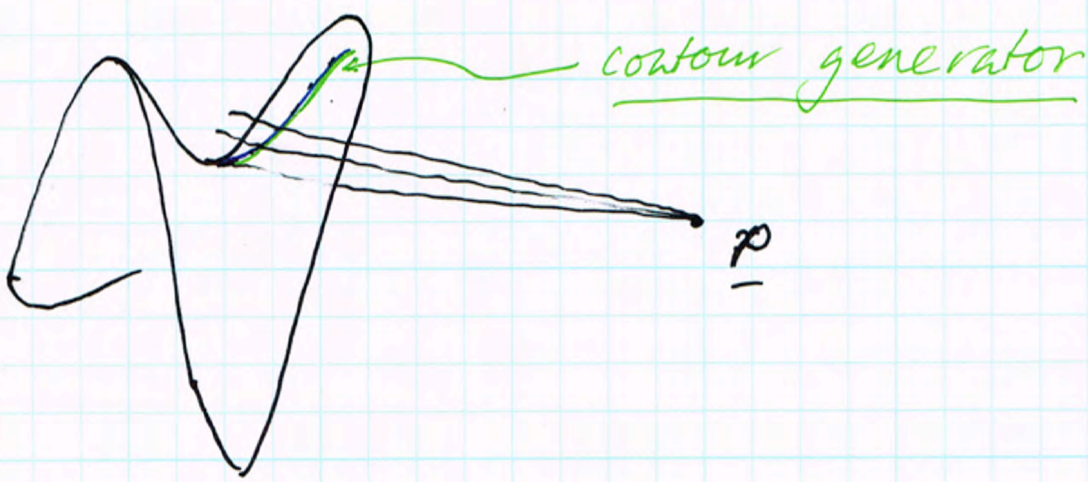
Contour generators and outlines

A contour generator on S is a curve where rays from $\underline{p}$ ← some chosen point are tangent to the surface



$\underline{p}$ could be infinitely far away, in which case all rays to $\underline{p}$ are ∥

∴ so rays in dir $\underline{V}$ are tangent to surf

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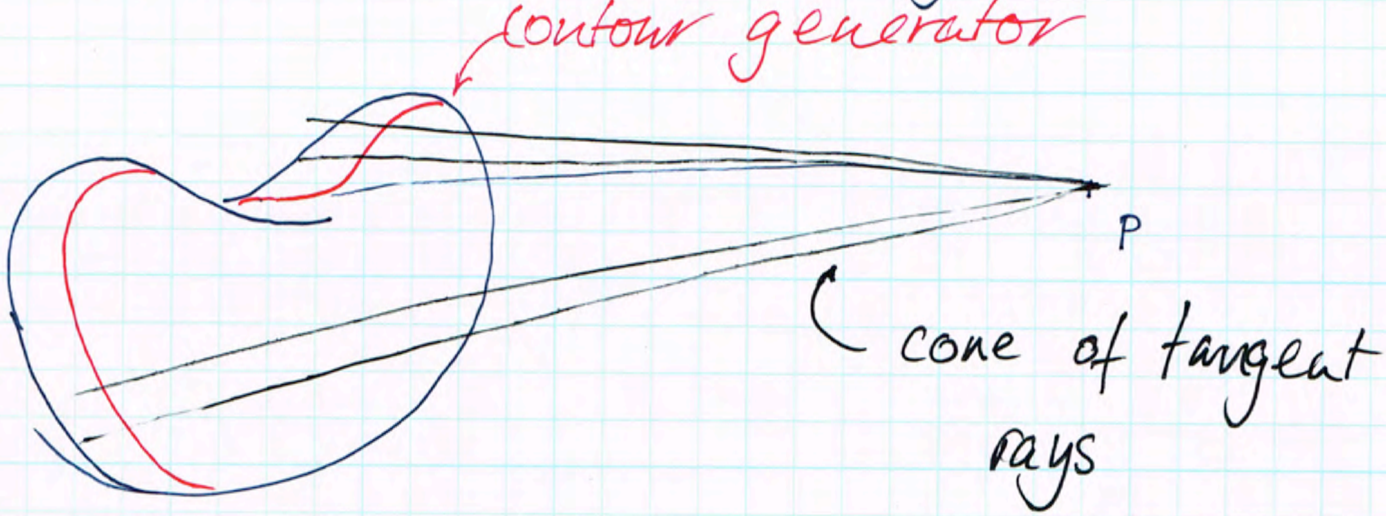
An outline is the projection into the image of a contour generator

- Often helpful to think of this as a cone of tangent rays



- Image is obtained by slicing this cone with a plane
- thinking about the cone avoids confusion caused by rotation about the focal point.

