

Some more on Christoffel symbols

notice we have

$$(x_{uu} \cdot x_u) = \Gamma_{11}^1 (x_u \cdot x_u) + \Gamma_{11}^2 (x_v \cdot x_u)$$

etc:

now we need to simplify notation. I'll write x_i for x_u for $(x_u \cdot x_u)$ etc.

then

$$\begin{pmatrix} (x_{uu} \cdot x_u) \\ (x_{uu} \cdot x_v) \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} \Gamma_{11}^1 \\ \Gamma_{11}^2 \end{pmatrix}$$

but this has full rank.

now notice $\frac{\partial}{\partial u} (x_u \cdot x_v) = (x_{uu} \cdot x_v) + (x_{uv} \cdot x_u)$

etc