Ray Intersections

CS 319

Advanced Topics in Computer Graphics

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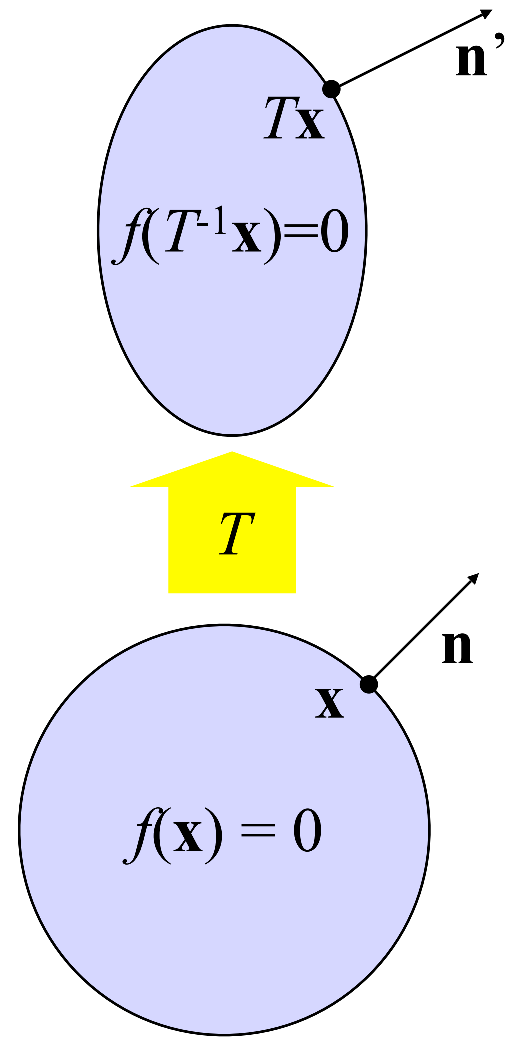
What about the normal?

- Let $\mathbf{n} = [a\ b\ c\ d]$ be a tangent plane
- Let $\mathbf{x} = [x\ y\ z\ 1]^T$ be a point
- Plane-point duality
  - Planes are row vectors
  - Points are column vectors
- Point $\mathbf{x}$ in plane $\mathbf{n} \Leftrightarrow \mathbf{n}\ \mathbf{x} = 0$
- Need to find $\mathbf{n}'$ such that $\mathbf{n}'\ T\ \mathbf{x} = 0$
- Notice $\mathbf{n}\ T^{-1}\ T\ \mathbf{x} = 0$
- New normal $\mathbf{n}' = \mathbf{n}\ T^{-1} = (T^{-1})^T\ \mathbf{n}^T$
- Could also use the adjoint $\mathbf{n}' = \mathbf{n}\ T^*$
  - $\mathbf{n}'$ not necessarily unit length even if $\mathbf{n}$ is
  - But we'll need the inverse anyway



Normals and implicit surfaces

- Affine coordinates
- Homogenous coordinates

Matrix Inverse

$$A^{-1} = \begin{bmatrix} a & b & c & x \\ d & e & f & y \\ g & h & i & z \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = (ST)^{-1} = \left(\begin{bmatrix} a & b & c & 0 \\ d & e & f & 0 \\ g & h & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1}$$

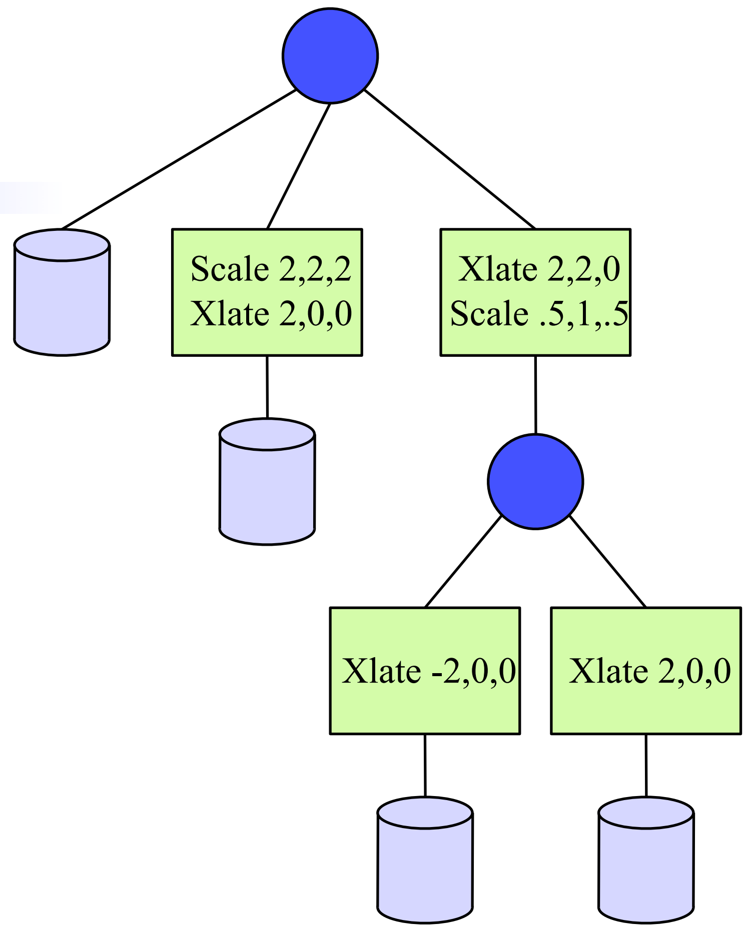
$$= \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} a & b & c & 0 \\ d & e & f & 0 \\ g & h & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & -z \\ 0 & 0 & 0 & 1 \end{bmatrix} \frac{1}{|S|} \begin{bmatrix} ei-hf & ch-bi & bf-ce & 0 \\ fg-di & ai-cg & cd-af & 0 \\ dh-eg & bg-ah & ae-bd & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