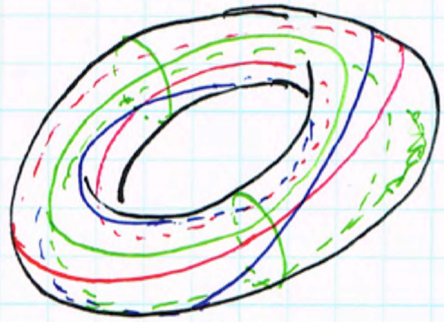
- We usually think about all components (not just visible) of these curves

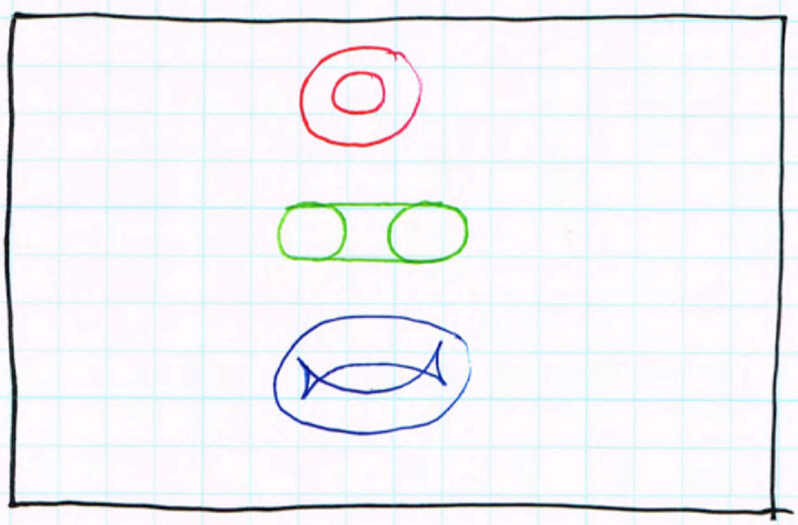
← outline of bean in previous image

- outlines exhibit complex behavior w/ change of viewpoint, as do contour generators (next p.)

- a <u>silhouette</u> consists of the visible exterior points of outline

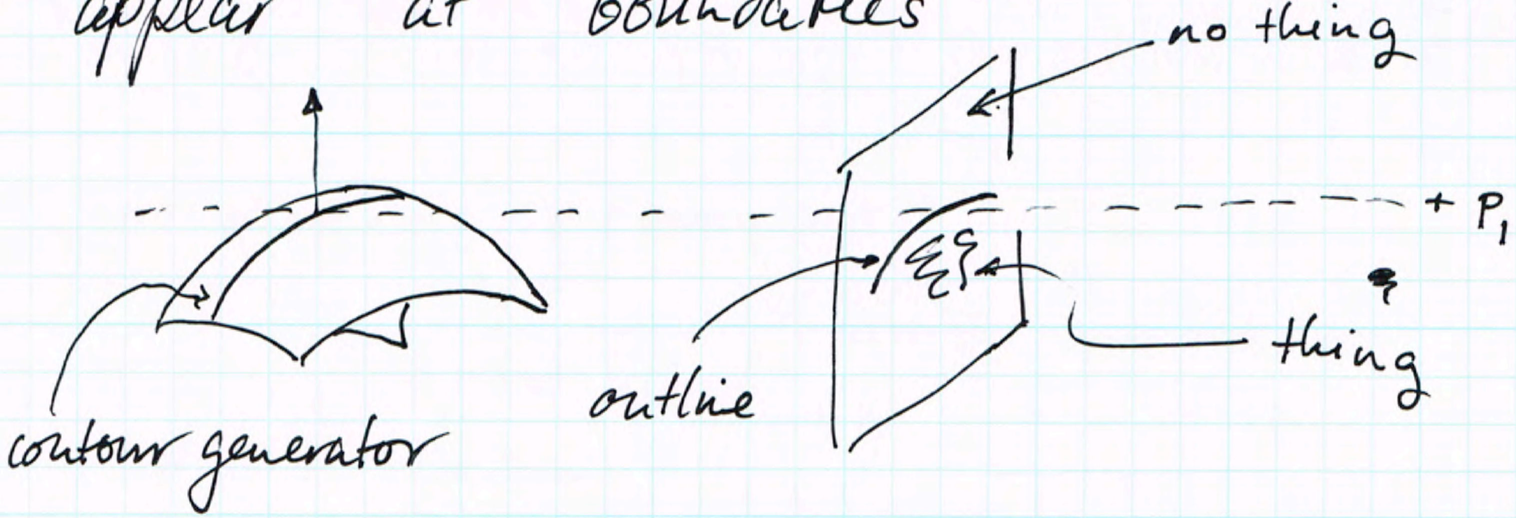
← silhouette of bean.



