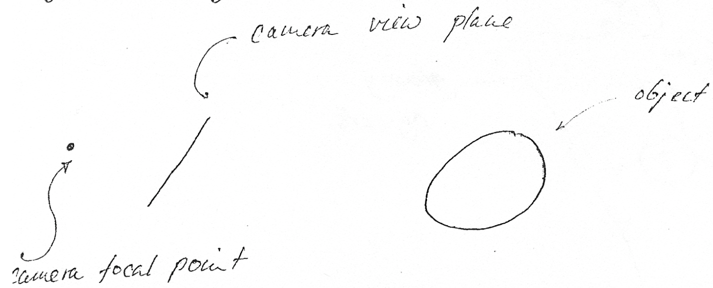
Ray Tracing: - General ideas



Intensity at Pixel (i, j) = Intensity at closest ∩ along ray from fp through (i, j)

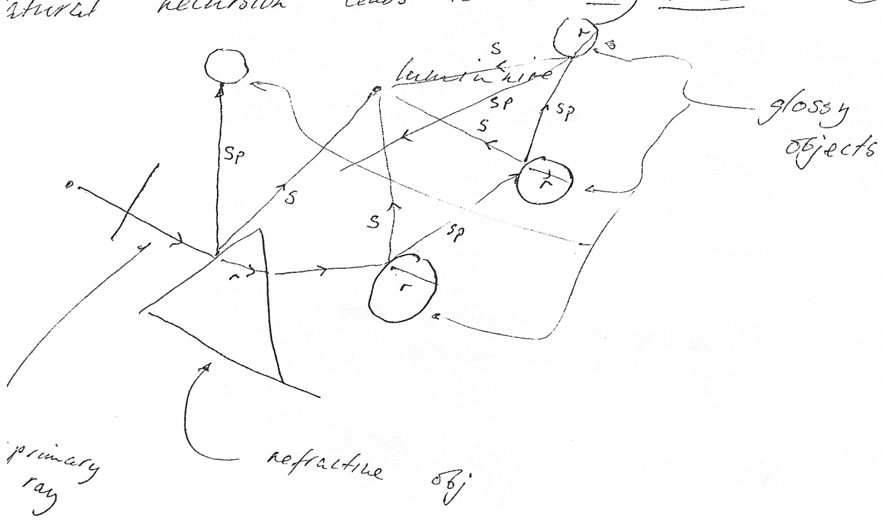
Obtain this in several possible ways

Local Shading: • only luminaires contribute
• can intersection see luminaires?
  - test this by checking line segments for intersections

Propagation: • does ray propagate?
  • Specular reflection
  • refraction
• test by spawning new ray.

Global Shading: • intensity at distant surfaces contributes as well
• how much?
  - interrogate by spawning rays

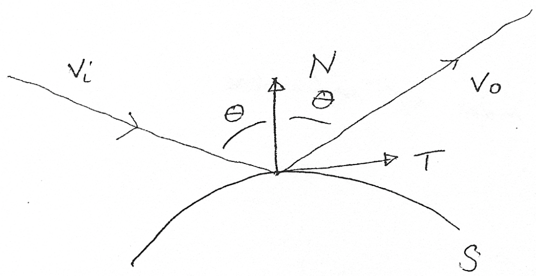
atural recursion leads to a ray tree



ore issues

- Mechanics of reflection, refraction
- Details of shading computation
- Ray intersection
  - Object representation
- Managing the ray tree.

Specular reflection



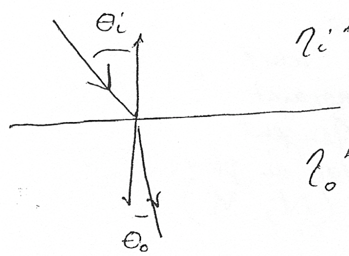
- Standard Specular surface
- N is unit normal
- T is unit tangent vector in plane of (N, V_i)

$$V_o \cdot N = -V_i \cdot N$$
$$V_o \cdot T = V_i \cdot T$$
$$(V_o \cdot T)T = V_i - (V_i \cdot N)N$$
$$\therefore \; V_o = (V_o \cdot N)N + (V_o \cdot T)T$$
$$= \boxed{V_i - 2(V_i \cdot N)N}$$

Notice that some special surfaces have <u>off-specular glints</u>
– different calc of spawned ray, otherwise no change.