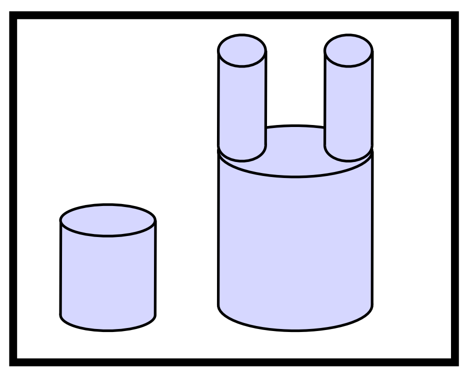
S^{-1} = [minors of S]^T

Don't need 1/|S| if just need direction of transformed normal. Will have to renormalize anyway is S not special unitary.

Scene Graph

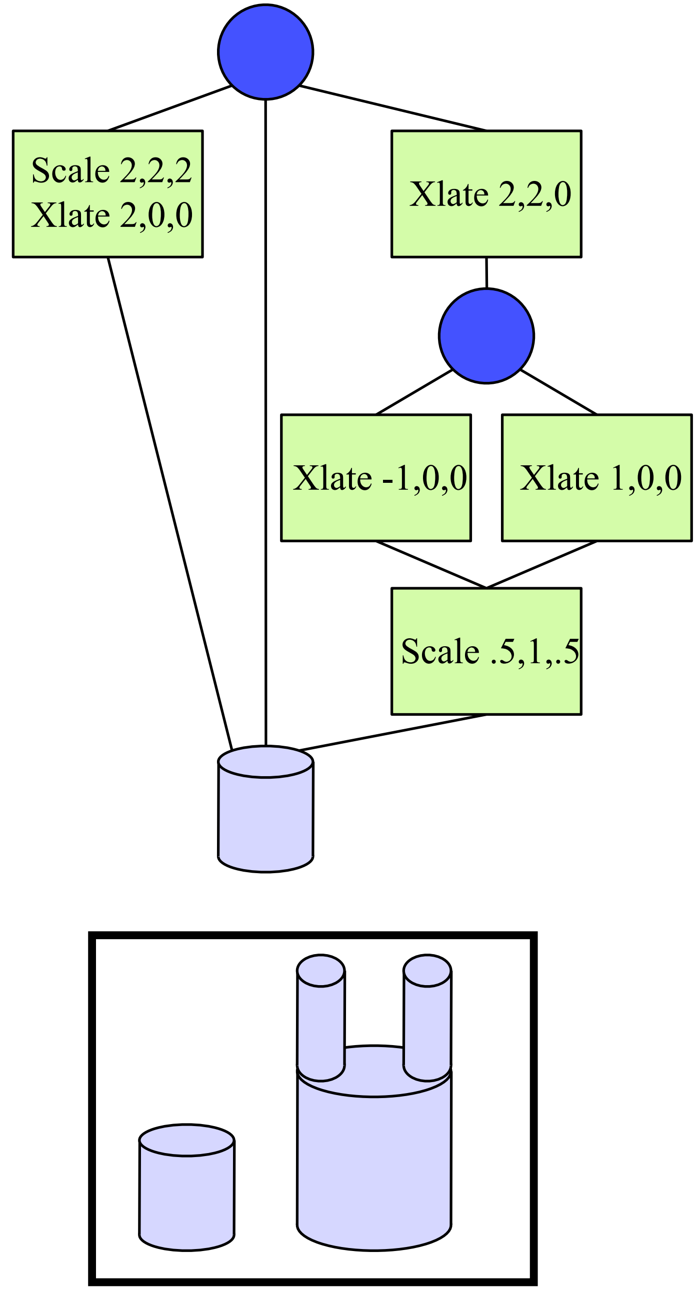
- Hierarchical representation of all objects in scene
- Transformation nodes
  – Intersect kids by T^{-1} $\mathbf{r}$
  – Returned normal $(T^{-1})^T$ $\mathbf{n}$
  – Maintain T^{-1} (not T)





