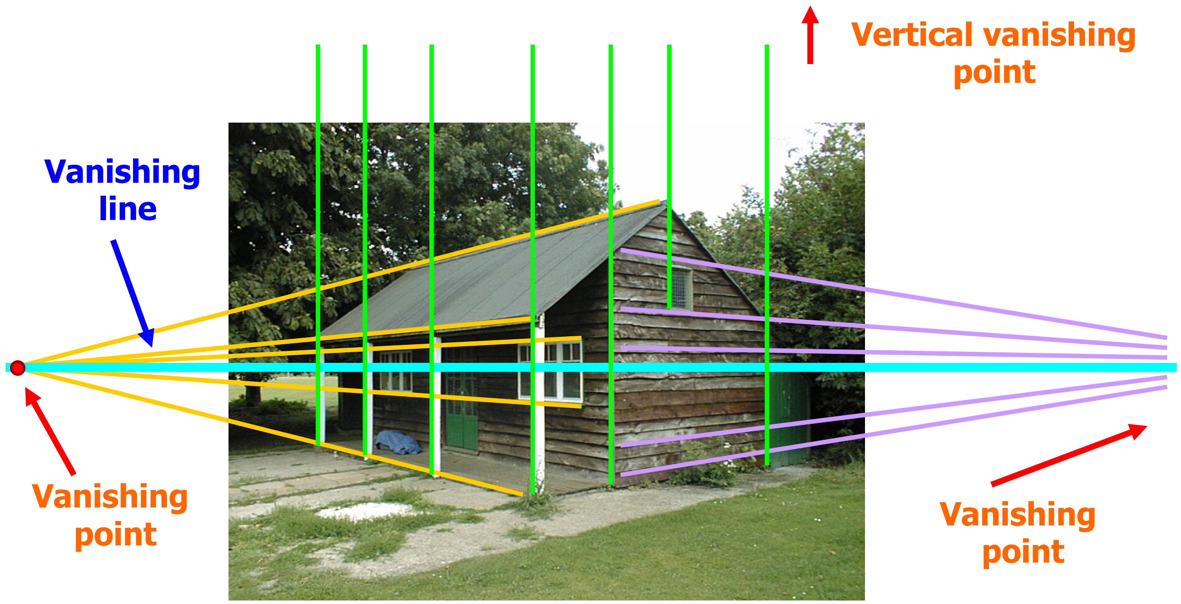
Camera calibration using vanishing points

- If world coordinates of reference 3D points are not known, in special cases, we may be able to use vanishing points

Source: A. Efros, A. Criminisi

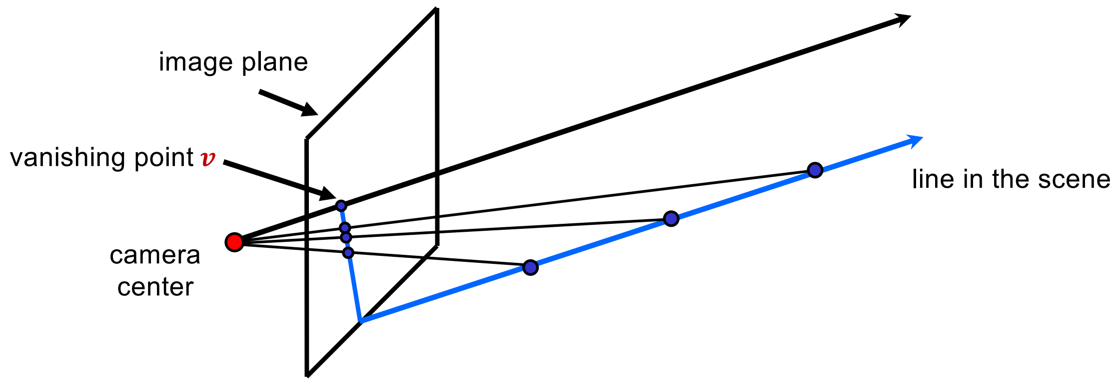
Camera calibration using vanishing points

- If world coordinates of reference 3D points are not known, in special cases, we may be able to use vanishing points



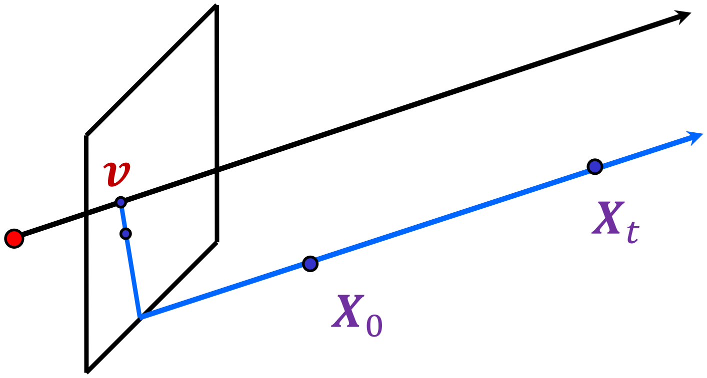
Source: A. Efros, A. Criminisi

Review: Vanishing points



- All lines having the same direction share the same vanishing point

Computing vanishing points



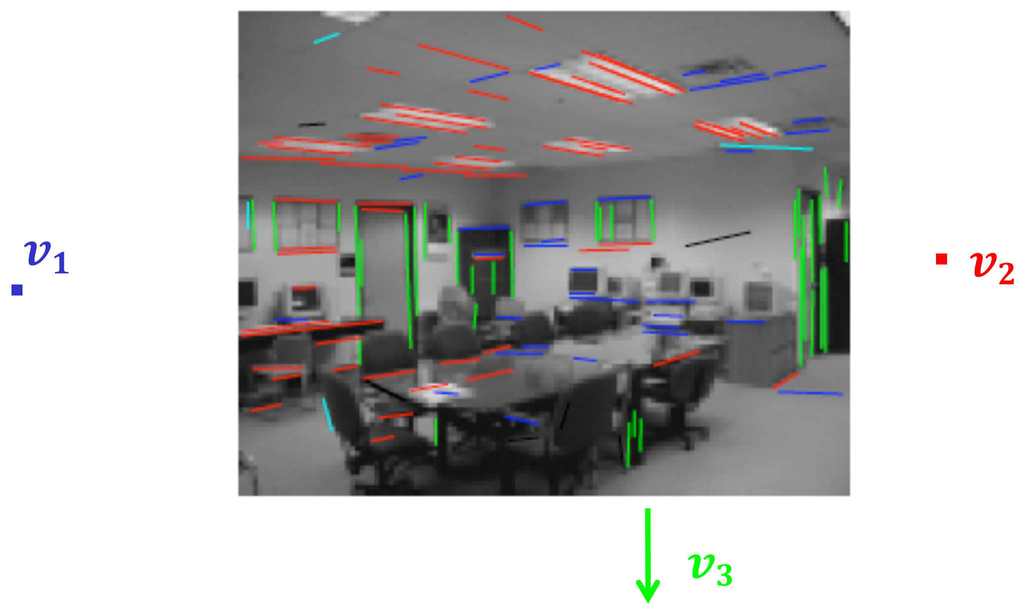
- Let's parameterize the line using point $\boldsymbol{X}_0 = (X_0, Y_0, Z_0, 1)^T$ and direction vector $\boldsymbol{D} = (D_1, D_2, D_3)^T$:

$$\boldsymbol{X}_t = \begin{pmatrix} X_0 + tD_1 \\ Y_0 + tD_2 \\ Z_0 + tD_3 \\ 1 \end{pmatrix} \cong \begin{pmatrix} X_0/t + D_1 \\ Y_0/t + D_2 \\ Z_0/t + D_3 \\ 1/t \end{pmatrix} \qquad \boldsymbol{X}_\infty = \begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ 0 \end{pmatrix}$$

- $\boldsymbol{X}_\infty$ is a *point at infinity,* $\boldsymbol{v}$ is its projection: $\boldsymbol{v} \cong \boldsymbol{P}\boldsymbol{X}_\infty$

