Q1 :

an <u>implicit surface</u> is given by

$$g(x,y,z) = 0 \quad , \quad \text{where } g = \text{some function}$$

a) assume that g is polynomial. Show that, at point u, v, w <u>on the surface</u> (i.e $g(u,v,w) = 0$) the normal N is <u>parallel</u> to

$$\left(\left.\frac{\partial g}{\partial x}\right|_{u,v,w} , \left.\frac{\partial g}{\partial y}\right|_{u,v,w} , \left.\frac{\partial g}{\partial z}\right|_{u,v,w} \right)$$

b) now consider the surface

$$x^2 + y^2 - (z^2+1)$$

show that, at each point (u,v,w) on the surface, there are two lines lying entirely on the surface and passing throug that point.

to do so, recall you can parametrize a line as

$$(u,v,w) + t(a,b,c)$$

plug in, and reason.

c) What is the sign of the Gaussian curvature at each pt. of surface? (do this by thinking + drawing, not formulae or you'll go mad)

d) Why are b and c not contradictory?

Q2: a surface containing a line through each pt. is a ruled surface

a) does a ruled surface necessarily have

$K = 0$?

b) a Developable Surface has the form

$X(s,t) = a(s) + t\,w(s)$.

with $\|w(s)\| = 1$. where

$$(w \times w') \cdot a' = 0 = \det[w, w', a']$$

Show that, for a Developable Surface

$K = 0$.

Q3:

a surface of revolution is given by

$$X(u,v) = \begin{matrix} f(v)\cos u \\ f(v)\sin u \\ g(v). \end{matrix}$$

a) Show that the first fundamental form is given by

$$g_{11} = f^2(v) \qquad g_{12} = 0 \qquad g_{22} = f_v^2 + g_v^2$$

b) Show that the equations for the geodesics are:

I $\quad u_{ss} + \dfrac{2 f f_v}{f^2} u_s v_s = 0$

II $\quad v_{ss} - \dfrac{f f_v}{(f_v^2 + g_v^2)} (u_s^2) + \dfrac{f_v f_{vv} + g_v g_{vv}}{(f_v^2 + g_v^2)} (v_s)^2 = 0$

c) Show that I is equivalent to

$$f^2 u_s = c \qquad \text{for some const } c$$

d) Show the angle θ between a geodesic and a parallel ($v = \text{const}$) is given by

$$\cos\theta = \frac{|X_u \cdot (X_u u_s + X_v v_s)|}{\| X_u \|} = |f u_s|$$

e) Use c and d to make a qualitative statement about geodesics on SOR's (hint: $\cos\theta = 1$)