Instancing

- Scene graph is a hierarchy
- Not necessarily a tree
- Directed acyclic graph (DAG)
- Nodes may have multiple parents
- *Instance*: Appearance of each node's geometry in scene



Fun with Instancing

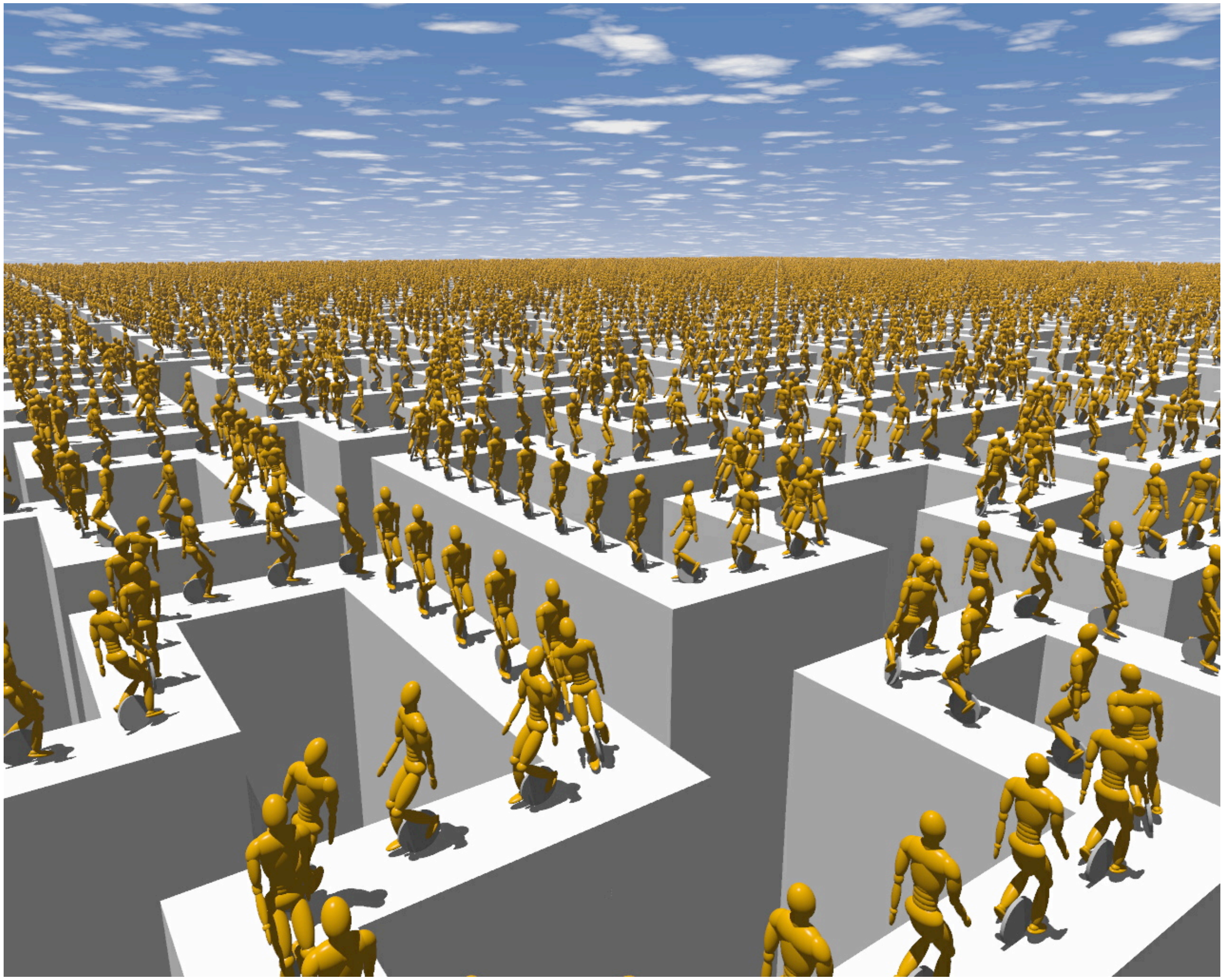
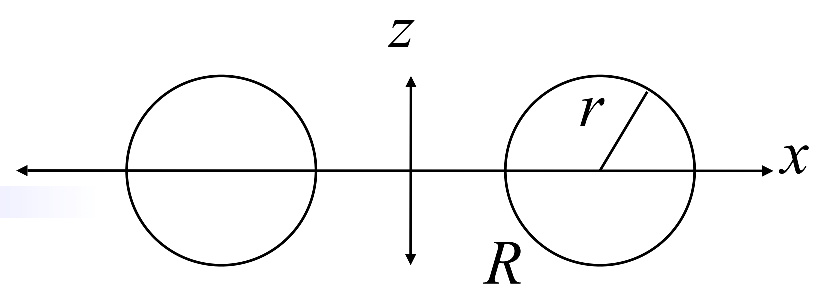