Calibration from vanishing points

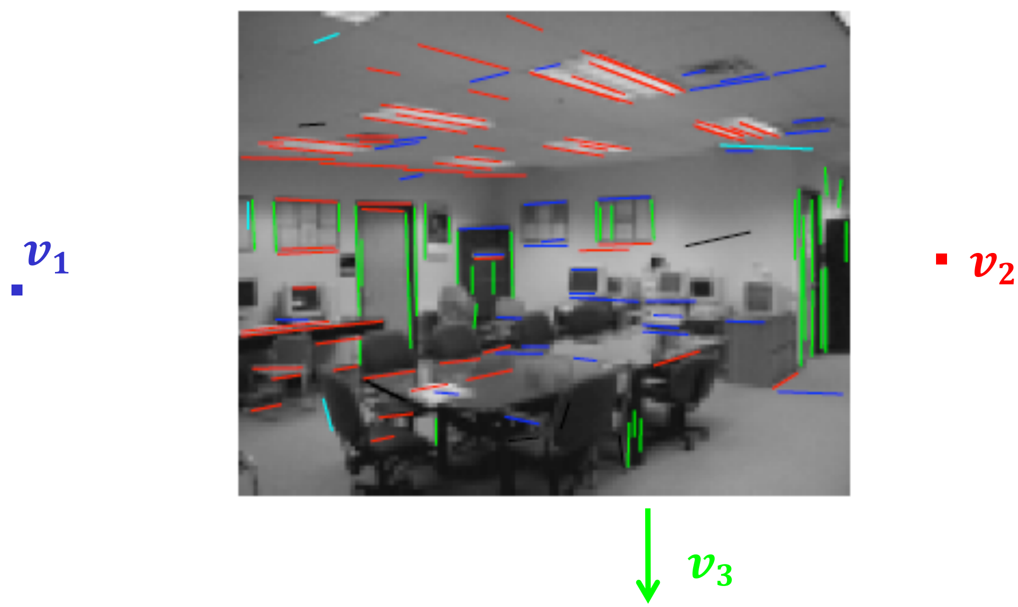
- Consider a scene with three orthogonal vanishing directions:



- Note: v_1, v_2 are *finite* vanishing points and v_3 is an *infinite* vanishing point

Calibration from vanishing points

- Consider a scene with three orthogonal vanishing directions:



- We can align the world coordinate system with these directions

Projection of the world coordinate system

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}$$

Projection of the world coordinate system

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \boldsymbol{p}_1$$

$\boldsymbol{p}_1$ $\boldsymbol{p}_2$ $\boldsymbol{p}_3$ $\boldsymbol{p}_4$

Projection of the world coordinate system

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \boldsymbol{p}_2$$

$\boldsymbol{p}_1 \quad \boldsymbol{p}_2 \quad \boldsymbol{p}_3 \quad \boldsymbol{p}_4$

Projection of the world coordinate system

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \boldsymbol{p}_3$$

$$\underbrace{\boldsymbol{p}_1 \quad \boldsymbol{p}_2 \quad \boldsymbol{p}_3}_{} \quad \boldsymbol{p}_4$$

Vanishing points in
x, y, z directions
(i.e., $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, $\boldsymbol{v}_3$)

Projection of the world coordinate system

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \boldsymbol{p}_4$$

$\boldsymbol{p}_1 \quad \boldsymbol{p}_2 \quad \boldsymbol{p}_3 \quad \boldsymbol{p}_4$

Vanishing points in x, y, z directions (i.e., $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, $\boldsymbol{v}_3$)

Projection of world origin

- Problem: this only gives us the four columns up to independent scale factors, additional constraints needed to solve for them

Calibration from vanishing points

- Let us align the world coordinate system with three orthogonal vanishing directions in the scene:

$$\boldsymbol{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \boldsymbol{v}_i \cong \boldsymbol{K}[\boldsymbol{R}|\boldsymbol{t}] \begin{pmatrix} \boldsymbol{e}_i \\ 0 \end{pmatrix}$$

Calibration from vanishing points

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$$\boldsymbol{v}_i \cong \boldsymbol{K}\boldsymbol{R}\boldsymbol{e}_i$$

$$\boldsymbol{e}_i \cong \boldsymbol{R}^T\boldsymbol{K}^{-1}\boldsymbol{v}_i$$

- Orthogonality constraint: $\boldsymbol{e}_i^T\boldsymbol{e}_j = 0$

$$\underbrace{\boldsymbol{v}_i^T\boldsymbol{K}^{-T}\boldsymbol{R}}_{\boldsymbol{e}_i^T}\underbrace{\boldsymbol{R}^T\boldsymbol{K}^{-1}\boldsymbol{v}_j}_{\boldsymbol{e}_j} = 0$$

Calibration from vanishing points

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- Orthogonality constraint: $\boldsymbol{e}_i^T\boldsymbol{e}_j = 0$

$$\boldsymbol{v}_i^T\boldsymbol{K}^{-T}\boldsymbol{K}^{-1}\boldsymbol{v}_j = 0$$

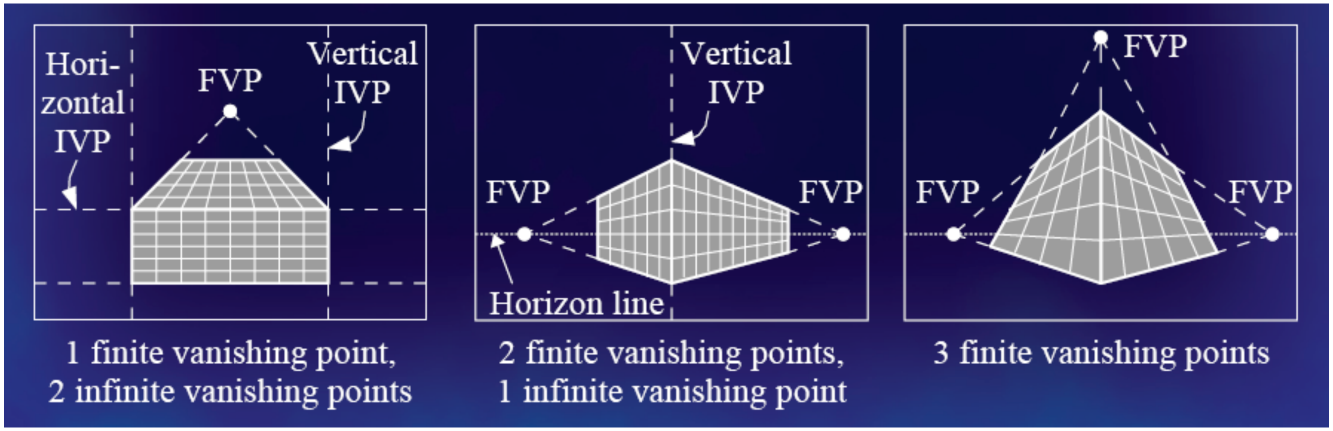
- Extrinsic parameter matrix ($\boldsymbol{R}$) disappears and we are left with constraints on the calibration matrix!

Calibration from vanishing points

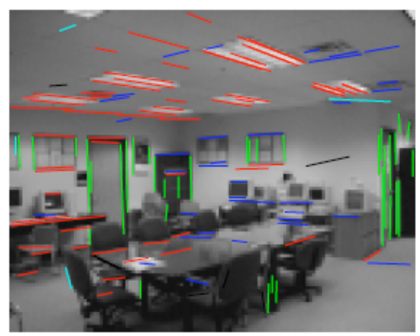
$$\boldsymbol{v}_i^T \boldsymbol{K}^{-T} \boldsymbol{K}^{-1} \boldsymbol{v}_j = 0$$

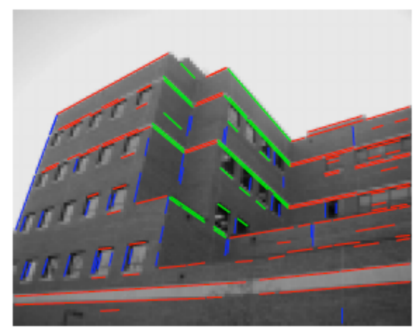
- How many constraints do we get?
  - Three: one for each pair of vanishing points
- How many unknown parameters does $\boldsymbol{K}$ have?
  - Three: f, p_x, p_y
- A couple of complications:
  - The constraints are nonlinear, but it's not hard to do the algebra (omitted)
  - At least two *finite* vanishing points are needed to solve for both focal length and principal point

Calibration from vanishing points



Cannot recover focal length, principal point is the third vanishing point





Can solve for focal length, principal point

Rotation from vanishing points

- Constraints on vanishing points: $\boldsymbol{v}_i \cong \boldsymbol{K}\boldsymbol{R}\boldsymbol{e}_i$
- We just used orthogonality constraints to solve for $\boldsymbol{K}$
- Now we have:

$$\boldsymbol{K}^{-1}\boldsymbol{v}_i \cong \boldsymbol{R}\boldsymbol{e}_i$$

Notice: $\boldsymbol{R}\boldsymbol{e}_1 = [\boldsymbol{r}_1 \quad \boldsymbol{r}_2 \quad \boldsymbol{r}_3]\begin{pmatrix}1\\0\\0\end{pmatrix} = \boldsymbol{r}_1$