Refraction

- change in ray direction due to change in speed of light



η_i — index of refraction

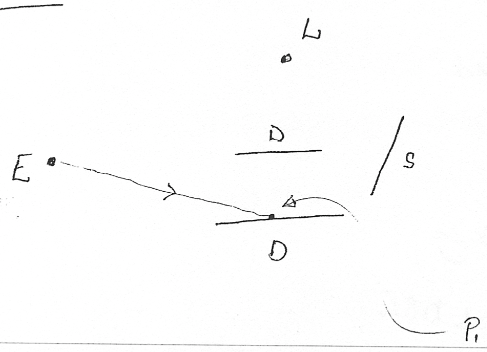
η_o

SNELLS LAW

$$\boxed{\eta_i \sin\theta_i = \eta_o \sin\theta_o}$$

ading details:

- Watch this space!
- for the moment, use what is familiar to compute shading values
- But this isn't good enough (lots of reasons)



- P_1 should be bright ($L \to S \to P_1$) but our alg won't find this
- Q: What are rays?
  A: paths down which light can propagate
  Q: Why ~~tra~~ not trace from light?
  A: may not find camera
  Q: Why trace only from eye?
  A: actually it isn't a great idea.
- Specular to Diffuse transfer

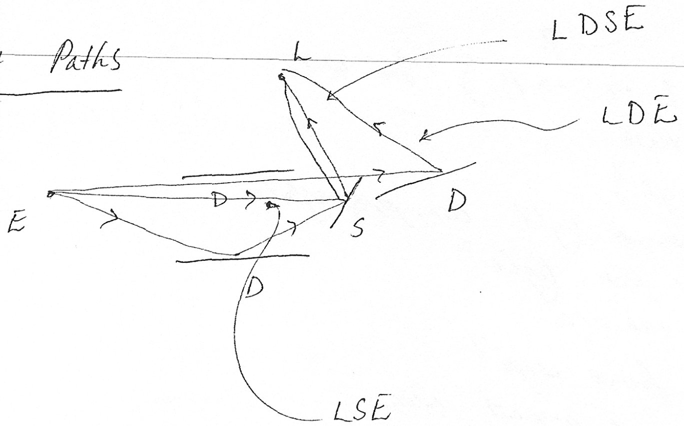
trategy:

① trace rays from light → first diffuse
(i.e. L-S-S-S . . . D)
2 and make a record

② trace rays from eye
- do what has been described
- also sample record.

tails:
- once we've discussed aliasing

ight Paths



etc:

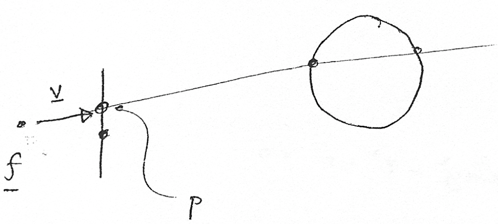
general path	$L\{S\|D\}^*E$
eyerays	LDS^*E
light rays	LS^*DE
bidirectional	LS^*DS^*E

these three don't cover all alternatives !

Ray Intersection

- Sphere. $(x - x_0)^2 + (y-y_0)^2 + (z-z_0)^2 - r^2 = 0$

(Implicit Surface)



- point on ray $\quad \underline{f} + t\underline{v} \quad , \quad t \geq 0$
- this point is on the sphere if eqn above vanishes
- eqn $\quad (\underline{x} - \underline{x}_c)\cdot(\underline{x} - \underline{x}_c) - r^2 = 0$

Substitute

$$(\underline{f} + t\underline{v} - \underline{x}_c)\cdot(\underline{f} + t\underline{v} - \underline{x}_c) - r^2 = 0$$

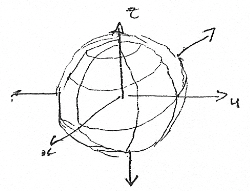
$$t^2(\underline{v}\cdot\underline{v}) + t\left(2\,\underline{v}\cdot(\underline{f} - \underline{x}_c)\right) + \left[(\underline{f} - \underline{x}_c)\cdot(\underline{f} - \underline{x}_c) - r^2\right] = 0$$

quadratic eqn.

Q: What is geometric meaning of

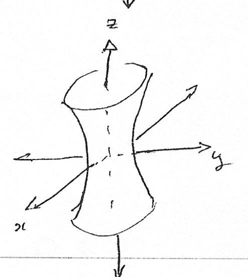
$$\text{disc} \begin{array}{c} < \\ = \\ > \end{array} 0 \quad ?$$

- General Quadratic form
  - includes spheres, ellipsoids, hyperboloids of revolution, etc
  - eg.