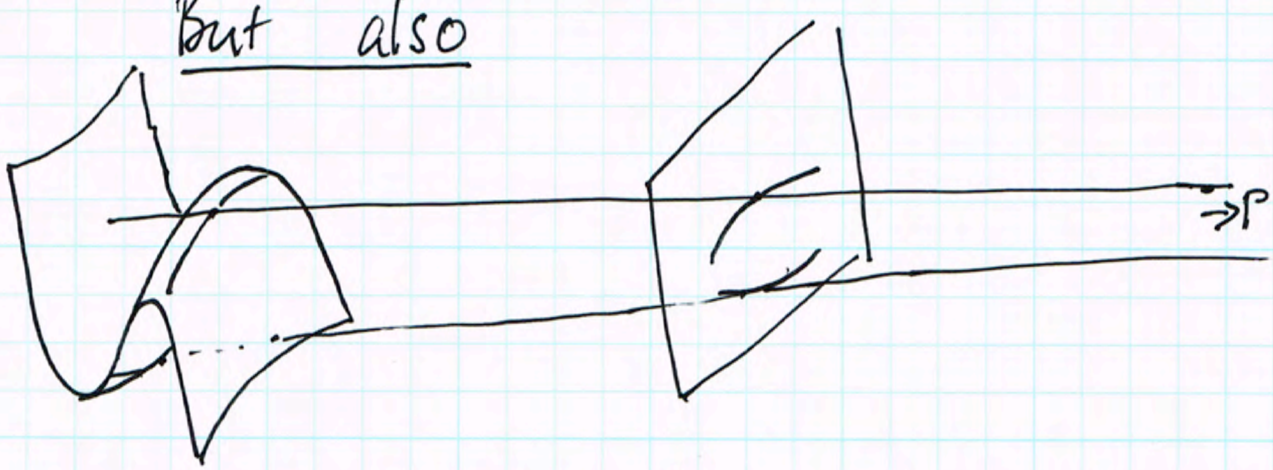


<u>NOTE</u>: outlines + C.G.s are notoriously confusing – famous names have made major errors in print; ~~they~~
C.G.'s are <u>hardly</u> <u>ever</u> plane, a point of much confusion.

These curves are interesting because they appear at boundaries



<u>But also</u>



We tend to keep track of all components because counting visibility is <u>hard</u>

Some properties of C.G.'s + outlines

- The contour generator for a sphere is plane. The outline is a plane section of a right circular cone.

- <u>Proof</u>: symmetry does the work. The geometry is rotationally symmetric about the line from p to center of sphere; C.G. must have this symmetry so be section of sphere by plane normal to axis. Rays form a cone.

- <u>Corollary</u>: perspective views of spheres could be ellipses (in principle, you could calibrate a camera like this)

<u>Q</u>: Why do spheres always look round?

write $\phi = 0$ for an <u>implicit surface</u>

then at x_0, y_0, z_0, Normal is

$$\nabla\phi\Big|_{x_0,y_0,z_0}$$

and tangent plane is

$$\nabla\phi\Big|_{x_0 y_0 z_0} \cdot \underline{x} - \nabla\phi\Big|_{x_0 y_0 z_0} \cdot \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = 0$$

(i.e. plane through x_0, y_0, z_0 w/ normal $\nabla\varphi$.)

then C.G. is curve (in $\underline{x} = (x, y, z)$)

$$\phi(\underline{x}) = 0$$

$$\nabla\phi^T p - \nabla\phi^T \underline{x} = 0$$

if ϕ is polynomial, all sorts of neat stuff happens. (See my paper, and good luck to you!)

One application:

- The C.G. of a generic quadric is plane

- generic quadric:

$$x^T \frac{A}{2} x + b^T x + c = 0 = \phi$$

for $\det A \neq 0$, $A - A^T = 0$

so C.G. is :

$$\phi = 0$$

$$x^T A p + b^T p - \left[x^T A x + b^T x \right] = 0$$

!

BUT

$$x^T A x + b^T x = 2\left[0 - c \right] - b^T x$$

and result follows

- an <u>Algebraic surface</u> of degree d.

is a set of all points x st

$\phi(x) = 0$ where ϕ is polynomial of degree d.

- a generic ~~ray~~ line intersects this surface in d points

(subst. $\underline{x} = \underline{u} + t\,\underline{v}$)

- The C.G., ~~and hence the~~ ~~is~~ can be given by two polynomial eqns

$$\phi = 0 \quad \leftarrow \text{degree } d$$
$$\psi = 0 \quad \leftarrow \text{degree } (d-1)$$

which means the C.G. intersects a generic plane in $d(d-1)$ pts (use Bézout)

The *global theory* of C.G.'s, outlines is poorly understood in all but a few special cases (algebraic; SOR; some others). Good evidence this is because those cases (which are nasty + complicated) are the easy ones.

Local theory is fairly easy BUT probably not much use (? these curves have global structure, in *ALL* cases ?)