Image courtesy John Amanatides

Torus



- Product of two implicit circles

$$(x - R)^2 + z^2 - r^2 = 0$$

$$(x + R)^2 + z^2 - r^2 = 0$$

$$((x - R)^2 + z^2 - r^2)((x + R)^2 + z^2 - r^2)$$

$$= (x^2 - 2Rx + R^2 + z^2 - r^2)\ (x^2 + 2Rx + R^2 + z^2 - r^2)$$

$$= x^4 + 2x^2z^2 + z^4 - 2x2r^2 - 2z2r^2 + r^4 - 2x^2R^2 +$$

$$2z^2R^2 - 2r^2R^2 + R^4$$

$$= (x^2 + z^2 - r^2 - R^2)^2 + 4z^2R^2 - 4r^2R^2$$

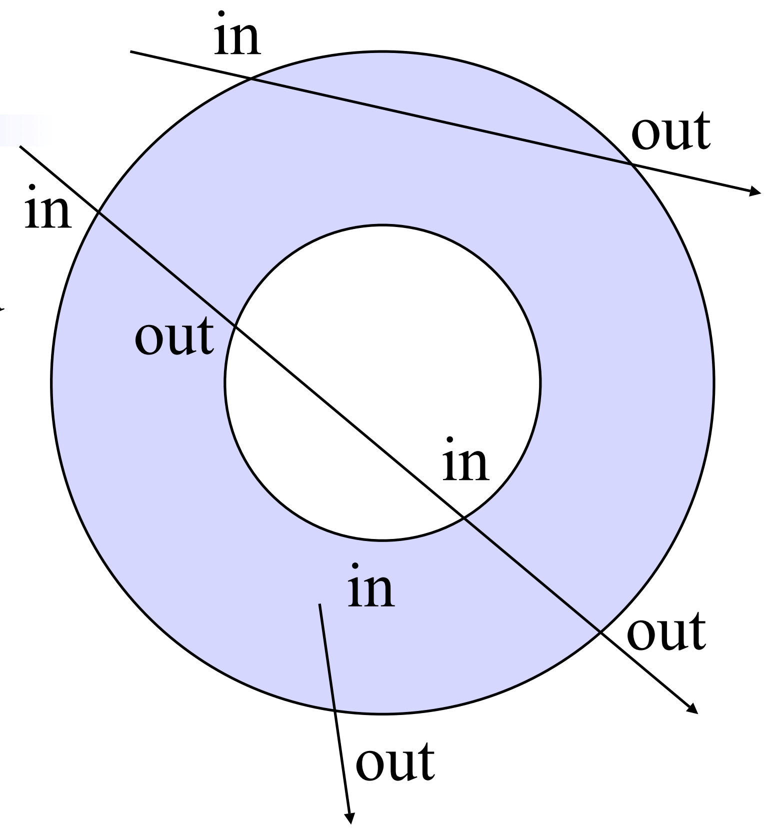
- Surface of rotation: replace x^2 with $x^2 + y^2$

$$f(x,y,z) = (x^2 + y^2 + z^2 - r^2 - R^2)^2 + 4R^2(z^2 - r^2)$$

- Quartic!!!
- Up to four ray torus intersections

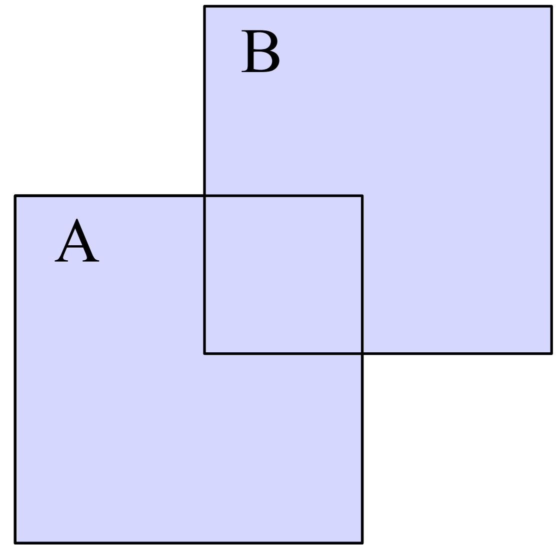
Ray-Object Intersection

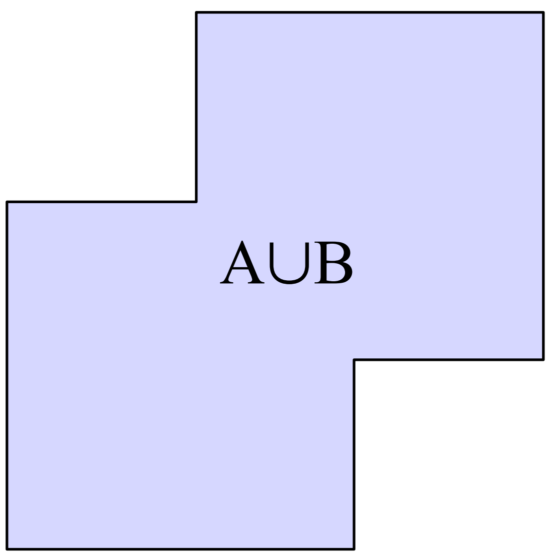
- Returns intersection in a hit record
- "Next" field enables hit record to hold a list of intersections
- List only non-negative intersection parameters
- Ray always originates outside
  - If first $t = 0$ then ray originated inside
- Parity classifies ray segments
  - Odd segments "in"
  - Even segments "out"

