

Now I want to differentiate vector fields

$$\nabla_X V = \left[ \begin{array}{l} \text{Directional derivative of} \\ V, \text{ proj onto } \text{surf} \\ \text{tangent plane} \end{array} \right]$$

$$= \uparrow \left[ m \frac{\partial}{\partial u} [a \underline{x}_u + b \underline{x}_v] + n \frac{\partial}{\partial v} [a \underline{x}_u + b \underline{x}_v] \right]$$

= project to surface tangent plane

$$= \left[ \begin{array}{l} m a_u + m a \Gamma_{11}^1 + m b \Gamma_{12}^1 \\ + n a_v + n a \Gamma_{12}^1 + n b \Gamma_{22}^1 \end{array} \right] \underline{x}_u +$$

$$\left[ \begin{array}{l} m b_u + m a \Gamma_{11}^2 + m b \Gamma_{12}^2 \\ + n b_v + n a \Gamma_{12}^2 + n b \Gamma_{22}^2 \end{array} \right] \underline{x}_v$$

We can cook up a cleaner notation for this mess, but for the moment, nice to know can do; operation is called:

COVARIANT DERIVATIVE