Thus, $\boldsymbol{r}_i \cong \boldsymbol{K}^{-1}\boldsymbol{v}_i$

- The scale ambiguity goes away since we require $\|\boldsymbol{r}_i\|^2 = 1$

Calibration from vanishing points: Summary

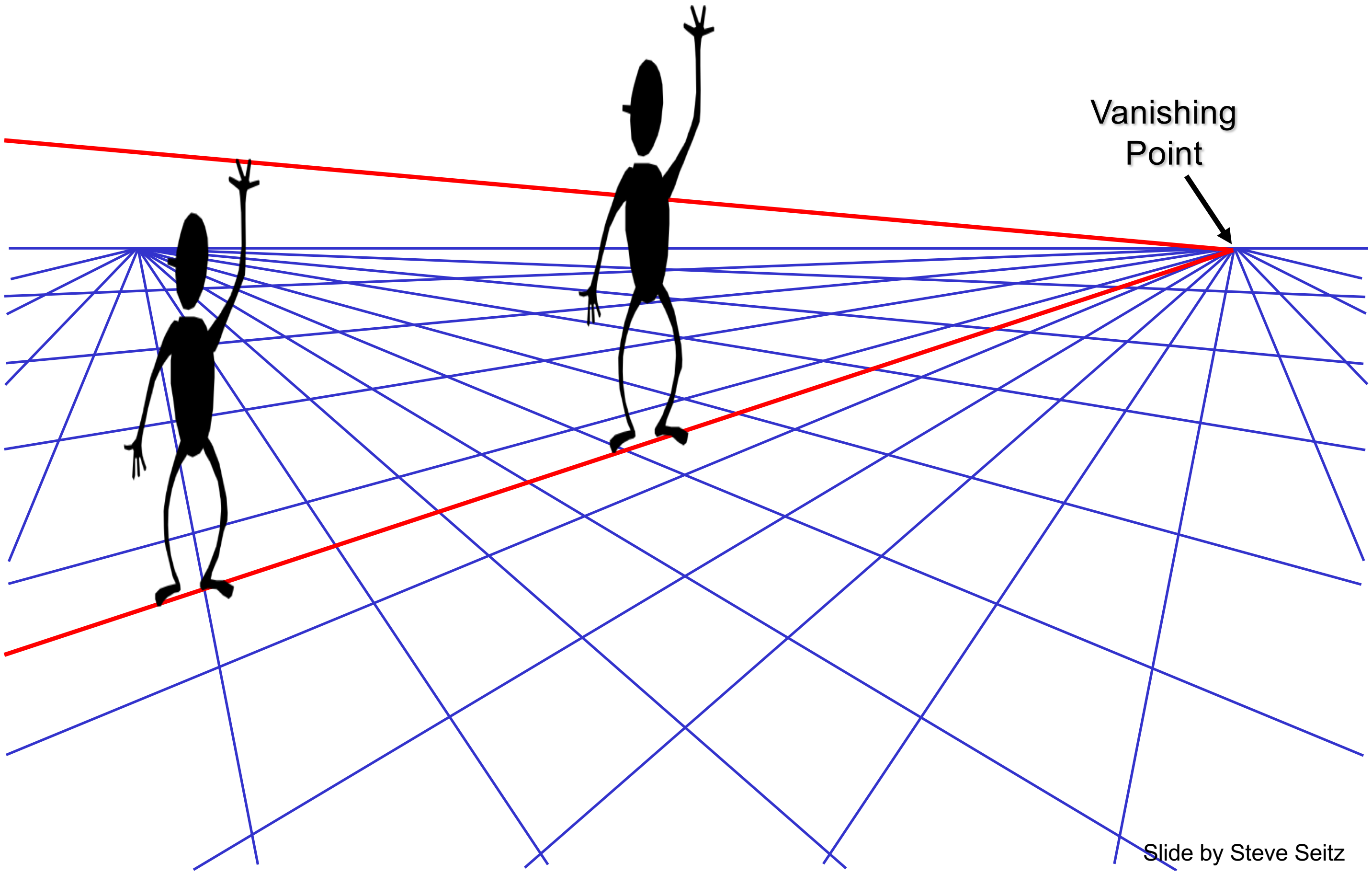
1. Solve for intrinsic parameters (focal length, principal point) using three orthogonal vanishing points
2. Get extrinsic parameters (rotation) directly from vanishing points once calibration matrix is known

- Advantages
  - No need for calibration chart, 2D-3D correspondences
  - Could be completely automatic
- Disadvantages
  - Only applies to certain kinds of scenes
  - It is tricky to accurately localize vanishing points
  - Need at least two finite vanishing points

Outline

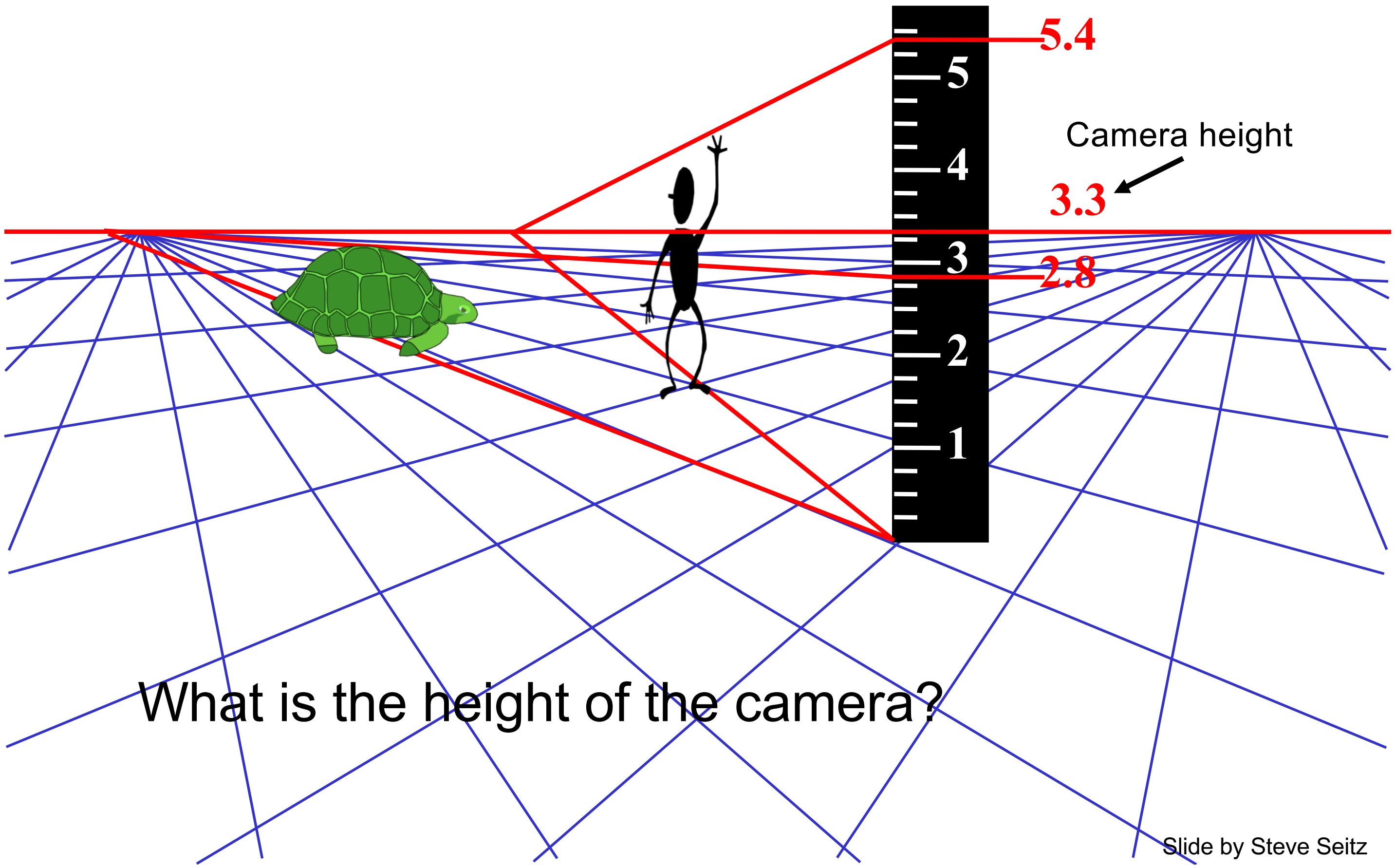
- Camera calibration using vanishing points
- Measurements from a single image
  - Measuring height above the ground plane
  - Measuring within planes

Comparing heights



Slide by Steve Seitz

Measuring height with a ruler

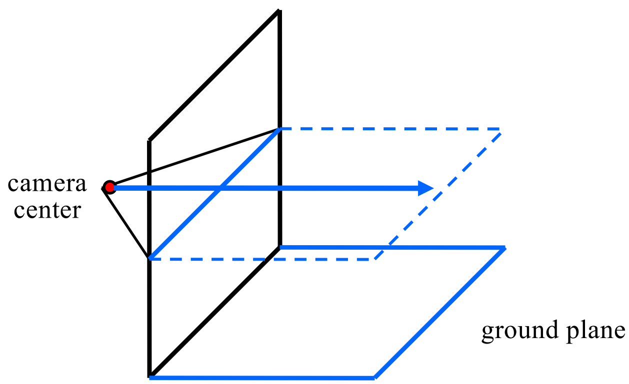


What is the height of the camera?

Slide by Steve Seitz

Horizon: Vanishing line of the ground plane

- What can the horizon tell us about the relative height of scene points and the camera?