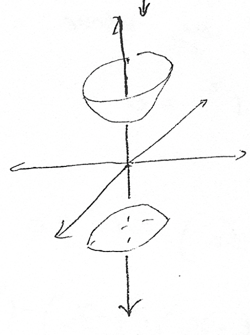
$$x^2+y^2+z^2-r^2=0$$

  - eg



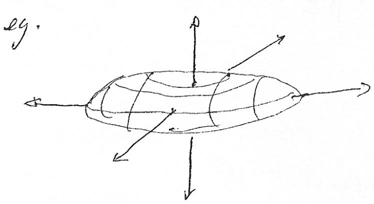
$$x^2+y^2-z^2-r^2=0$$

  - eg



$$z^2-(x^2+y^2)=r^2$$

  - eg.



$$\lambda_1^2x^2+\lambda_2^2y^2+\lambda_3^2z^2-1=0$$

e can combine S.S w/ interval ... to
locate roots

tice:

- coefficients can grow fast (try some e.g.'s w/ Mathematica)
- One S.S. per pixel?

merical root finding

- to polish root
- to exploit coherence

- Interval halving
  - If we know there is one isolated root in $[a, b]$, we can use sign of $Q(t)$, split intervals
  - very robust
  - relatively slow

- Newtons method

$$Q(t_i + \delta t) \approx Q(t_i) + \delta t \cdot \left.\frac{dQ}{dt}\right|_{t=t_i}$$

now if $t_i + \delta t$ is a root,

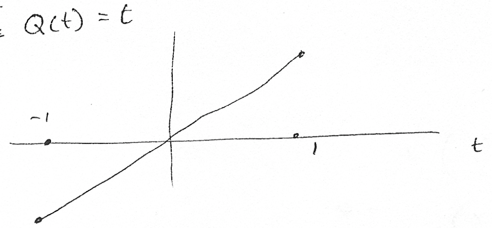
then $$\delta t = \frac{-Q(t_i)}{\left.\left(\frac{dQ}{dt}\right)\right|_{t=t_i}}$$

Iteration

$$t_{i+1} = t_i + \left[\frac{-Q(t_i)}{\left.\left(\frac{dQ}{dt}\right)\right|_{t=t_i}} \right]$$

Examples:

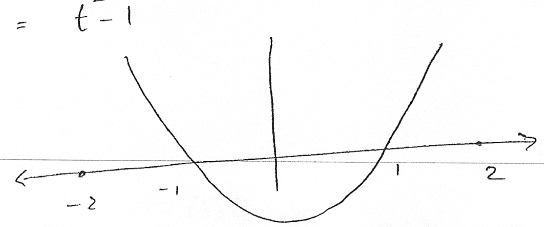
$Q(t) = t$



$Q(-1) = -1 \quad ; \quad Q(1) = 1$

$\Rightarrow$ must be a root $\in [-1, 1]$

$Q(t) = t^2 - 1$



$\left.\begin{array}{l} Q(-2) = 3 \\ Q(2) = 3 \end{array}\right]$ $\therefore$ is there a root?

look at $\frac{dQ}{dt} = 2t$

$\frac{dQ}{dt}(-2) = -4$ $\therefore$ falling

$\frac{dQ}{dt}(2) = 4$ $\therefore$ rising

Q: does it pass through 0?

A: $\frac{d^2Q}{dt^2}$ could tell us.

$\underline{x}'\underline{\underline{A}}\underline{x} + \underline{b}'\underline{x} + c = 0$

symmetric

. root finding on

$$t^2(\underline{v}'\underline{\underline{A}}\underline{v}) + t(2\underline{f}'\underline{\underline{A}}\underline{v} + \underline{b}'\underline{v}) + (\underline{f}'\underline{\underline{A}}\underline{f} + \underline{b}'\underline{f} + c) = 0$$

again, straightforward.

<u>More general algebraic surface</u>:

$p(x, y, z) = 0$

polynomial.

$Q(t) = 0$, by substitution

<u>Problem 1</u>: find smallest t, $t > 0$, s.t. $Q(t) = 0$
(primary ray)

<u>Problem 2</u>: is there a $0 \leq t \leq 1$ such that $Q(t) = 0$
(Shadow cast).

<u>Root finding techniques</u>:

Exact (Sturm Seqs, etc)
Numerical (Newton's method).
Coherence

$\underline{t^2-1}$

$f_0 :\quad t^2-1$

$f_1 :\quad 2t$

$f_2 :\quad 2 \qquad\qquad$ (because $tf_1 - 2 = 2f_0$)

$-2: \quad +,\ -,\ + \qquad\qquad 2: \quad +,\ +,\ +$

$V(-2) = 2 \qquad\qquad V(2) = 0 \qquad \therefore\ 2$ distinct roots

---

$\underline{t^3-t}$

$f_0 :\quad t^3-t$

$f_1 :\quad 3t^2-1$

$f_2 :\quad 2t \qquad\qquad$ (because $tf_1 - 2t = 3f_0$)

$f_3 :\quad 2 \qquad\qquad$ (because $3tf_2 - 2 = 2f_1$)

$-2: \quad -,\ +,\ -,\ + \qquad\qquad 2: \quad +,\ +,\ +,\ +$

$V(-2) = 3 \qquad\qquad V(2) = 0 \qquad \therefore\ 3$ distinct roots

---

$\underline{t^4-t^2}$

$f_0 :\quad t^4-t^2$

$f_1 :\quad 4t^3-2t$

$f_2 :\quad 2t^2 \qquad\qquad$ (because $tf_1 - 2t^2 = 4f_0$)

$f_3 :\quad 2t \qquad\qquad$ ($2tf_2 - 2t = f_1$)

$f_4 :\quad 0 \qquad\qquad$ ($tf_3 = f_2$)

$-2: \quad -,\ +,\ -,\ + \qquad\qquad 2: \quad +,\ +,\ +,\ +$

$V(-2) = 3 \qquad\qquad V(2) = 0 \qquad \therefore\ 3$ dist roots.