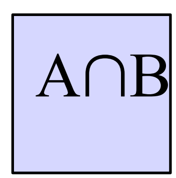


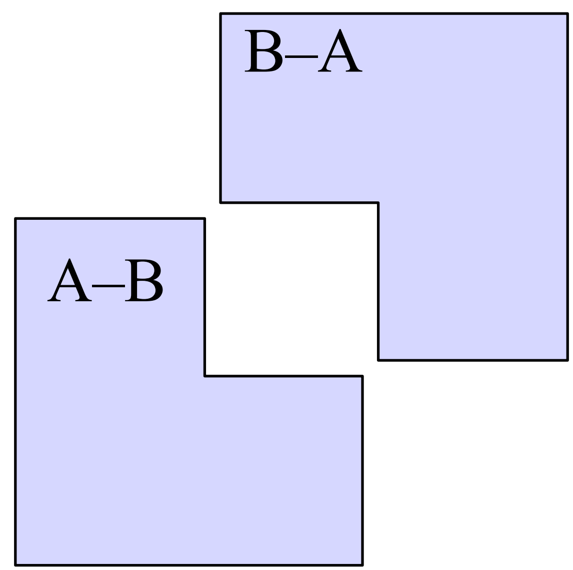
Constructive Solid Geometry

- Construct shapes from primitives using boolean set operations
- Union: A∪B, A + B, A or B
- Intersection: A∩B, A*B, A and B
- Difference: A\B, A–B, A and not B

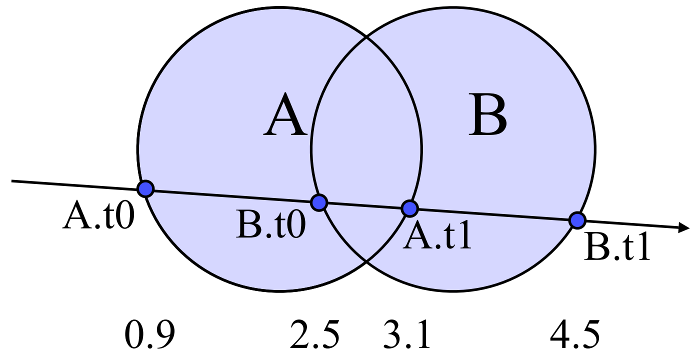








CSG Intersections



- List of *t*-values for A, B w/in-out classification

  A.t_list = {0.9, 3.1} = {0.9in, 3.1out}

  B.t_list = {2.5, 4.5} = {2.5in, 4.5out}

  – Use dot(r.d,n) to determine in,out
- Merge both lists into a single *t*-ordered list

  { 0.9 Ain Bout,
  2.5 Ain Bin,
  3.1 Aout Bin,
  4.5 Aout Bout }

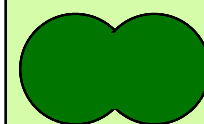
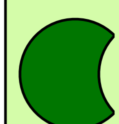
  – Keep track of A and B in/out classification
- Use Roth table to classify *t*-values

A+B = {0.9in, 2.5in, 3.1in, 4.5out} = {0.9, 4.5}

A*B= {0.9out, 2.5in, 3.1out, 4.5out} = {2.5, 3.1}

A-B = {0.9in, 2.5out,3.1out, 4.5out} = {0.9, 2.5}

Roth Table

Op	A	B	Res
+	in	in	in
	in	out	in
	out	in	in
	out	out	out
*	in	in	in
	in	out	out
	out	in	out
	out	out	out
–	in	in	out
	in	out	in
	out	in	out
	out	out	out