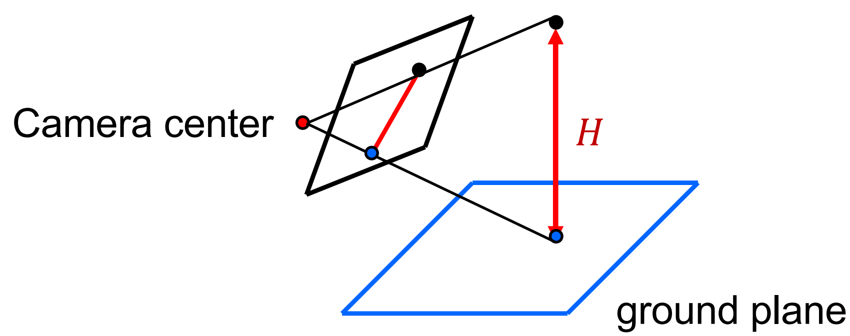


What is higher: The camera or the parachutist?

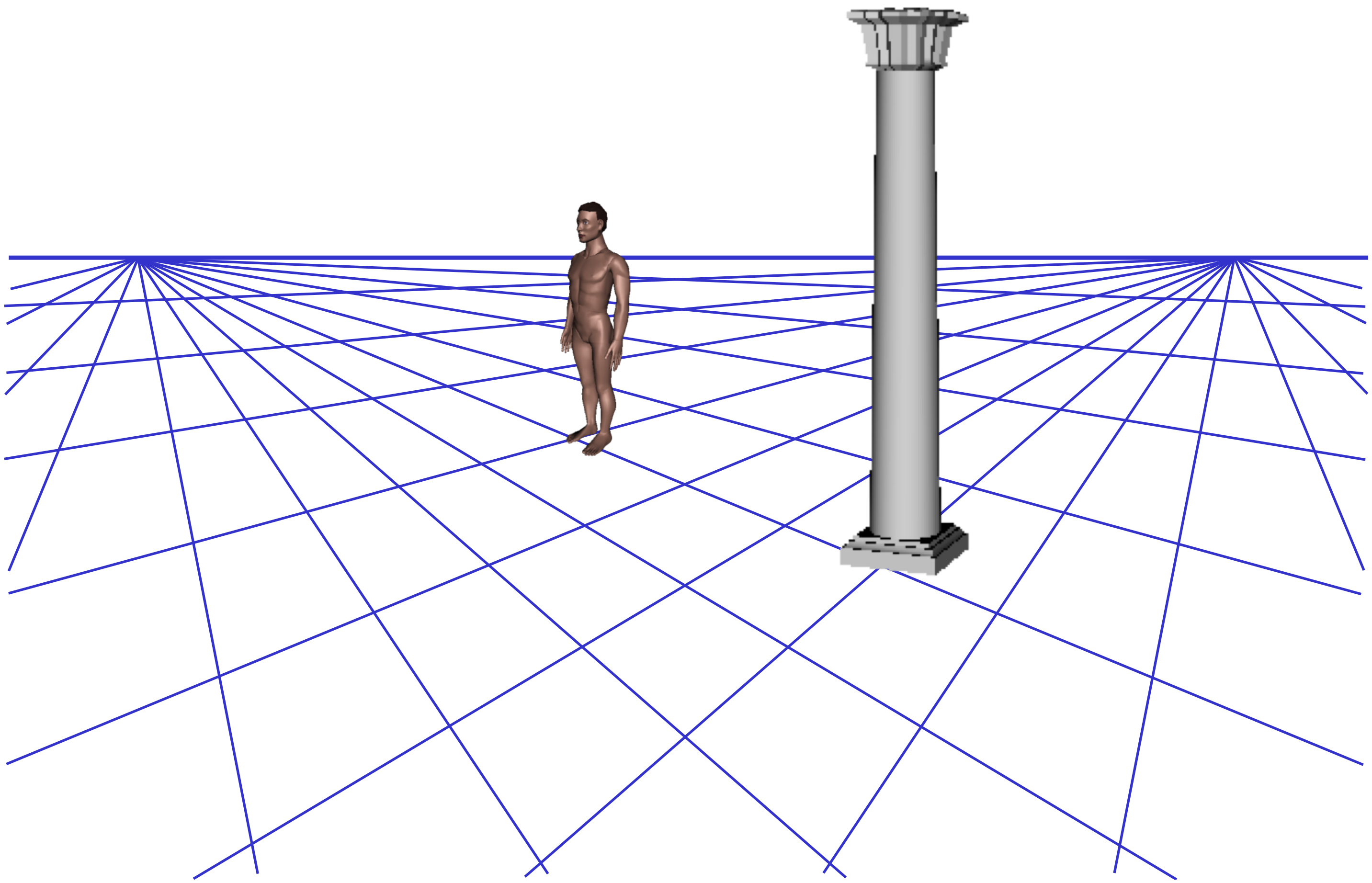
Image source: S. Seitz

Measuring height without a ruler

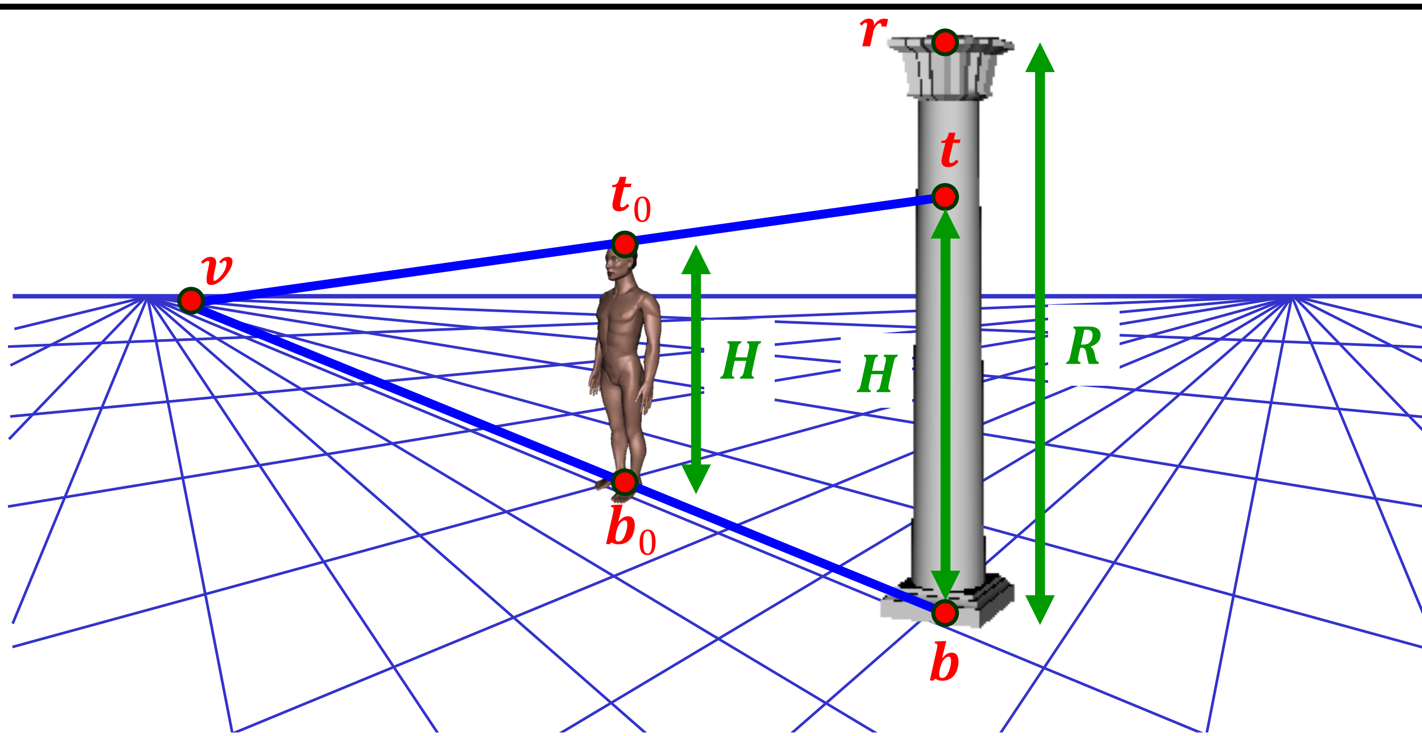
- Goal: compute height H from image measurements



