now this will allow me to extract an expression for Γ's.

let $\partial_1 \equiv \frac{\partial}{\partial u}$; $\partial_2 \equiv \frac{\partial}{\partial v}$

(Note: we are looking forward to when there are many params.)

then $(x_{uu} \cdot x_u) = \frac{1}{2}\left(\partial_1 g_{11} + \partial_1 g_{11} - \partial_1 g_{11}\right)$

$$(x_{uv} \cdot x_u) = \frac{1}{2}\left(\partial_1 g_{21} + \partial_2 g_{11} - \partial_1 g_{12}\right)$$

just write it out

This brings us to:

$$\boxed{\sum_i g_{ri} \Gamma^i_{jk} = \frac{1}{2}\left(\partial_j g_{kl} + \partial_k g_{lj} - \partial_l g_{jk}\right)}$$

So: Christoffel symbols depend on metric, its derivatives