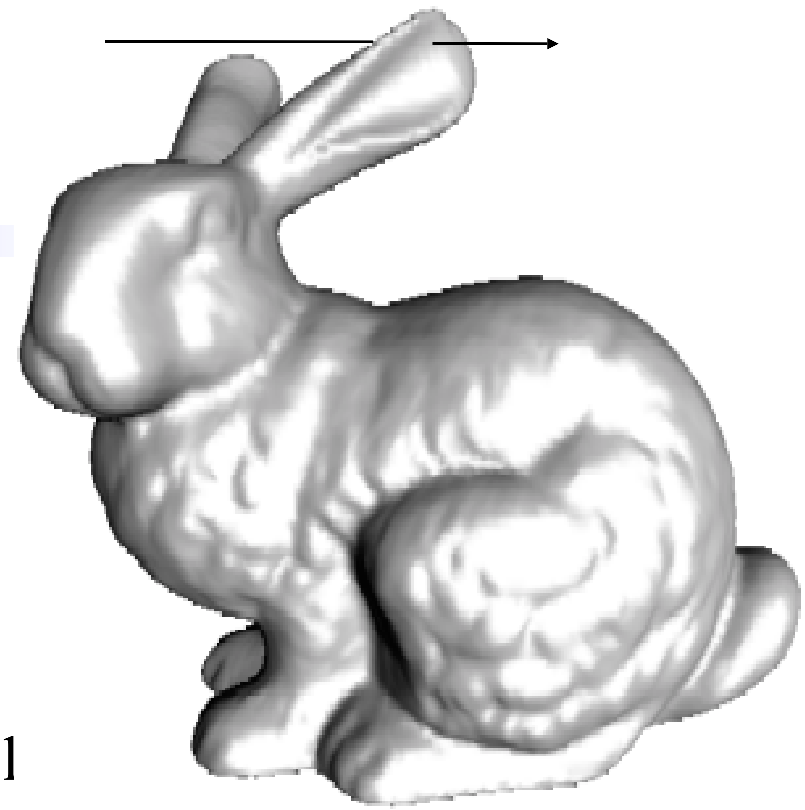
Accelerating Ray Intersections

- Q: Why is basic ray tracing so slow?
- A: It intersects every ray with every primitive in every object
- Q: How can we make ray tracing faster?
- A: Coherence

  Image coherence – neighboring pixel probably display same object

  Spatial coherence – neighboring points probably exhibit same appearance

  Temporal coherence – Pixels in neighboring frames probably display same object

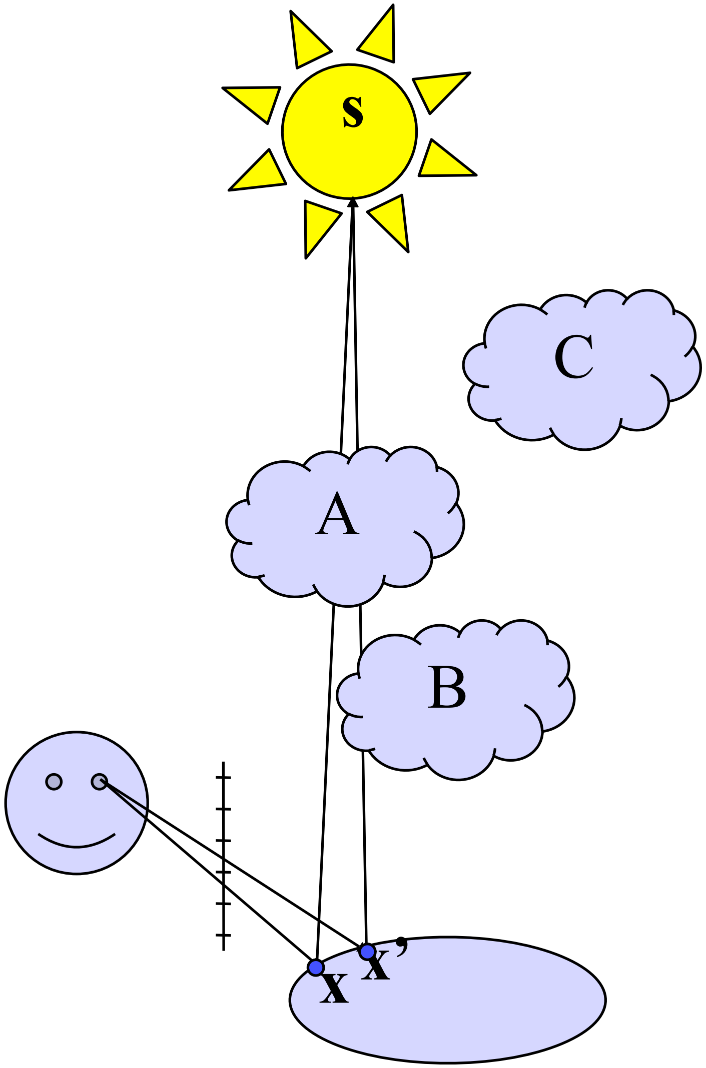


Stanford Bunny
~70K triangles

Do we need 70K ray-triangle intersections for each ray?