Measuring height without a ruler

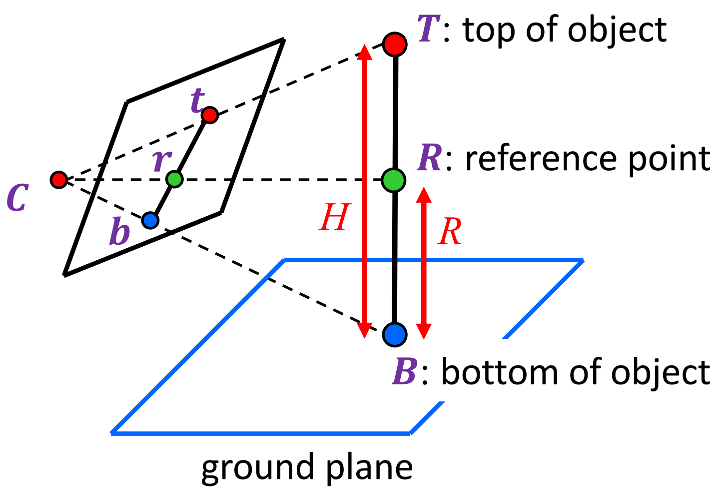


Measuring height without a ruler



- To find $\frac{H}{R}$, we need to do some extra work…

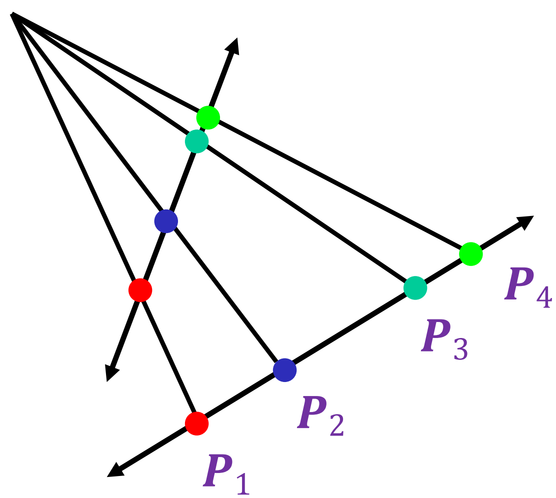
Measuring height



To make 3D measurements based on image observations, we need an invariant that does not change under perspective projection from 3D to 2D

Cross-ratio

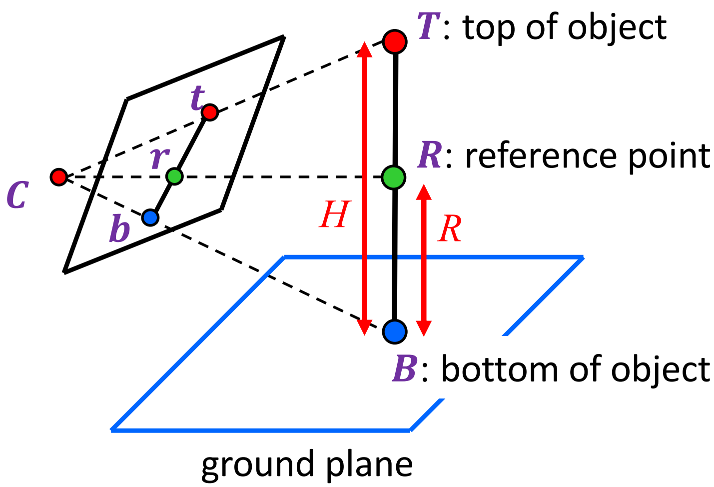
- Defined for four collinear points, invariant to projective transformations, including perspective projection from 3D to 2D
  - Also invariant to permutation of the points



$$\frac{\|\boldsymbol{P}_3 - \boldsymbol{P}_1\|}{\|\boldsymbol{P}_3 - \boldsymbol{P}_2\|}\frac{\|\boldsymbol{P}_4 - \boldsymbol{P}_2\|}{\|\boldsymbol{P}_4 - \boldsymbol{P}_1\|}$$

Measuring height

- We need a fourth point to compute a cross-ratio – what could it be?



Measuring height

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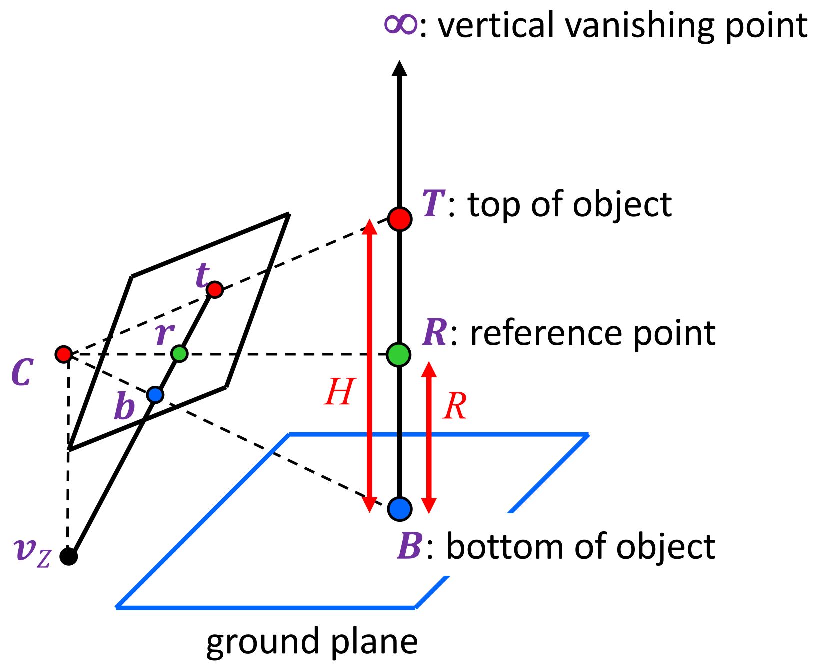


Scene cross ratio:

$$\frac{\|T-B\|}{\|R-B\|}\frac{\|\infty-R\|}{\|\infty-T\|}=\frac{H}{R}$$

Image cross ratio:

$$\frac{\|t-b\|}{\|r-b\|}\frac{\|v_Z-r\|}{\|v_Z-t\|}=\frac{H}{R}$$

How do we compute the coordinates of $\boldsymbol{v}$ and $\boldsymbol{t}$?

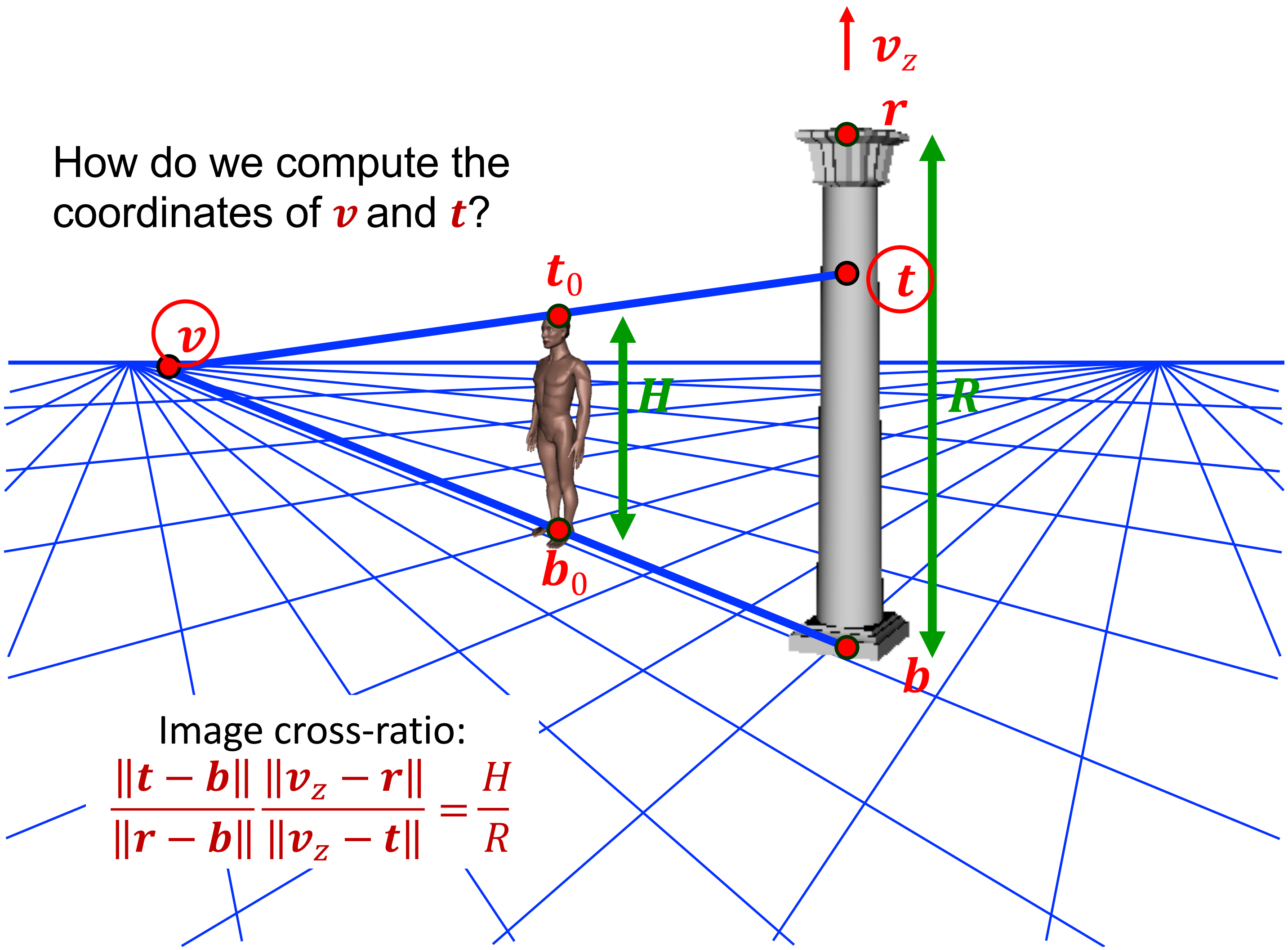


Image cross-ratio:

$$\frac{\|\boldsymbol{t}-\boldsymbol{b}\|}{\|\boldsymbol{r}-\boldsymbol{b}\|}\frac{\|\boldsymbol{v}_z-\boldsymbol{r}\|}{\|\boldsymbol{v}_z-\boldsymbol{t}\|}=\frac{H}{R}$$