1) View direction and Tangent to Contour generator are *conjugate*

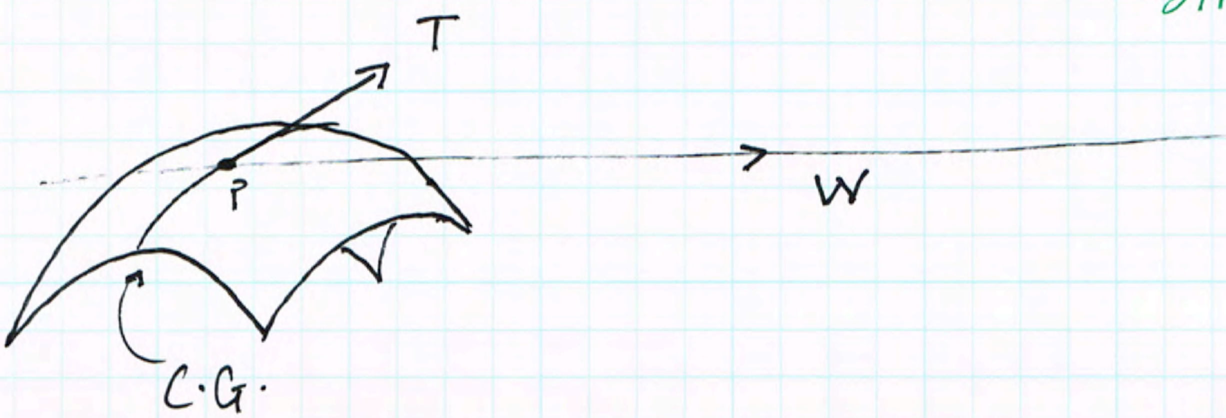
Write W for view dir^n (which is a tangent vector at

T for tangent to C.G. point of interest

Then

$$\underbrace{\mathrm{II}(T, W) = 0}$$ at any pt. on C.G.

this condition = T, W are conjugate dir'ns



at p, $N \cdot W = 0$ (defn of C.G.)

so $\nabla_T (N \cdot W) = 0$

directional derivative of $N \cdot W$ in T dir'n

so

$$(\nabla_T N) \cdot W + \underline{N \cdot (\nabla_T W)} = 0$$

$$\mathrm{II}(T, W) + 0 = 0$$

see next p.

consider $\nabla_T W$; two cases;

(a) View is orthographic $\therefore$ W is const

$\therefore \nabla_T W = 0$

(b) View is perspective, so

$$W = \underset{\text{P.O.I.}}{p} - \underset{\text{focal point}}{f}$$

$\nabla_T W$: slide p very slightly along T to $p + \varepsilon T$, then form

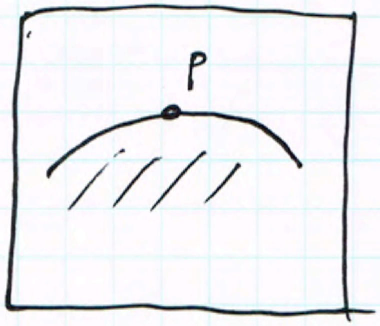
$$\lim_{\varepsilon \to 0} \frac{(p + \varepsilon T - f) - (p - f)}{\varepsilon}$$

this is clearly a tangent vector, so we are done.

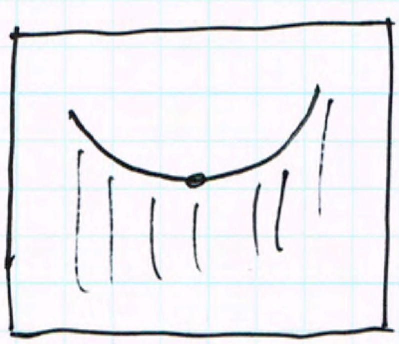
$$\boxed{II(T, W) = 0}$$

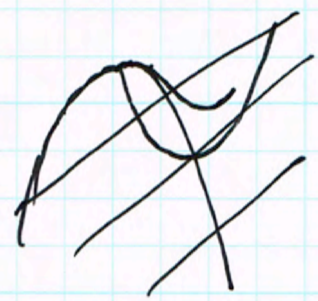
we will get Koenderink's thm out of this; worth remembering.

