Notice something here:

we can now differentiate vector fields on surfaces in a meaningful way

- You may not have noticed, but previously we couldn't — derivative wasn't necessarily on surface, which often doesn't make sense.

  - e.g. a ball is moving on a surface / velocity $v$; what acceleration occurs wrt the surface?

A: compute acceleration in the usual way

THEN project to its tangent plane.