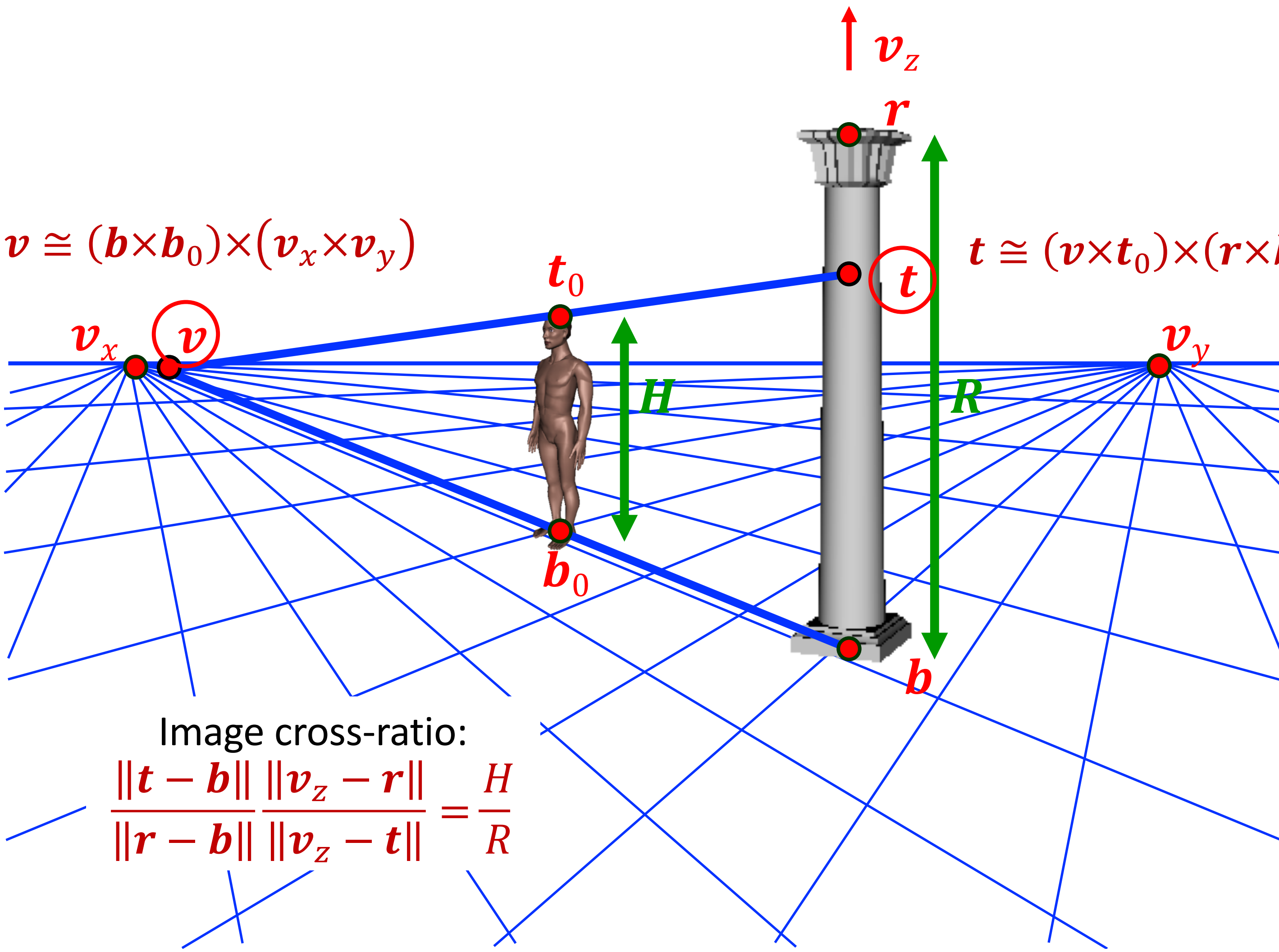
2D lines in homogeneous coordinates

- Line equation: $ax + by + c = 0$

$$\boldsymbol{l}^T \boldsymbol{x} = 0, \text{ where } \boldsymbol{l} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \boldsymbol{x} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

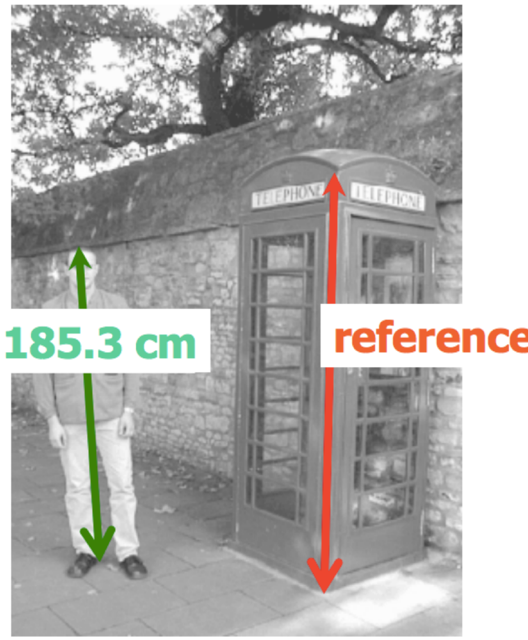
- Line passing through two points: $\boldsymbol{l} = \boldsymbol{x}_1 \times \boldsymbol{x}_2$

- Intersection of two lines: $\boldsymbol{x} = \boldsymbol{l}_1 \times \boldsymbol{l}_2$
  - What is the intersection of two parallel lines?



v_z
r
$v \cong (b \times b_0) \times (v_x \times v_y)$
$t \cong (v \times t_0) \times (r \times b)$
t_0
t
v_x
v
v_y
H
R
b_0
b
Image cross-ratio:
$\frac{\|t - b\|}{\|r - b\|} \frac{\|v_z - r\|}{\|v_z - t\|} = \frac{H}{R}$

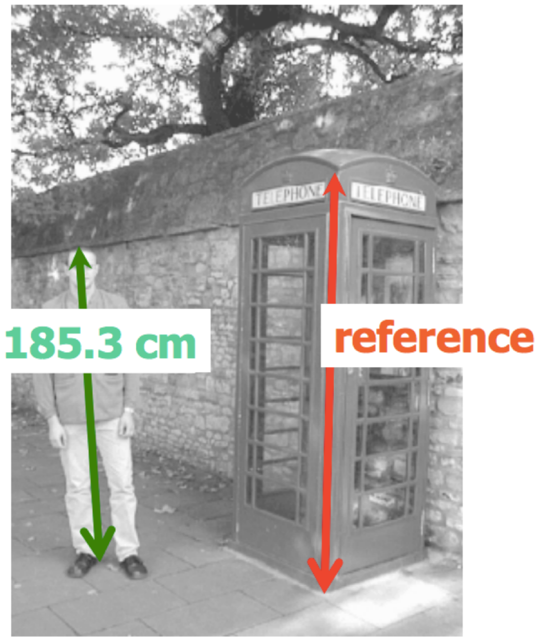
Single-view measurement examples

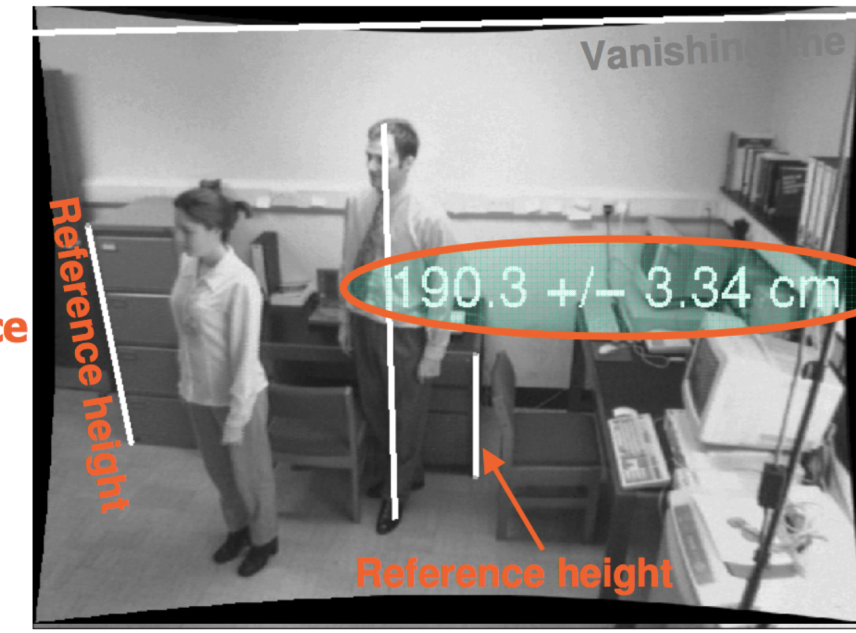


That booth is still there! (Oxford, September 2022)