The shape of an outline clearly says something about Gaussian curvature

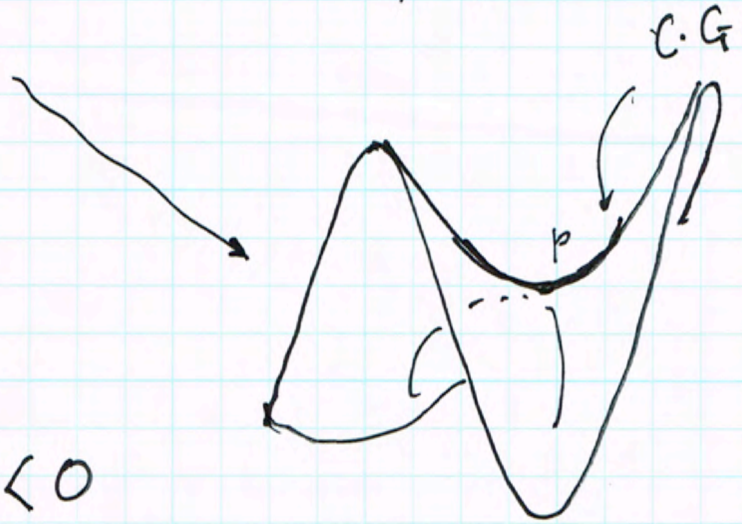


$K > 0$, clearly





SORRY

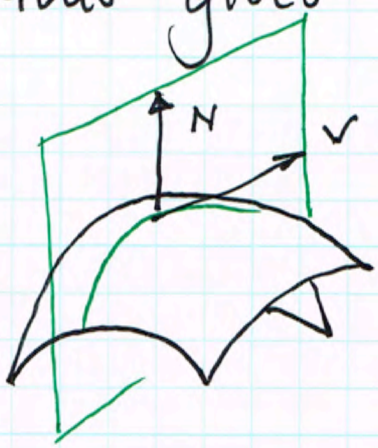


$K < 0$

To go further, we need a new notion

at a pt p. on a surface, and tangent vector V, slice surface with a plane spanned by N, V

– this gives a plane curve with tangent V at p.



The curvature of this plane curve at p. is called the __directional__ curvature of the surface in dir V

__Ex__ Show it is $\dfrac{II(V,V)}{I(V,V)}$

You really should do this!

Koenderinks thm (orthographic version)

- We view a smooth surface in orthographic view. Choose a pt p on the <u>outline</u> in the image plane. Write κ_o for the curvature of ~~this curve~~ the <u>outline</u> at p. Write V for the view dir'n, and P for the pt on the surface csp to p.

then

$$K_P = \kappa_o \cdot \kappa_V$$

directional curvature at P in dir V.

Gaussian curvature of the surface at P

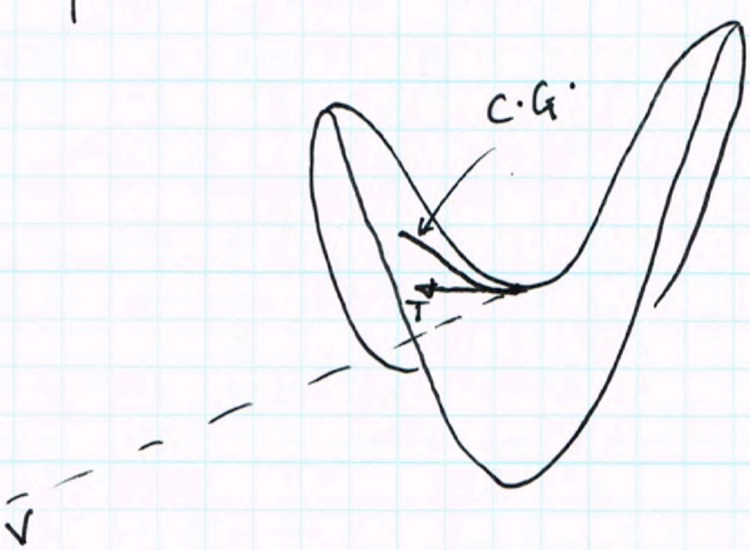
as in text, curvature of <u>outline</u> at p.

Proof:

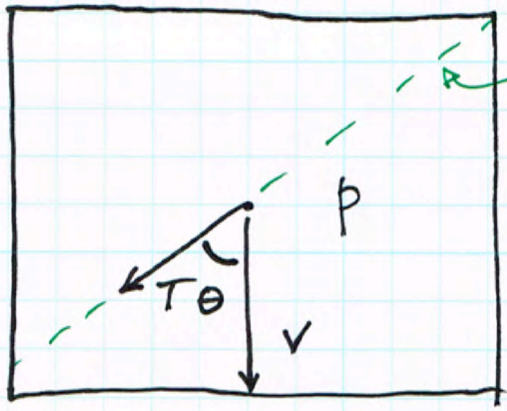
- I ignore issues of sign because pix on P 13 sort that out.

$$K_v = \frac{\mathbb{I}(v,v)}{\mathrm{I}(v,v)}$$

- Now notice that the tangent vector to the C.G. T is not necessarily ∥ to the image plane → the C.G. is foreshortened to form the outline



easiest to look at this in tangent plane



T, N plane, seen from above

tangent plane to Surf at p

Image plane seen from above

Q: directional curvature at p in dirn T

is $\dfrac{\mathrm{II}(T,T)}{\mathrm{I}(T,T)} = \kappa_T \longrightarrow$ but this <u>isn't</u> κ_0,

which is <u>foreshortened</u>. Assume p is origin

. the curve is $uT + \frac{\kappa_T}{2}u^2 N$

in T, N plane.