Sturm seq.

~~Sturm~~ $f_0(t) = Q(t)$

$f_1(t) = \frac{dQ}{dt}$

$\vdots$

$f_{i+1}(t) = -\underbrace{f_{i-1} \bmod f_i}_{\text{remainder from division alg,}}$

~~keeping rest~~

$f_{s+1} = 0$

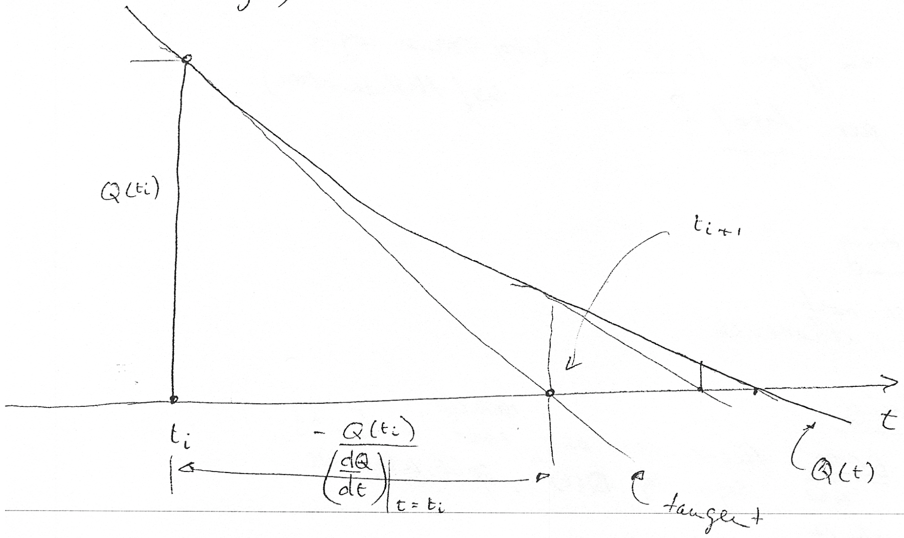
Define $V(f_0, f_1 \cdots f_s; a)$ to be the number of changes in sign of:

$f_0(a)$
$f_1(a)$ (See example)
$f_2(a)$
$\vdots$
$f_s(a)$

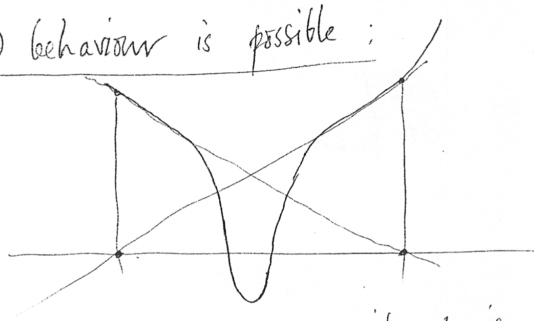
i) $V(f_0, f_1, f_2 \cdots f_s; a) - V(f_0, f_1, f_2, f_s; b)$

= # of DISTINCT roots in $[a, b]$ (as long as a, b are not roots) is method is

(but can diverge!)



y weird behaviour is possible :



– but isn't common if t is close to root.

ve can combine ... w/ interval ... to
locate roots

...tice:

- coefficients can grow fast (try some e.g.'s w/ Mathematica)
- One S.S. per pixel?

...merical root finding

- to polish root
- to exploit coherence

- Interval halving
  - If we know there is one isolated root in [a, b], we can use sign of $Q(t)$, split intervals
  - very robust
  - relatively slow

- Newtons method

$$Q(t_i + \delta t) \approx Q(t_i) + \delta t \cdot \left.\frac{dQ}{dt}\right|_{t=t_i}$$

now if $t_i + \delta t$ is a root,

then $$\delta t = \frac{-Q(t_i)}{\left.\left(\frac{dQ}{dt}\right)\right|_{t=t_i}}$$

Iteration

$$t_{i+1} = t_i + \left[\frac{-Q(t_i)}{\left.\left(\frac{dQ}{dt}\right)\right|_{t=t_i}} \right]$$