A. Criminisi, I. Reid, and A. Zisserman, Single View Metrology, IJCV 2000
Figure from UPenn CIS580 slides

Single-view measurement examples





A. Criminisi, I. Reid, and A. Zisserman, Single View Metrology, IJCV 2000
Figure from UPenn CIS580 slides

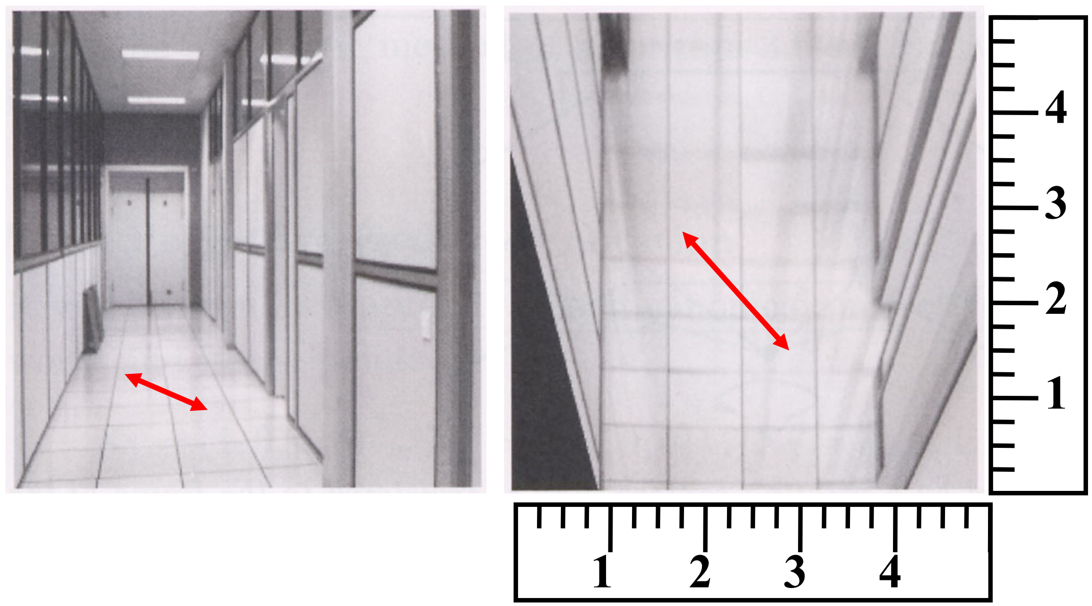
Another example

- Are the heights of the two groups of people consistent with one another?
  - Measure heights using Christ as reference

Piero della Francesca*, Flagellation,* ca. 1455

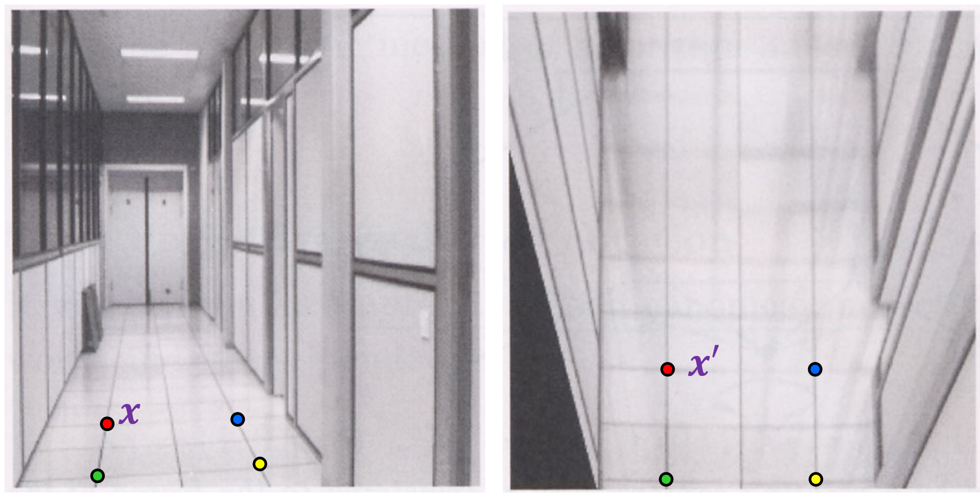
A. Criminisi, M. Kemp, and A. Zisserman, Bringing Pictorial Space to Life: computer techniques for the analysis of paintings, *Proc. Computers and the History of Art*, 2002

Measurements within planes



- Simplest approach: unwarp then measure
- What kind of warp is this?

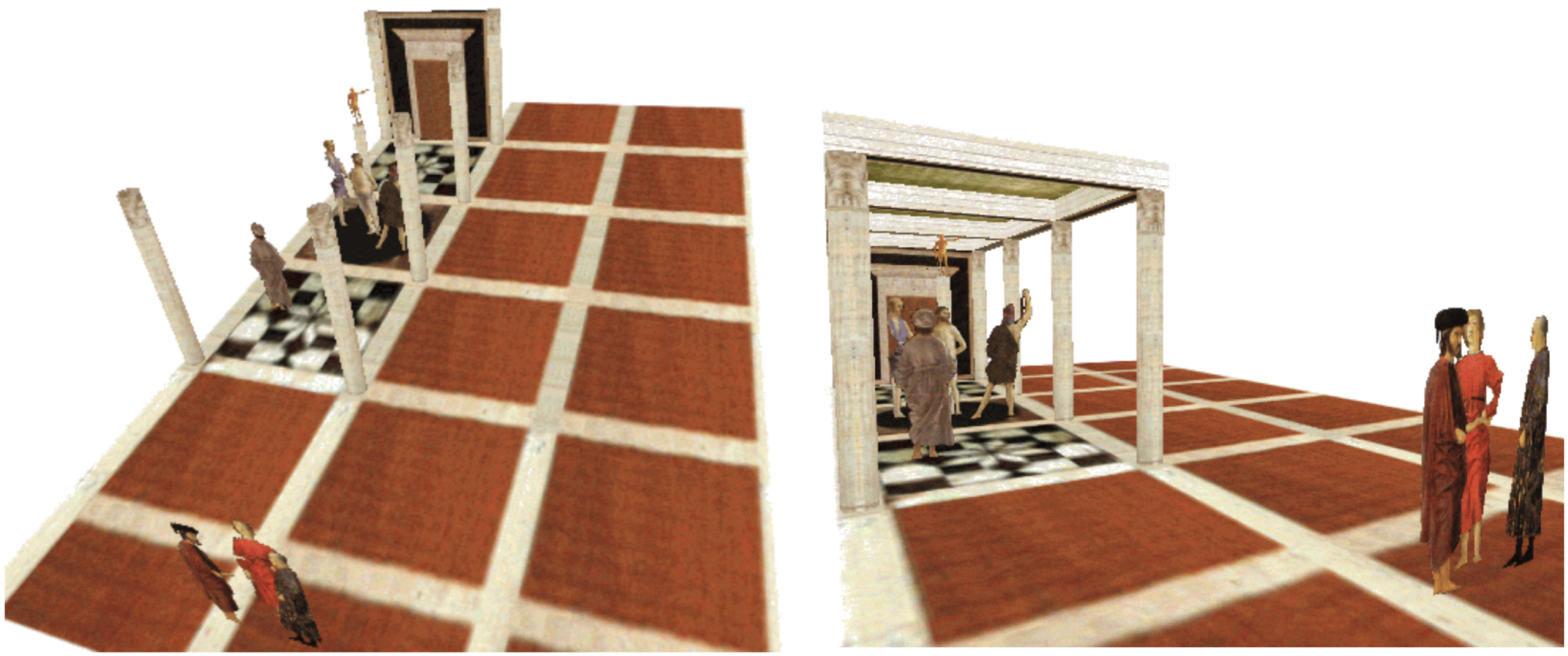
Image rectification



- To unwarp (rectify) an image, solve for homography $\boldsymbol{H}$ given four pairs of matches assumed to have a known configuration (e.g., square)