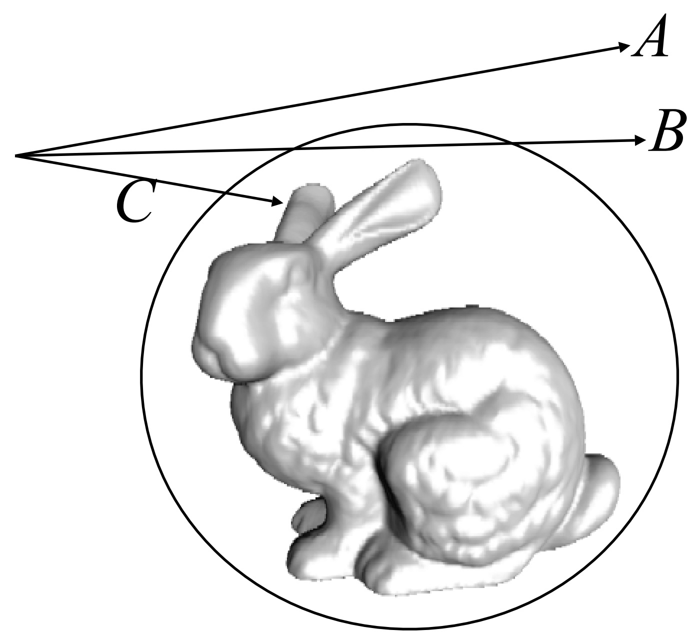
Shadow Caching

- Any interloper between surface point **x** and the light source **s** will cast a shadow
  - Doesn't matter how many
  - Doesn't matter which is closest
  - Stop ray intersections once *any* intersection found
- Neighboring shadowed surface points **x** and **x**' probably shadowed by the same object
  - Start shadow ray intersection search with object intersected in last shadow search

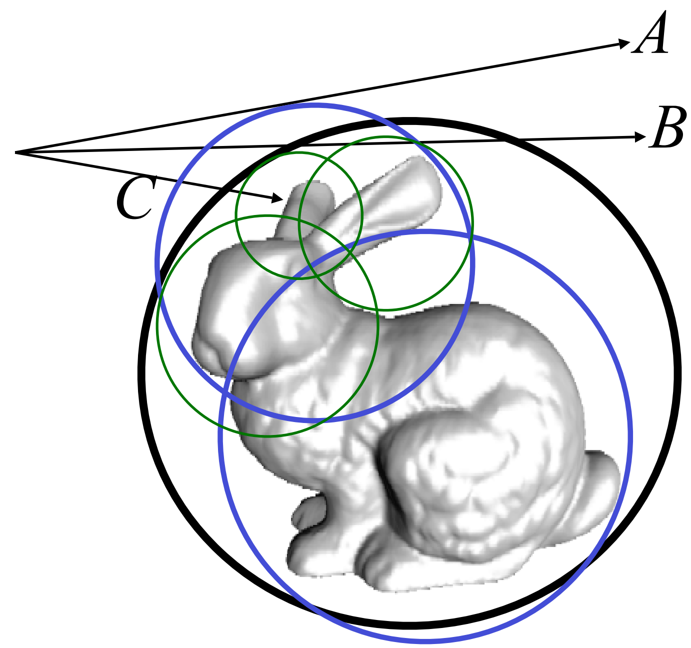


Bounding Volume

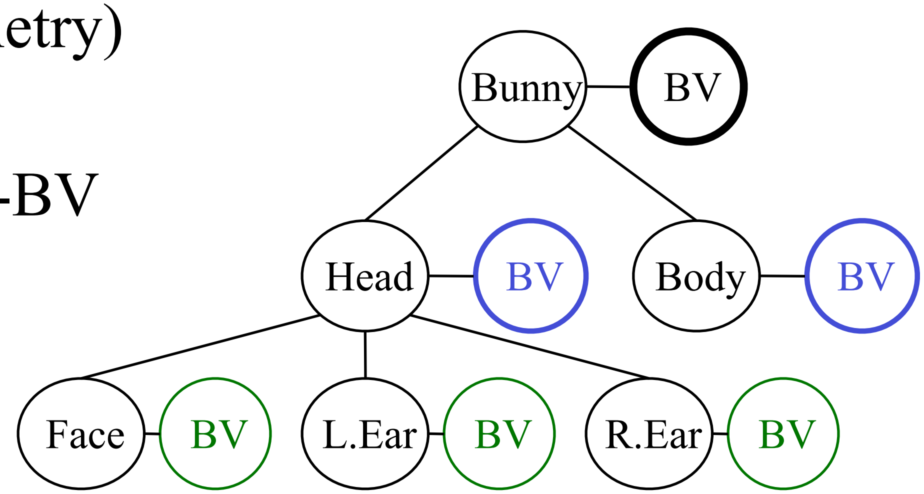
- Ray-bunny intersection takes 70K ray-triangle intersections even if ray misses the bunny
- Place a sphere around bunny
  - Ray *A* misses sphere so ray *A* misses bunny without checking 70K ray-triangle intersections
  - Ray *B* intersects sphere but still misses bunny after checking 70K intersections
  - Ray *C* intersects sphere and intersects bunny
- Can also use axis-aligned bounding box
  - Easier to create for triangle mesh