• in <u>image</u> plane, it is

$$(u\,\overset{\sin}{\cancel{\cos}}\,\theta)\,X + \frac{\kappa_T}{2}u^2\,Y$$

write $u \, \cancel{\cos} \sin\theta = \omega$, then

$$\omega X + \frac{K_{\cancel{I}}}{2 \, \cancel{\cos^2} \sin^2\theta} \omega^2 Y$$

so $K_0 = \cancel{\frac{K_I}{\cos\theta}} \; \frac{K_T}{\sin^2\theta}$

Now defn means $K = \dfrac{II(u,u)\,II(v,v) - II(u,v)^2}{I(u,u)\,I(v,v) - I(u,v)^2}$

for <u>any</u> u, v st. $\det(u,v) \neq 0$

(<u>Check</u> this)

Now assume that u, v are <u>conjugate</u>.

then

$$K = \frac{II(u,u)\,II(v,v)}{I(u,u)\,I(v,v)\left[1 - \cos^2\phi\right]}$$

angle between u, v

$$= \frac{II(u,u)\,II(v,v)}{I(u,u)\,I(v,v)\sin^2\theta} = K_0 K_v$$

$\square$

Koenderink's thm, perspective case.

- Now assume that focal pt. is r along V
- then

$$K_p = \frac{K_o \cdot K_v}{r}$$

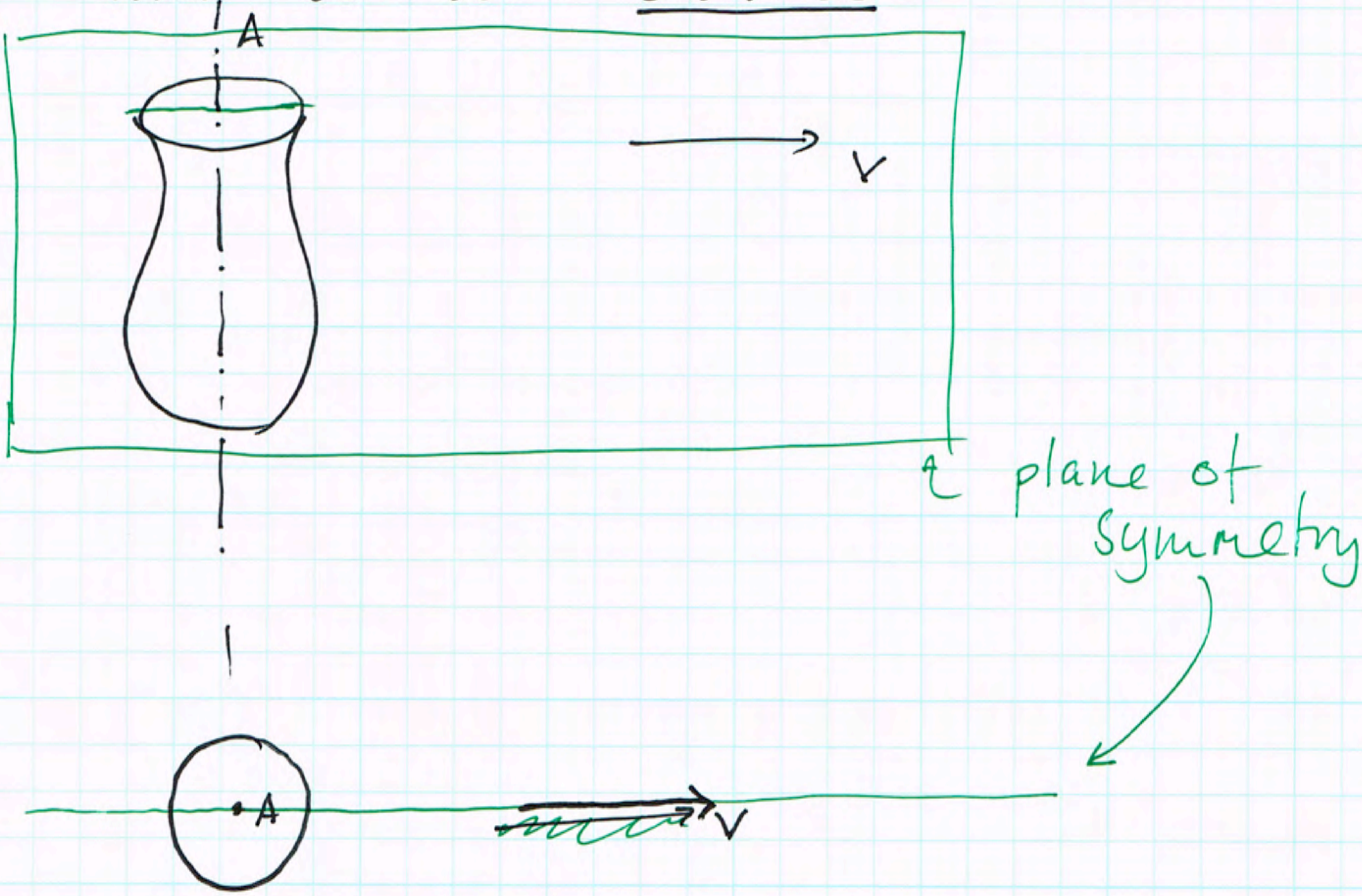
(easy version of proof above; note K_o in this case is $\frac{r K_T}{\sin^2\theta}$)

To go much further, we need a (very) little singularity theory — then we can investigate Gauss maps and curvature further.