Image rectification: Example

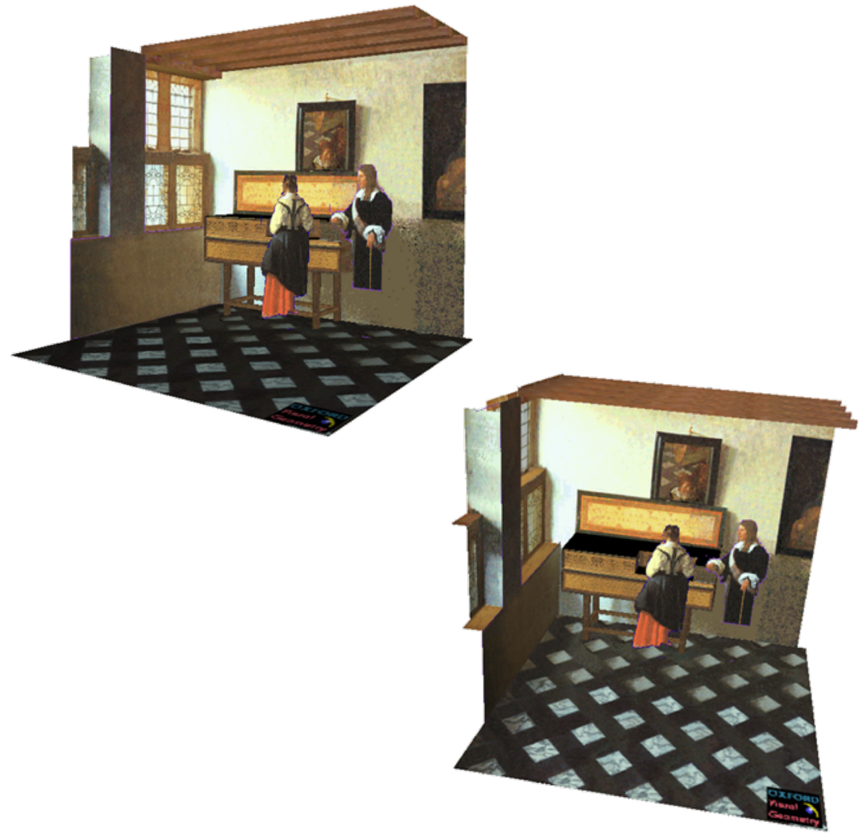
Putting everything together: Single-view modeling



A. Criminisi et al. [Bringing Pictorial Space to Life: computer techniques for the analysis of paintings](), *Proc. Computers and the History of Art*, 2002

Putting everything together: Single-view modeling

J. Vermeer, Music Lesson, 1662



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Proc. Computers and the History of Art, 2002

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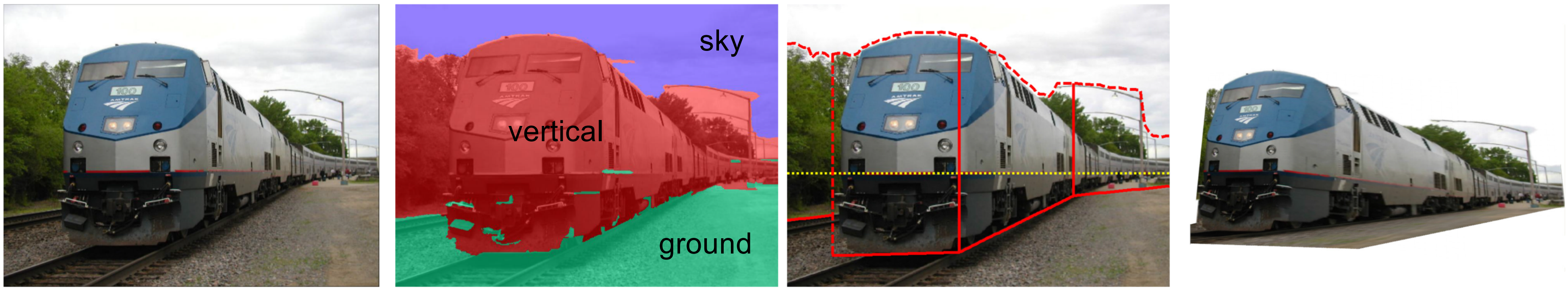
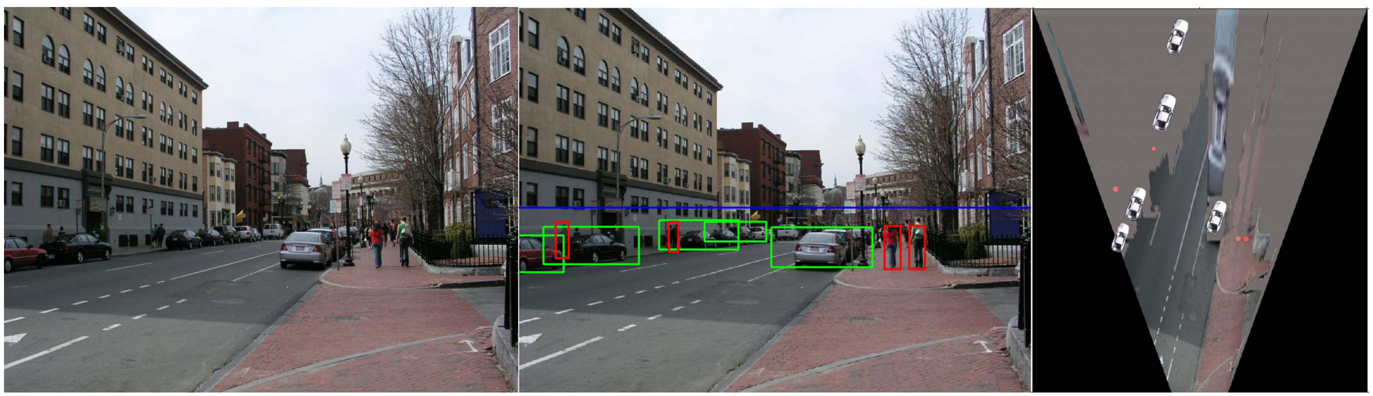
Application: Fully automatic modeling



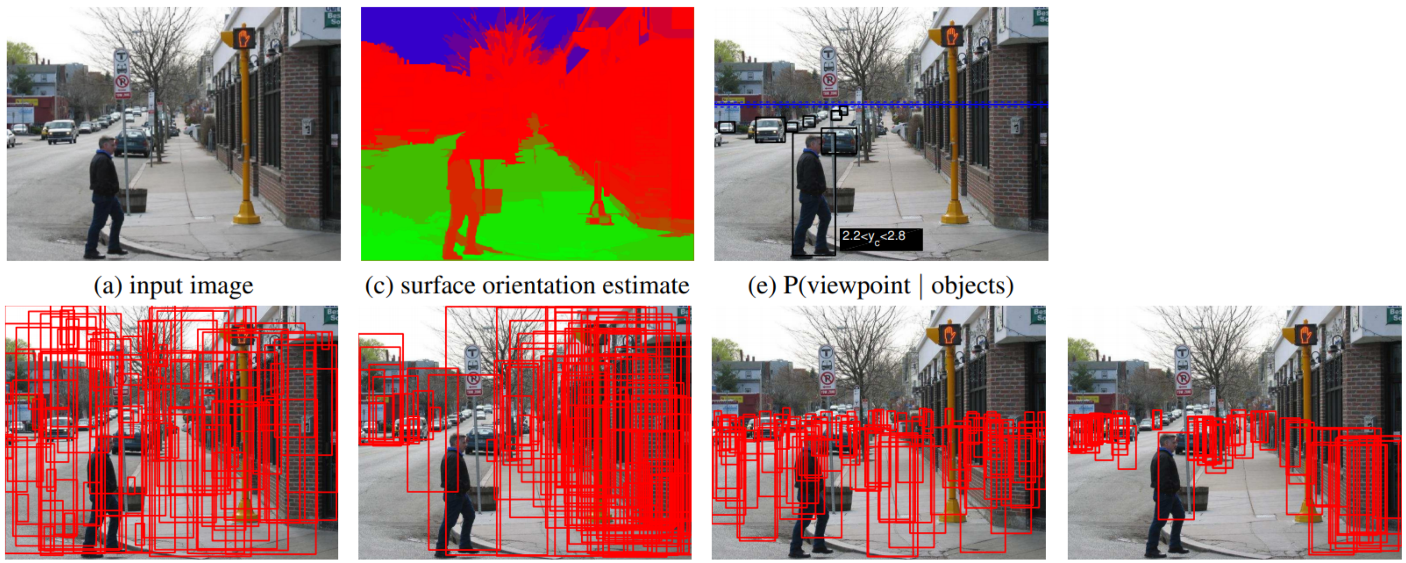
D. Hoiem, A.A. Efros, and M. Hebert, [Automatic Photo Pop-up](), SIGGRAPH 2005

Application: Object detection



D. Hoiem, A.A. Efros, and M. Hebert. Putting Objects in Perspective. CVPR 2006

Application: Object detection



(a) input image (c) surface orientation estimate (e) P(viewpoint | objects)

(b) P(person) = uniform (d) P(person | geometry) (f) P(person | viewpoint) (g) P(person|viewpoint,geometry)

D. Hoiem, A.A. Efros, and M. Hebert. [Putting Objects in Perspective](). CVPR 2006

Application: Image editing

- Inserting synthetic objects into images

K. Karsch and V. Hedau and D. Forsyth and D. Hoiem. [Rendering Synthetic Objects into Legacy Photographs]. *SIGGRAPH Asia* 2011

Application: Image editing

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