Bounding Volume Hierarchy

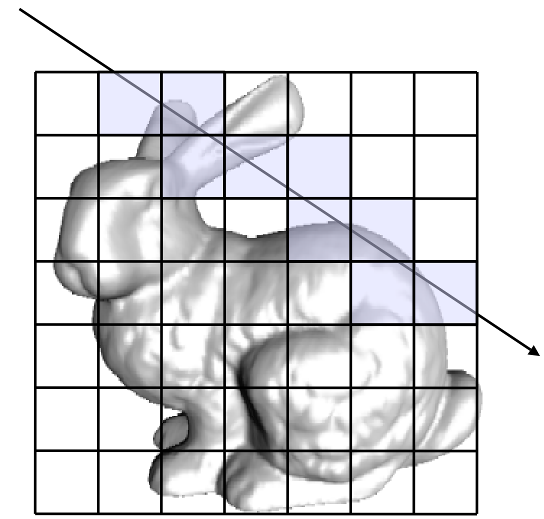


- Associate bounding volume with each node of scene graph
- If ray misses a node's bounding volume, then no need to check any node beneath it
- If ray hits a node's BV, then replace it with its children's BV's (or geometry)
- Breadth first search of tree
  - Maintain heap ordered by ray-BV intersection *t*-values
  - Explore children of node w/least pos. ray-BV *t*-value



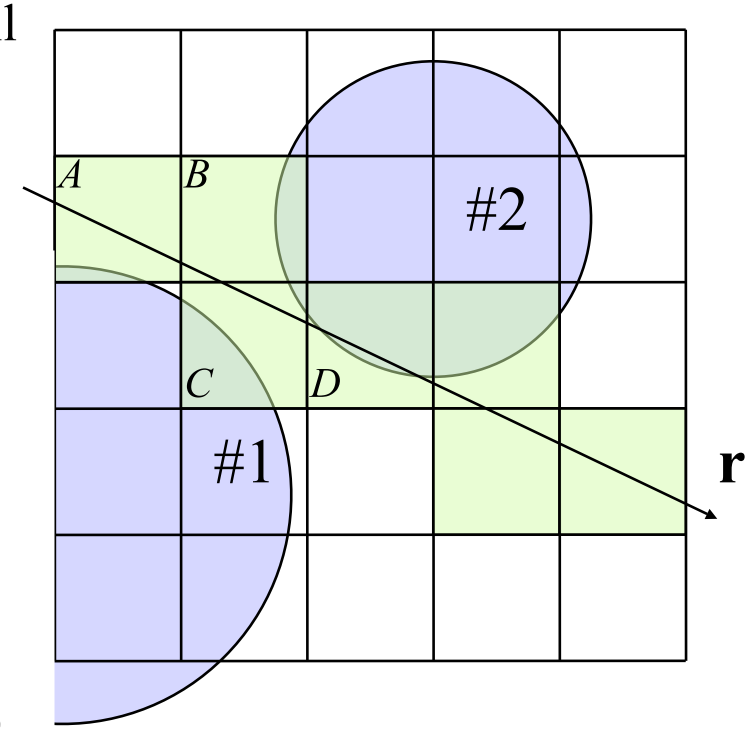
Grids

- Encase object in a 3-D array of cubic cells
- Each cell contains list of all triangles it contains or intersects
- Rasterize ray to find which cells it intersects
  - 3D Bresenham algorithm
  - All cells that contain any part of ray
- Working from first ray-cell to last…
  - Find least positive intersect of ray with triangles in cell’s list
  - If no intersection, move on to next cell

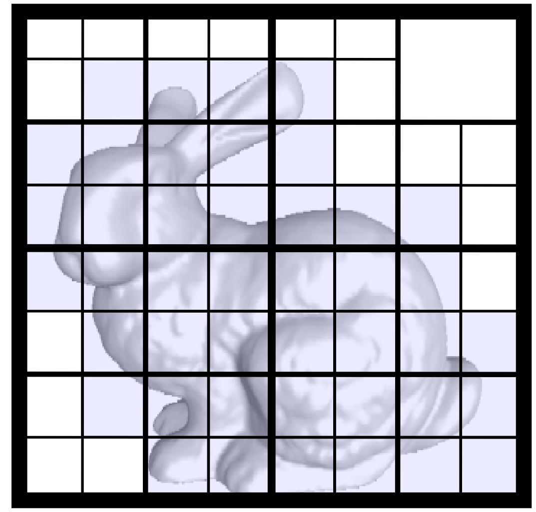


Tagging

- Ray-object intersection test valid for ray with entire object
  - not just portion of object inside current cell
- Need only intersect object once for each ray
- In cell *A* – list = {#1}
  - Intersect **r** with #1? Yes
    - Miss Tag #1 with no-intersection
- In cell B – list = {#2}
  - Intersect **r** with #2? Yes
    - ray **r** hits object #2 but later in cell *C*
    - Tag object #2 with intersection-at-*C*
- In cell *C* – list = {#1,#2}
  - Intersect **r** with #1? No (no-intersection)
  - Intersect **r** with #2? No (intersection-at-*D*)
- In cell *D* – list = {#2}
  - Intersect **r** with #2? No (intersection-at-*D*)



Other Partitioning Structures



- Octree
  - Ray can parse through large empty areas
  - Requires less space than grid
  - Subdivision takes time
- Binary Space Partition (BSP) Tree
  - Planes can divide models nearly in half
  - Trees better balanced, shallower
  - Added ray-plane intersections