But first, SOR.

Consider an SOR (surface of revolution) w/ axis A.

- in <u>orthographic</u> view, view dir'n V, C.G. has a <u>reflectional</u> symmetry about plane given by A and V
  - and so does <u>outline</u>



Now drawing above makes it <u>obvious</u> that $\underbrace{\text{C.G. is } \underline{\text{plane}}}$ — most unusual phenomena

~~Plane~~ Outline w/ reflectional symmetry

$\Downarrow$

You might be able to find outline of SOR in an image <u>w/o</u> knowing its geometry.

- <u>Very attractive idea</u> - you can find likely instances of S.O.R.'s with very little prior knowledge
  - <u>Difficulty</u>: few SOR's, <u>many</u> symmetric curve frag.'s

SOR - perspective view

- C.G. still have a symmetry, about plane thru A and f
- Outline has a form of symmetry (image plane is not necessarily $\perp$ to plane of symmetry)

SHGC

- take a closed plane curve, C
- Choose a point p on the plane; construct an axis A through p, $\perp$ to the plane
- make a surface by sweeping C along A, scaling by some fn of A

Parametric form

WLOG, put curve on x, y plane

$$C = (f(t), g(t), 0), \quad A = (0, 0, 1)$$

scaling by σ gets

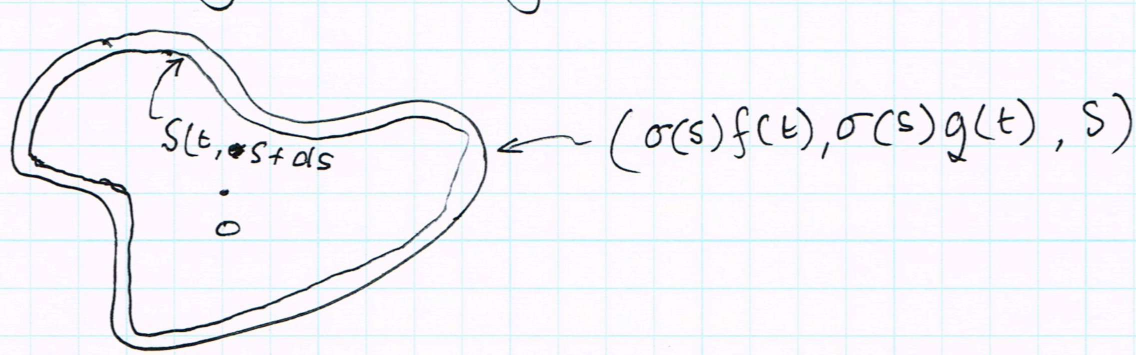
Surface

$$(\sigma(s) f(t), \sigma(s) g(t), s)$$

locally for any small range of s, $s \in [s, s+ds]$

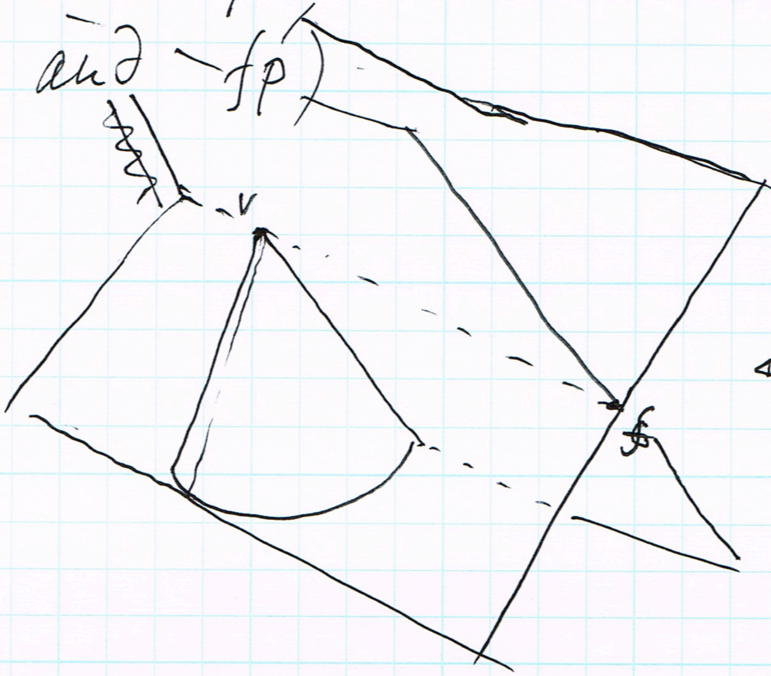
this surface is a cone

(easiest seen by looking from above

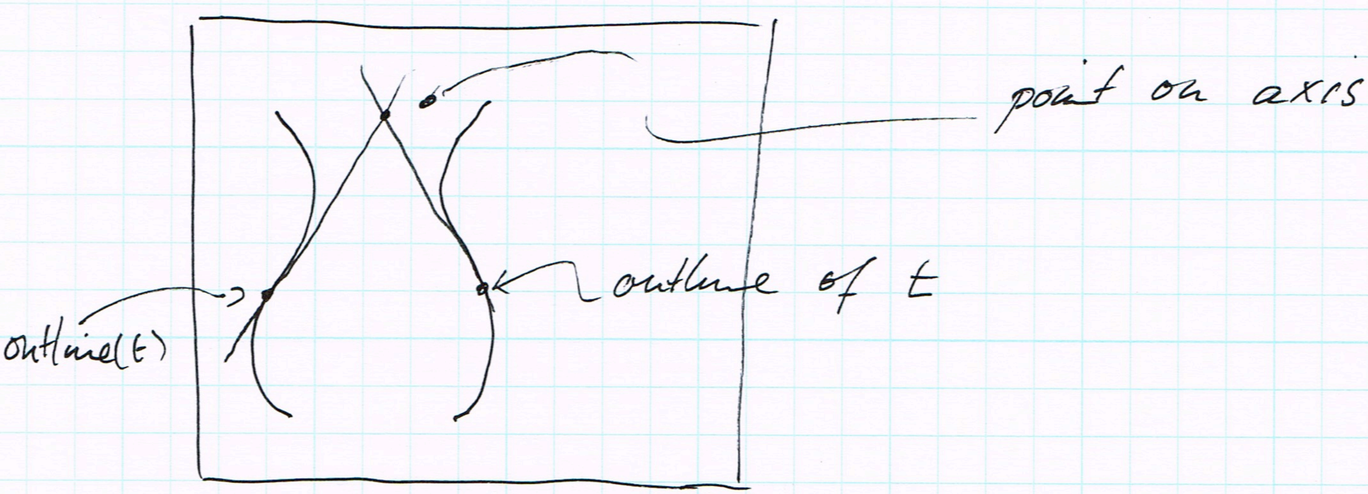


Now join $S(t, s)$ to $S(t, s+ds)$ w/ a line — this must pass thru $(0, 0, 1)$

Now the outline of a cone is easy (on planes thru line connecting vert and fp)



Now this means I can reconstruct the axis if I have <u>correspondence</u>



There are a variety of hacks to obtain csp, none particularly reliable

IF you see the crosse-section at top or bottom, you might be able to reconstruct, with some more geometric constraint.

ATTRACTION OF THESE THEORIES:

category independent links between outline and geometry, sometimes allowing grouping and reconstruction.

DISADVANTAGES:

- often v. weak constraints, results
- surface model is dubious, at best
- Doesn't interact w/ useful notions of category well
- category notions not recovered by ~~modern~~ language of primitives, composition

Challenge: let us try to ressurrect an easy/useful bit with modern machinery

- Most accessible
  - You can identify the contour of an SOR by image symmetry
- Obstacles
  - Nothing is a true SOR
  - contour points are hard to find
- positives
  - we're really good at building and applying detectors
  - our C.G. is much better than 20 years ago
  - we have mean shift

· replace SOR with "pinche"

$$\Sigma = \{ x \mid \phi_{SOR} \geq 0 \}$$ ← solid model of SOR.

$$\Sigma_o = \{ x \mid \phi_o \geq 0 \}$$ ← outside pinch

$$\Sigma_i = \{ x \mid \phi_i \geq 0 \}$$ ← inside

choose ϕ st for all x $\phi_o \geq \phi_{SOR} \geq \phi_i$

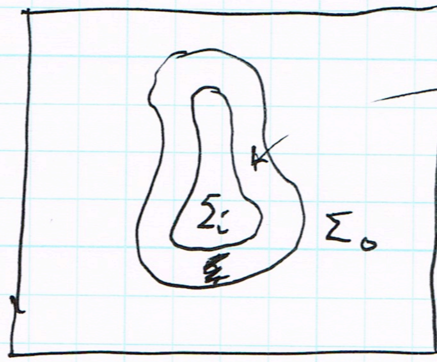
· now there are many pinches, choose one with a plausible gap

· consider some focal point f "outside"

then any ray from f that enters Σ_o but not Σ_i could be on a silhouette.

- Now we get an

image



anything here could be outline

~~Q~~: Strategy build a detector that responds to points in this volume $\Sigma_o - \Sigma_i$

Strategy do this for "few" pieces of SOR, and link detector hits w/ mean shift

Strategy: Search to find ~~decom~~ appropriate fp's to train detectors

→ HOW